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Механика на твърдото деформируемо тяло

On the behaviour of high-pressure pipelines under seismic loading

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The seismic loading of high-pressure pipelines is of crucial importance for their design. In spite of the extensive studies on the Pipeline Dynamics, performed so far [1] to the author's knowledge the solution of the corresponding earthquake-vibration problem is still missing. This embarrasses the earthquake resistant design of pipelines.

An essential feature of the pipeline is that it is a long structure, i. e. it is necessary not only to study the structure response due to the base shaking, but also the interaction of its different parts because the pipeline-length is of the same order as that of the seismic wave.

In this paper the problem of the earthquake-induced vibrations of an open horizontal pipeline with constant cross section characteristics is considered. The liquid is in equilibrium. This condition is assumed because the paper is considered to be an early-stage investigation of the whole problem. In this case the moving mass is large, whereas the resisting part of the cross section is small. The seismic loading is transmitted to the pipeline through base blocks which are located in equal distances from each other (6-12 m). Only horizontal seismic input is taken into account. Besides, the seismic

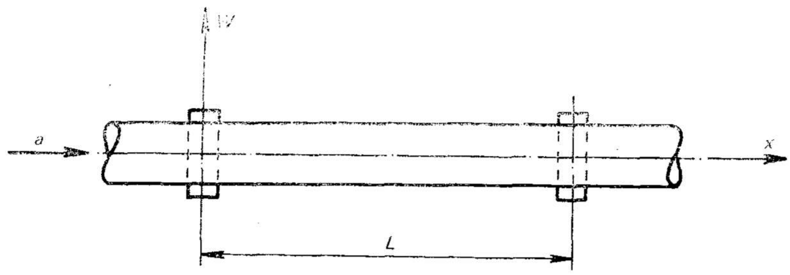

Fig. 1. One-span pipe

wave is assumed to run along the pipe which yields maximum loading. The solution is based on the usual Earthquake Engineering assumptions: visco-elastic range for the material, pseudovelocity and pseudoacceleration, Housner's spectrum.

Let us first consider one-span pipe, whose layout is shown in fig. 1.

The velocity of seismic wave is a, so the time for covering the distance between two blocks will be $\beta = \frac{L}{a}$. If a seismic pulse acts on support 1 in the time moment t, the same pulse will reach support 2 in moment $t+\beta$.

It is well known [2]-p. 302 that the motion of a one-span beam due to kinematic support exitations and with dissipation is governed by the equation:

(1) $$\frac{\partial^2}{\partial x^2}\left(\mathrm{EI}\frac{\partial^2 w}{\partial x^2} + c_s \mathrm{I}\frac{\partial^3 w}{\partial x^2 \partial t}\right) + m\frac{\partial^2 w}{\partial t^2} + c\frac{\partial w}{\partial t} = p_{eff},$$

where

(2) $$p_{eff} = -[m\varphi_1(x)\ddot{\delta}(t) + m\varphi_2(x)\ddot{\delta}(t-\beta)]$$

is the "effective" load.

Here m is the mass per unit length, EI — the bending stiffness; c, c_s are the coefficients of external and internal viscous damping respectively; w — the additional transverse displacement which accounts for the dynamic character of the loading. $\varphi_i(x)$ is the displacement pattern due to unit value of the static displacement of support i. This latter can be easily obtained using standard static beam-deflection procedures.

In Eq. (2) are included only the terms connected with support displacements. The terms arised from the rotations are neglected because of the following considerations. The velocities of P- and S-waves for different soils are 300-2500 m/s and 100-500 m/s respectively (see [3]-p. 72). This means that the distance between two adjacent supports would be covered for time which is much less than one second. The past earthquake records [4]-p. 227 show that for such a short time interval the displacements remain almost constant, i. e. the variation of the relative displacements between each two adjacent supports would be so small that the influence of the support rotations may be neglected.

The seismic input may be introduced in terms of $\ddot{\delta}(t)$, i. e. the real accelerogram which is the same for both supports but shifted in time with β.

We will seek an approximate solution of Eq. (1) in the form:

(3) $$w^* = f(t)\sin\frac{\pi x}{L}$$

according to the assumptions introduced.

