БЪЛГАРСКА АКАДЕМИЯ НА НАУКИТЕ
ТЕОРЕТИЧНА И ПРИЛОЖНА МЕХАНИКА
София, 1990, год. XXI, № 4

BULGARIAN ACADEMY OF SCIENCES
THEORETICAL AND APPLIED MECHANICS
Sofia, 1990, Year XXI, No 4

Analytical and Numerical Investigations on the Hydroelasticity of One-Span Pipings in Their Parametric Space

D. Dermendjian-Ivanova, V. Dzhupanov

I. Introduction and review

The pipes are universal structural components used in many technologies as transporting means for gases, liquids and powders, (aircraft, ship structures, off-shore structures, chemistry, nuclear power technology, etc).

The basic principles governing the design of piping configuration are: (*i*) space economy; (*ii*) metal economy; (*iii*) correspondence between the loadings and design decision accepted.

In most cases of aforementioned technologies the complicated isometry is main characteristic of the pipings, (e. g. the multitude of *L*-bows and one-span sections, different supportings, etc.).

The stream of publications on the piping investigations connected with the modern technologies is quite dense, viscous and large. Due to these qualities and to appearing of some unsteady vortices shedding, this stream is exclusively difficult to be kept under observation. Its history is followed and systematized by some monographs, e. g. [1, 2, 8, 9, 15, 16, etc.], by the special reviews [5, 10, 12, 13, etc.], and (at the end) by the short reviews of the special publications referred to in the present contribution.

It is important to be mentioned preliminarily the attempts for parallel (theoretical and experimental) investigation of the dynamical behaviour of pipelines under liquid flow, reported in Ref. [1, 6, 7, 11, 12, 13, 14, etc.], and the fact that the experimental results confirm the theoretical ones in a wonderful degree.

Exclusively methodical exposition of the flow-induced vibration philosophy one can find in Blevin's monograph (1977) [1]. The equation describing the beam-like vibrations was deduced in the form:

$$(EI)w^{IV}+\rho AU^2 w''+2\rho AU\dot{w}'+(\bar{m}+\rho A)\ddot{w}=0, \tag{1}$$

where (EI) — rigidity factor, ρ — liquid density, U — velocity of liquid flow, A — cross section, $\bar{m}$ — tube mass per unit length.

The cases of pinned-pinned and cantilevered tubes under the action of liquid flow with velocity U are considered in details.

The three reviews of Païdoussis (1982) [12] and Dzhupanov (1985) [4] & (1987) [5] describe general aspects of the flow-induced vibrations and the classical hydroelasticity. The reviews of Païdoussis (1974) [10] & (1987) [13] are devoted partially to the special questions of the pipe instability under inner liquid flow.

II. The problem

Five schemes of one-span pipes conveying longitudinally flowing inviscid, compressible and heavy liquid are shown in Fig 1. The differences between them are in the supportings. Following the terminology accepted one shall call them: **A**) hinged-hinged supportings; **B**) hinged-clamped supportings; **C**) clamped-clamped supportings $+N$ apparatus; **D**) clamped-clamped supportings $+Q$ apparatus; **E**) cantilevered pipe. The velocity of the liquid flow U is one and same in all points of the cross-section.

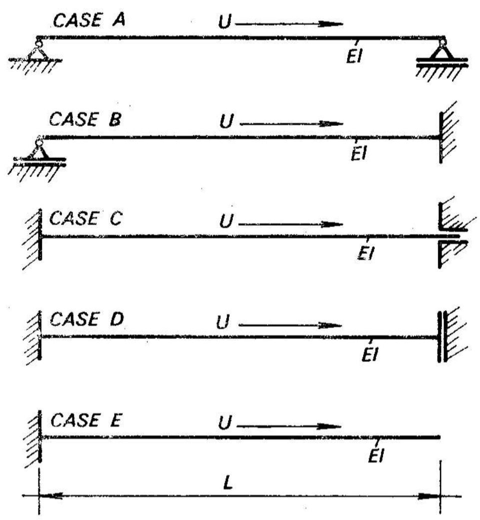

Fig. 1. Supporting types.