This form is justified by the zero points coinciding with the supports and since the adjacent spans have to deflect in different directions. Besides, this curve corresponds to minimum potential energy.

Then:

(4) $$\mathrm{EI}\frac{\partial^4 w^*}{\partial x^4} + c_s \mathrm{I}\frac{\partial^5 w^*}{\partial x^4 \partial t} + m\frac{\partial^2 w^*}{\partial t^2} + c\frac{\partial w^*}{\partial t} - p_{eff} = \mathrm{R} \neq 0.$$

Introducing (4) we get:

(5) $$m\ddot{f}\sin\frac{\pi x}{\mathrm{L}} + \left(c_s \mathrm{I}\frac{\pi^4}{\mathrm{L}^4} + c\right)\dot{f}\sin\frac{\pi x}{\mathrm{L}} + \mathrm{EI}\frac{\pi^4}{\mathrm{L}^4} f\sin\frac{\pi x}{\mathrm{L}} - p_{eff} = \mathrm{R}.$$

According to the Bubnov-Galerkin's method we orthogonalize the remainder and the coordinate function over the interval [0, L]:

(6) $$\int_0^L R\sin\frac{\pi x}{\mathrm{L}}\,dx = 0$$

$$(7)\qquad \int_0^L \left[m\ddot{f}\sin\frac{\pi x}{L}+\dot{f}\sin\frac{\pi x}{L}\left(c_s I\frac{\pi^4}{L^4}+c\right)+EI\frac{\pi^4}{L^4}f\sin\frac{\pi x}{L}-p_{eff}\right]\sin\frac{\pi x}{L}\,dx=0$$

$$(8)\qquad m\ddot{f}\int_0^L \sin^2\frac{\pi x}{L}\,dx+\left(c_s I\frac{\pi^4}{L^4}+c\right)\dot{f}\int_0^L \sin^2\frac{\pi x}{L}\,dx+EI\frac{\pi^4}{L^4}f\int_0^L \sin^2\frac{\pi x}{L}\,dx$$

$$-\int_0^L p_{eff}\sin\frac{\pi x}{L}\,dx=0.$$

The solution of the first three integrals is trivial. However the fourth cannot be solved directly. Let us denote it by I_4. Because of the Betty's theorem we may replace the deflection curve due to unit support displacement by the influent for the reaction force in the same support. Thus we may use the table values for the influent of a continuous beam with five equal spans [5]. The actual solution of I_4 is performed numerically following the trapezoidal rule dividing the [0, L] interval to 10 equal subintervals:

$$(9)\qquad I_4=\int_0^L \left[m\varphi_1(x)\ddot{\delta}(t)+m\varphi_2(x)\ddot{\delta}(t-\beta)\right]\sin\frac{\pi x}{L}\,dx$$

$$=m\ddot{\delta}(t)\int_0^L \varphi_1(x)\sin\frac{\pi x}{L}\,dx+m\ddot{\delta}(t-\beta)\int_0^L \varphi_2(x)\sin\frac{\pi x}{L}\,dx$$

$$=m\ddot{\delta}(t)0{,}36674\,L+m\ddot{\delta}(t-\beta)\,0{,}36674\,L.$$

So after some mathematics Eq. (8) gets the appearance:

$$(10)\qquad \ddot{f}+\left(c_s I\frac{\pi^4}{L^4}+c\right)\frac{1}{m}\dot{f}+EI\frac{4\pi^4}{mL^4}f=0{,}73348\ddot{\delta}(t)+0{,}73348\,\ddot{\delta}(t-\beta),$$

or

$$(11)\qquad \ddot{f}+2\xi\omega\dot{f}+\omega^2 f=0{,}73348\,\ddot{\delta}(t)+0{,}73348\ddot{\delta}(t-\beta),$$

where

$$(12)\qquad \omega=\frac{2\pi^2}{L^2}\sqrt{\frac{EI}{m}};\quad \xi=\left(c_s I\frac{\pi^4}{L^4}+c\right)\frac{L^2}{4\pi^2\sqrt{EIm}},\quad \text{and}$$

$$\omega_d=\omega\sqrt{1-\xi^2}=\frac{1}{2mL^2}\sqrt{16\pi^4 EIm-\left(c_s I\frac{\pi^4}{L^4}+c\right)L^4}.$$

In this form Eq. (11) for the unknown coefficient $f(t)$ from (3) is the equation of forced vibrations for a SDOF system. It is essential however that in its right-hand side stands an accelerogram rather than sin-function. Moreover, the accelerograms are two, shifted in time to each other though identical in appearance. Such an equation with an accelerogram in the right-hand side is typical for the Earthquake Engineering and its solution may be written in terms of Duhamel integral in the form (see [2]-p. 101):

$$(13)\qquad f(t)=\frac{1}{\omega_d}\int_0^t [0{,}73348\ddot{\delta}(\tau)+0{,}73348\,\ddot{\delta}(\tau-\beta)]e^{-\xi\omega(t-\tau)}\sin[\omega_d(t-\tau)]d\tau$$

Here an essential feature must be noted: the loading is represented not by one series of successive pulses but by two such series: $\ddot{\delta}(\tau)$ and $\ddot{\delta}(\tau-\beta)$. They are identical as time evolution but are shifted to each other in time interval β. This fact leads

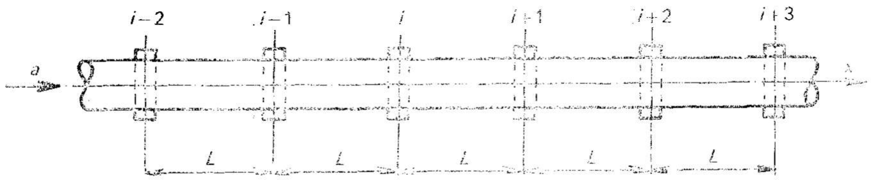

Fig. 2. Continuous pipe supported in equidistant points

to a natural generalisation of the Duhamel integral for more series of successive pulses.

Let us now consider an arbitrary span $(i)-(i+1)$ belonging to a continuous pipe supported in many equidistant points — fig. 2:

Considering this particular span we will take into account the influence of the nearest two spans in every direction. Because of the strong attenuation of the influent the effect of the supports located after these two spans may be neglected. In this case the "effective" load will be:

(14)
$$v_{eff} = -m\,[\ddot{\delta}(t+2\beta)\varphi_{i-2}(x) + \ddot{\delta}(t+\beta)\varphi_{i-1}(x) + \ddot{\delta}(t)\varphi_{i}(x) + \ddot{\delta}(t-\beta)\varphi_{i+1}(x) + \ddot{\delta}(t-2\beta)\varphi_{i+2}(x) + \ddot{\delta}(t-3\beta)\varphi_{i+3}(x)].$$

After an orthogonalisation similar to the above performed and numerical integration according to trapezoidal rule instead of Eq. (10) we get:

(15)
$$\ddot{f} + \left(c_s I\frac{\pi^4}{L^4} + c\right)\frac{1}{m}\dot{f} + EI\frac{4\pi^4}{mL^4}f = 0{,}0445\ddot{\delta}(t+2\beta) + 0{,}1332\ddot{\delta}(t+\beta) + 0{,}73348\ddot{\delta}(t) + 0{,}73348\,\ddot{\delta}(t-\beta) + 0{,}1332\ddot{\delta}(t-2\beta) + 0{,}0445\ddot{\delta}(t-3\beta) = F(t).$$

Or:

(16)
$$\ddot{f} + 2\xi\omega\dot{f} + \omega^2 f = F(t)$$

with the notations (12).