The natural frequencies of the hydroelastic system (**HES**) "tube + liquid" are under question, (i); the vibration modes corresponding to the natural frequences are sought also (ii); the lowest critical velocities U_I^* have to be determined, (iii). The components under consideration are beam-like vibrating tubes with given cross-section A. We shall presuppose that A is bounded by the circular cylindrical middle surface of the tube wall. The Bernoulli-Navier assumption for the tube cross-section is taken into account. The tube material is accepted as lineary elastic Hookian one. Another assumption affirms that small dynamical deflections $w(x, t)$ generate small perturbations of the initial potential liquid motion. Here is accepted that the perturbations have plane character and can be described keeping in mind only the motion along the tube axis.

In case under consideration, independently of the fact that the axis deflections $w(h, t)$ are small, the Koriolis acceleration is considerable and must be taken into account, Ref. [1, 5, etc.].

III. Solution

The function $w(x, t)$ has to be a solution of Eq. (1) and to satisfy the boundary conditions corresponding to the supporting types for every one of the five cases. The difficulties for the solution originate from the existing of dissipative member $[2\rho AU/(EI)]\dot{w}^{I}$. Due to the dissipation a solution of Eq. (1) is convenient to be sought in a form of running waves

(2) $$w(x, t) = A \exp i(kx - \omega t), \quad \text{or} \quad w(x, t) = B \exp i(kx + \omega t)$$

and to take the real part of the solution. Substituting these expressions into (1) one can obtain both dispersion equations

(3) $$k^4 - 2\bar{q}k^2 \pm 2\bar{K}\omega k - \bar{\Omega}^4 \omega = 0, \quad \bar{q} = \frac{\rho AU^2}{2EI}, \quad \bar{K} = \frac{\rho AU}{EI}, \quad \Omega^4 = \frac{\bar{m} + \rho A}{EI},$$

where k is wave number. We choose the Bubnov — Galerkin's method as most successful for our needs. Expressing the solution in the form

(4) $$w(\xi, t) = W(\xi) \,.\, e^{i\sigma t}, \quad (\xi = x/L),$$

and substituting into Eq. (1), we obtain the differential equation

(5) $$W^{IV} + 2qW^{II} + 2iKW^{I} - \Omega^4 W = 0, \quad \text{where}$$

(6) $$q=\frac{\rho AU^2}{2EI}L^2,\quad K=\frac{\rho AU}{EI}L^3\sigma,\quad \Omega^4=\frac{\overline{m}+\rho A}{EI}\sigma^2L^4.$$

The trial functions $W(\xi)$ are considered by using the modes obtained for the "dry" statical buckling of one-span pipes (for the five cases of supportings) under Eulerian longitudinal force P.

Table 1
Trial functions

Case	Trial functions series	Formula
CASE A	$W(\xi)=\sum_m A_m.\sin m\pi\xi$ $m=1, 2, 3, \ldots$ $\sin kL=0$; $k_m=m\pi$	(7)
CASE B	$W(\xi)=\sum_m A_m\,[(\sin k_m\,\xi/\cos k_m)-k_m\,\xi]$ $tg\,kL=kL$; $k_1=4,493$, $k_2=7.7252$ $k_3=10,904$; $k_4=14.066$. $k_5=17.221$	(8)
CASE C	$W(\xi)=\sum_m A_m\,(\cos 2m\pi\xi-1)$ $m=1, 2, 3, \ldots$ $\cos kL=1$; $k_m=2m\pi$	(9)
CASE D	$W(\xi)=\sum_m A_m\,(\cos m\pi\xi-1)$ $m=1, 2, 3, \ldots$ $\sin kL=0$; $k_m=m\pi$	(10)
CASE E	$W(\xi)=\sum_m A_m\,[\cos(m\pi\xi/2)-1]$ $m=1, 3, 5, \ldots$ $\cos kL=0$; $k_m=m\pi/2$	(11)

In Table 1 for every supporting case the trial series, the equation corresponding to Eq. $\Delta(kL)=0$, $(kL=k_1, k_2, k_3, \ldots, k_\infty)$ and the first five eigenvalues are given.