The solution in terms of Duhamel integral will be:

(17)
$$f(t) = \frac{1}{\omega_d}\int_0^t F(\tau)e^{-\zeta\omega(t-\tau)}\sin[\omega_d(t-\tau)]d\tau.$$

Now this generalized integral is actually a sum of six integrals:

(18)
$$f(t) = \frac{0{,}0445}{\omega_d}\int_0^t \ddot{\delta}(\tau+2\beta)e^{-\xi\omega(t-\tau)}\sin[\omega_d(t-\tau)]d\tau$$

$$+\frac{0{,}1332}{\omega_d}\int_0^t \ddot{\delta}(\tau+\beta)e^{-\xi\omega(t-\tau)}\sin[\omega_d(t-\tau)]\,d\tau + \frac{0{,}73348}{\omega_d}\int_0^t \ddot{\delta}(\tau)e^{-\xi\omega(t-\tau)}\sin[\omega_d(t-\tau)]\,d\tau$$

$$+\frac{0{,}73348}{\omega_d}\int_0^t \ddot{\delta}(\tau-\beta)e^{-\xi\omega(t-\tau)}\sin[\omega_d(t-\tau)]\,d\tau+\frac{0{,}1332}{\omega_d}\int_0^t \ddot{\delta}(\tau-2\beta)e^{-\xi\omega(t-\tau)}\sin[\omega_d(t-\tau)]d\tau$$

$$+\frac{0{,}0445}{\omega_d}\int_0^t \ddot{\delta}(\tau-3\beta)e^{-\xi\omega(t-\tau)}\sin[\omega_d(t-\tau)]\,d\tau.$$

It is noteworthy that the pulse function:

(19) $$h=\frac{1}{\omega_d}e^{-\xi\omega(t-\tau)}\sin[\omega_d(t-\tau)]$$

is the same in each of the six integrals, whereas the accelerations $\ddot{\delta}$ which are to be taken from the accelerogram of the real input are shifted in time in one or more intervals β with regard to the conventionally chosen starting moment τ. Keeping in mind that the seismic pulse reaches each one of the six supports at different time moments we have to involve in the whole seismic input on the pipe the different accelerations which act on the different supports at that particular moment.

It is necessary to pay attention to an error involved in the integration — namely the time shifting of the accelerogram with respect to the pulse function (β, 2β, 3β) in the different integrands. Because of the seismic wave velocity and the span length it is clear that the order of the interval β is hundredths of second. The mutual shifting of the two multipliers (the accelerogram and the pulse function) in such a short interval obviously will not cause any noteworthy error in the final result. Besides, with the exception of the cases with the very long free vibration periods the pseudovelocity which is usually involved in the Earthquake Engineering statistically is very close to the actual relative velocity. And it is well known that this quantity has been obtained after dropping a term containing a ξ multiplier and replacing cos by sin in the Duhamel integrand, i. e. after a change which does not contribute to the accuracy. That is why it may be concluded that the error due to the above mentioned shifting is small.

The question arises about the practical value of the result obtained. It is clear that we are finally interested in the maximum absolute value of $f(t)$ from Eq. (18). In the basic case when the Duhamel integral contains only one accelerogram, it is not difficult to calculate on a computer its value for any given frequency ω and fixed damping ζ as a function of time. Further, the variation of the maximum absolute value as a function of ω for different values of the damping can be followed. Thus the well known response spectra have been obtained.

In our case, however, there is a basic difference in the problem of constructing a response spectrum. Now we need the maximum absolute value of (18) but the latter is a sum of six Duhamel integrals. Since the parameter β is involved, the response spectrum is to be constructed for different values of this parameter. It is easy to realize what amount of calculations would be involved to get the response spectrum in this case for one given record. And this is evidently useless since the record of the future earthquake is not available. It is known ([2]-p. 541) that in order to overcome this problem Housner has suggested the standard spectrum obtained by averaging 8 strong motion records. That is why we suggest for our case to be used the statistical rule SRSS with the data from Housner's spectrum. For a given free vibration frequency (accordingly period), and for a given damping each of the six integrals in (18) would lead to the same ordinate of the Housner's spectrum. Applying the SRSS rule we will get an additional multiplier which is actually SRSS of the numerical coefficients standing in Eq. (18) which value is:

(20) $$\sqrt{2.0{,}0445^2+2.0{,}1332^2+2.0{,}73348^2}=\sqrt{1{,}11543}=1{,}056.$$

It is easy to see that if we neglect the influence of all the additional spans in both directions on the basical span between supports (i) and $(i+1)$ this coefficient will have the value:

$$\sqrt{1{,}076}=1{,}037 \tag{21}$$

i. e. this simplification makes difference of 1,8% which is fully acceptable for the design accuracy.

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