The system of equations for unknown constants determination is

(12) $$[a_{kl}].\{A_l\}=0,\quad \text{and taking}$$

(13) $$\mathbf{Det}\,[a_{kl}]=0$$

the equations of eigen values are obtained. In the enlarged variant of the present work the terms a_{kl} $(k=1, 2, 3, 4, 5\,\&\,l=1, 2, 3, 4, 5)$ are described in details. If we rewrite the matrix $[a_{kl}]$ in developed form, the considered approximations are shown in Table 2.

Table 2
Approximation orders

a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	...
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	...
a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	...
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	...
a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	...
.	.	.	.	.	
.	.	.	.	.	
.	.	.	.	.	

VI. Numerical investigations and analysis.

The characteristics of the testing pipe are: outer diameter: 200 mm; inner diameter: 190 mm; inertia momentum of cross-section: $I = 1{,}48395 \,.\, 10^{-5}\,{}^{4}\text{m}$; Youngs modulum of the steel material: $E = 206 \,.\, 10^{6}$ kN/m^2; cross-section rigidity: $EI = 3056{,}937$ kNm^2; cross-section area: $A = 3{,}0615 \,.\, 10^{-3}\, m^2$; empty pipe mass per unit length: $\overline{m} = 0{,}024498$ kN^{2}s/m^2; liquid mass per unit length; $M = 0{,}028902$ kN^{2}s/m^2; $L_{\min} = 3$ m and $L_{\max} = 12$ m.

The second group of parameters having **qualitative** character includes: (*i*) the circular frequency (σ); (*ii*) the liquid velocity (U); (*iii*) the span length (L). Eq. (14) can be repvesented as

$$(14) \qquad \mathbf{Det}\,[a_{kl}(U,\, L,\, \sigma,\, \text{——})] = 0,$$

where by a line the massive of all parameters of the first group are designated. We shall consider them as known. Then in the space of the parameters (σ, U, L), Eq. (14) determines the complicated surface

$$(15) \qquad \sigma = \sigma(U,\, L),$$

which is used for an investigation of the dynamical behaviour of the **HES**, Fig. 2. We would like to mention that to every pair of values of $U \& L$, Eq. (15) puts into correspondence fully determined value (values) of σ.

This value can be expressed by real, imaginary of complex number. The multitude of all real σ obtained by Eq. (15), if $U \& L$ are given, forms a "spectrum of HES natural frequences". Real importance have only the real σ_m, where $m = 1,2,3,\ldots,\infty$.

Eq. (14) produces real σ only if the velocity U is small. Then one shall obtain that the pipe vibration will have stationary character. We shall use the designation $\sigma = \sigma_1^* \pm i\,\sigma_2^*$, which clearly shows, the critical character of the case $U > U_1^*$ where U_1^* is the first critical velocity.

The appearance of $\mathbf{Im}\,\sigma = \sigma_2^*$ leads to infinitely increasing vibration amplitudes of whole **HES** [1, 2]. Really, putting $\sigma = \sigma_1^* \pm i\,\sigma_2^*$ into (4) one shall obtain

$$(20) \qquad w\,(x,\, t) = W(x) \,.\, \exp(i\,\sigma_1^* t) . \exp\,(\pm \sigma_2^*\, t)$$

which has two branches: $\exp\,(-\sigma_1^*\, t)\, \&$ $\exp\,(+\sigma_2^*\, t)$. The first of them provides the steady pipe motion, as with the time t, the amplitudes decrease to zero. The second branch provides unsteady motion of the **HES**, as with the time its amplitudes increase infinitely.

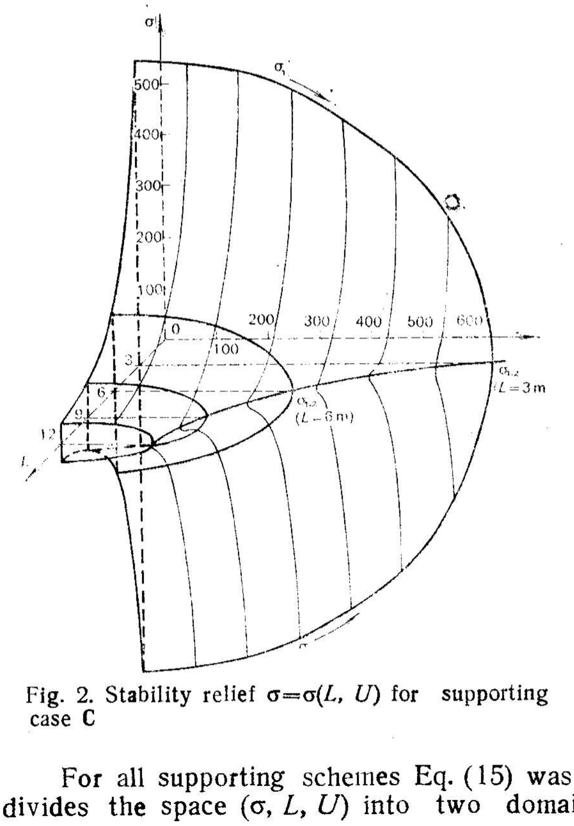

Fig. 2. Stability relief $\sigma = \sigma(L,\, U)$ for supporting case **C**

For all supporting schemes Eq. (15) was solved. The surface, which was obtained, divides the space (σ, L, U) into two domains: (*i*) the internal one ($U < U_1^* \,\&\, L < L_1^*$), considered as stability domain, and (*ii*) the external one, i. e. $U > U_1^* \,\&\, L > L_1^*$ — instability domain.

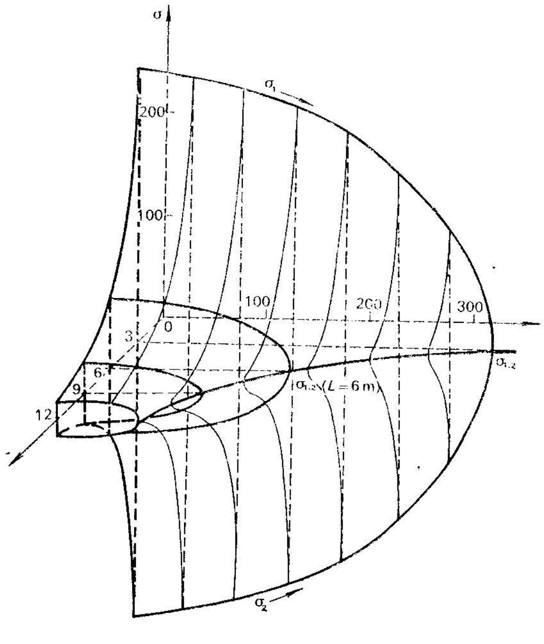

Fig. 3. Stability relief $\sigma=\sigma(L, U)$ for supporting case **A**

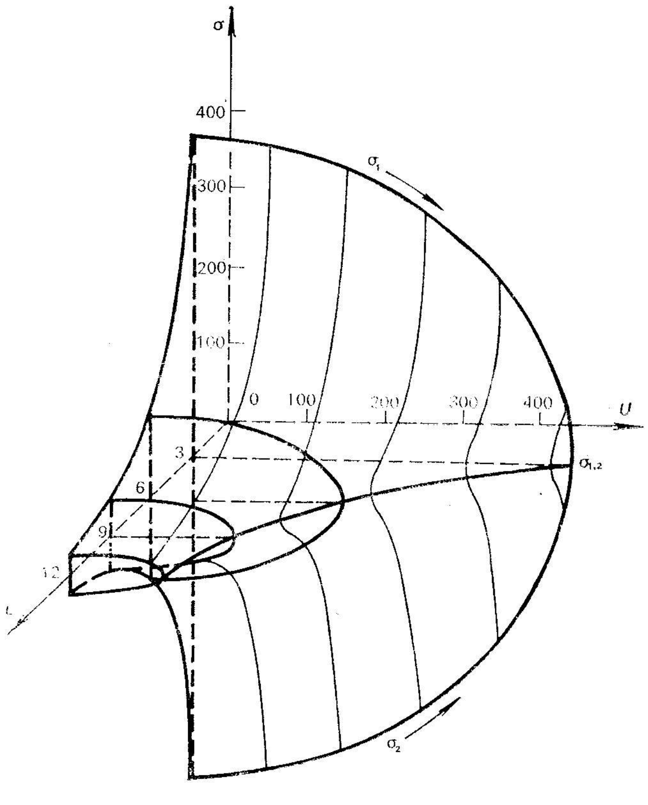

Fig. 4. Stability relief $\sigma=\sigma(L, U)$ for supporting case **B**

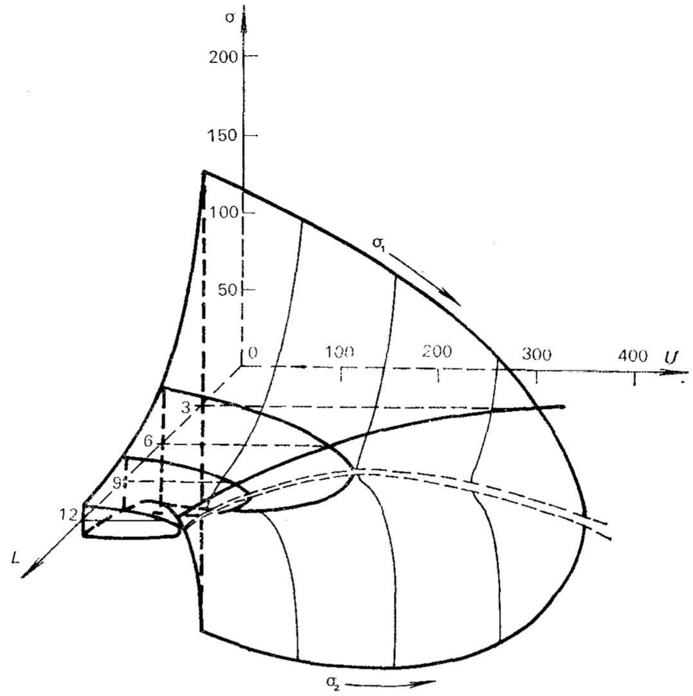

Fig. 5. Stability relief $\sigma = \sigma(L, U)$ for supporting case **D**

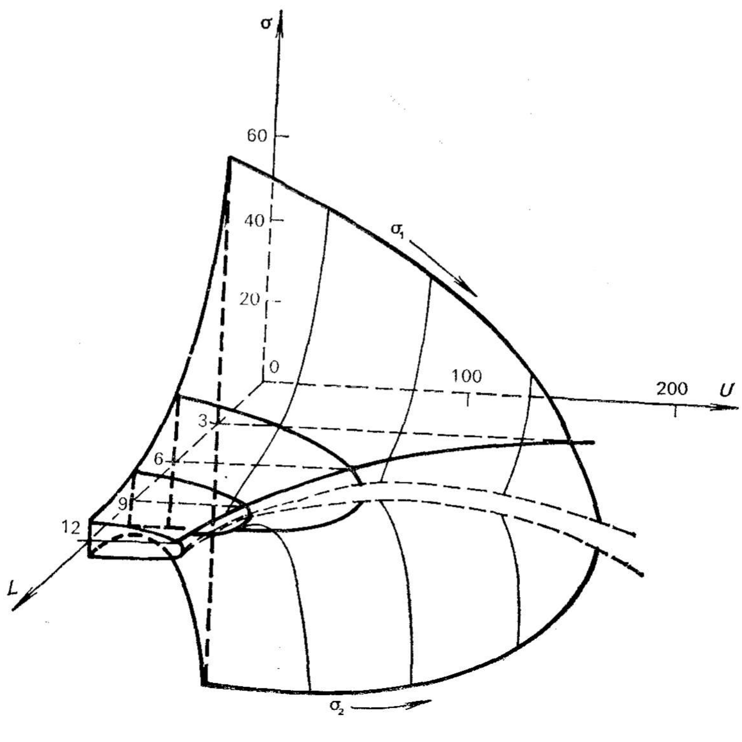

Fig. 6. Stability relief $\sigma = \sigma(L, U)$ for supporting case **E**

However all reliefs of the first loop are shown in the above figures following the order accepted in Table 1, see Figs 3, 4, 5, 6.

It is important to mention the assymmetry appearing in the last two cases (Fig. 5 & Fig. 6). We have waited to have any peculiarities in both cases, but the confluence of σ_1 & σ_2 in the 5-th octant of the space (σ, L, U) has been surprising. Trying to explain this as well as the fact that any indeterminacy in σ_{12} computing has appeared we have decided that this is due to the fact that we have used statitical trial function for both typical cases.

Obviously, the reliefs considered are the bearers of all important information connected with the determination of the lowest critical velocity of one-span pipings.

References

1. Blevins, R. D. Flow-Induced Vibration. — Van Nostr. Reinh. Co., 1977.
2. Bolotin, V. V. Nonconservative Problems of the Theory of Elastic Stability. — Moscow, Physics & Mathematics, 1961.
3. Chen, S.-S. Vibrations of Continuous Pipes Conveying Fluid. Proc. of IUTAM-IAHR Symp. on Flow-Induced Struct. Vibr., August, 1972, Karlsruhe, FRG, Springer Verlag 1974, 663-675.
4. Dzhupanov, V. A. Hydroelasticity of High Pressure Active Pipelines. — Fifth Nat. Congress on TAM, 23-29 September, 1985, Varna, Bulgaria, BAS, Proceedings, Vol. 1, 1973, 243-254.
5. Dzhupanov, V. A. Hydroelasticity in the Mech. of Nuclear Power Reactors. — Centre of Sci. Inform., BAS, Vol. 14, No 3, 1987.
6. Gregory, R., M. Païdoussis. Unstable Oscillation of Tubular Cantil. Conveying Fluid — Part I. Theory: 512-527; Part II. Exp.: 528-542). Proc. Roy. Soc., Ser. A, 293, London, 1966.
7. Hara, F. Two-phase Flow-Induced Parametric Vibrations in Structural Systems — Rep. of the Insts of Ind. Science, Univ. of Tokyo, Vol. 28, No 4 (serial No 1983), March 1980, 1-43.
8. Kartvelishvily, N. A. Dynamics of Press. Pipes. — M., Energ., 1979.
9. Leleev, H. S. Nonstationary Motion of the Heatcarrier in the Heated Tubes of the Great Steam-generators. — Moscow, Energia, 1978.
10. Païdoussis, M. P. The Dynamical Behaviour of Cylindrical Structures in Axial Flow. — In: "Annals of Nuclear Science & Engineering", Vol. 1, 1974, No 2, 83-106.
11. Païdoussis, M. P., N. T. Issid, M. Tsui. Parametric Resonance Oscillations of Flexible Slender Cylinders in Harmonically Perturbed Axial Flow. — Part 1. Theory: 709-714; Part 2. Experiments: (715-719). — J. of Appl. Mech., Vol. 47, 1980, 709-719.
12. Païdoussis, M. P. A Review on Flow-Ind. Vibr. in Reactors & Reactor Components. — Nucl. Eng. & Design, Vol. 74, 1982, 31-60.
13. Païdoussis, M. P. Flow-Induced Instabilities of Cylindrical Structures. — Appl. Mech. Reviews, Vol. 40, 1987, 163-175.
14. Païdoussis, M. P., F. C. Moon. Nonlinear and Chaotic Fluidelastic Vibrations of a Flexible Pipe Conveying Fluid. — Proceedings of the XVI IUTAM Congress of Mechanics, Grenauble, 1988.
15. Samarin, A. A. Vibration of the Piping in Power Plants and Methods for Their Elimination. — Moscow, Energia, 1979.
16. Volmir, A. S. Shells in Liquid and Gas Streams. — (Vol. 2. A Problem of Hydroelasticity). Moscow, Nauka, 1984.

Received 25. 04. 1989.