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On the Inverse Problems in Dynamics of Layered Systems*

Notation

$\sigma_x, \sigma_y, \sigma_{xz}$ — stresses
$\sigma_z^k, \tau_x^k, \tau_y^k$ — stresses in each layer
F_x, F_y, F_z — body forces
N_x, N_y, N_{xy} — strain forces
Q_x, Q_y, Q_{xy} — shear forces
M_x, M_y, M_{xy} — bending moments
F^k, f^k — functions, describing the distribution of the shear stress along the thickness of the layer
E_k — Young's modulus
μ_k — Poisson's ratio
G^k — shear modulus
ρ^k — material density
η — viscosity coefficient of the covering layer
$c_k=\sqrt{E_k/\rho^k}$ — velocity of the wave propagation in the k-th layer
D_k — flexural rigidity
γ_{xz}, γ_{yz} — shear deformations
α, β — bending deformations
h^k — thickness of the layer
H — thickness of the plate
l_x, l_y — lengths along X and Y axis
I^k — inertia moment
V_0 — initial velocity of the impact
Q_0 — amplitude of the impact force

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Introduction

Recently, the inverse problems in linear and nonlinear dynamics of different structures have become subject of increasing interest and they are considered by many authors [1, 2].

In the case of direct problems the response of the system has to be found out for given input data, while in the most general case of inverse problems the response of the system is known, but either the equations modelling the process or the input data are unknown. Although the inverse problems are more difficult for solution they are applicable to cases where complete measurements cannot be done. Because of these reasons considerable attention is paid to the problems of identification properties and of the input data and to the methods of their solution [3-8].

This paper presents some properties identification problems in the dynamics of layered systems. A method is proposed for determining the unknown coefficient in the governing equations of multilayered plates which depend on the properties of the material. It is based on Fourier series expansion with respect to the spatial co-ordinates. A numerical example is presented too for determining the parameters of the visco-elastic covering of multilayered plates at limited displacements under a pulse load.

1. Formulation of the direct problem

A multilayered plate composed of an arbitrary number of isotropic layers is considered. In the case of beams, plates and shells the equilibrium equations in three-dimensional elasticity in Cartesian co-ordinates

$$\begin{aligned}&\frac{\partial\sigma_x}{\partial x}+\frac{\partial\sigma_{yx}}{\partial y}+\frac{\partial\sigma_{zx}}{\partial z}+F_x=0,\\ (1)\qquad&\frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_y}{\partial y}+\frac{\partial\sigma_{zy}}{\partial z}+F_y=0,\\ &\frac{\partial\sigma_{xz}}{\partial x}+\frac{\partial\sigma_{yz}}{\partial y}+\frac{\partial\sigma_z}{\partial z}+F_z=0\end{aligned}$$

could be integrated with respect to thickness of thinwalled structures. The co-ordinate system is placed in such a way that $x\in[0, l_x]$, $y\in[0, l_y]$, $z\in[-h_1, h_2]$, and $z=0$ coincide with the midplane of the plate. Then neglecting the body forces F_x, F_y, F_z and integrating to corresponding thickness of the layers gives:

$$\begin{aligned}&\frac{\partial N_x^k}{\partial x}+\frac{\partial N_{xy}^k}{\partial y}+\tau_x^k-\tau_x^{k-1}=0,\\ (2)\qquad&\frac{\partial N_{xy}^k}{\partial x}+\frac{\partial N_y^k}{\partial y}+\tau_y^k-\tau_y^{k-1}=0,\\ &\frac{\partial Q_x^k}{\partial x}+\frac{\partial Q_y^k}{\partial y}+\sigma_z^k-\sigma_z^{k-1}=0;\end{aligned}$$

$$\begin{aligned}(3)\qquad&\frac{\partial M_x^k}{\partial x}+\frac{\partial M_{xy}^k}{\partial y}-Q_x^k+h^k\tau_x^k-h^{k-1}\tau_x^{k-1}=0,\\ &\frac{\partial M_{xy}^k}{\partial x}+\frac{\partial M_y^k}{\partial y}-Q_y^k+h^k\tau_y^k-h^{k-1}\tau_y^{k-1}=0,\end{aligned}$$

where k is the number of the layer.

The equilibrium equations for bending a plate in the (x, y) plane resulting from lateral loading are denoted by (3). In the dynamic problem the inertial forces due to rotation and deflection have to be added to the right of Eq. (3).

$$\frac{\partial Q_x^k}{\partial x}+\frac{\partial Q_y^k}{\partial y}+\sigma_z^k-\sigma_z^{k-1}=h^k\rho^k\frac{\partial^2 w}{\partial t^2},$$

$$(4)\qquad \frac{\partial M_x^k}{\partial x}+\frac{\partial M_{xy}^k}{\partial y}-Q_x^k+h^k\tau_x^k-h^{k-1}\tau_x^{k-1}=I^k\rho^k\frac{\partial^2\alpha}{\partial t^2},$$

$$\frac{\partial M_{xy}^k}{\partial x}+\frac{\partial M_y^k}{\partial y}-Q_y^k+h^k\tau_y^k-h^{k-1}\tau_y^{k-1}=I^k\rho^k\frac{\partial^2\beta}{\partial t^2}.$$

On the boundaries between the layers the following conditions are satisfied in the case of a contact without slipping:

$$5)\qquad \sigma_z^k=\sigma_z^{k-1},\quad \sigma_{xz}^k=\sigma_{xz}^{k-1}.$$

The plate is taken out from static equilibrium by impact with duration t^*, providing that the mechanical contact between the touching surfaces during the impact is not disturbed. Then the problem (4), (5) is integrated under the initial conditions:

$$(6)\qquad \dot{w}(x,y,0)=V_0(x,y),\quad w(x,y,0)=0.$$

In the case of a contact without slipping of the layers and provided that the shear deformations are proportional to the shear module in the respective layers, a summation over the number of the layers could be done in Eq. (4). Thus, the problem is reduced to a system of three partial differential equations. Accounting the effects of the shear deformations and the rotatory inertia, the governing equations and conditions on the layer boundaries could be written with respect to the displacements:

$$D_k\frac{\partial^2\alpha^k}{\partial x^2}+D_k\frac{1-\mu_k}{2}\frac{\partial^2\alpha^k}{\partial y^2}+D_k\frac{1+\mu_k}{2}\frac{\partial^2\beta^k}{\partial x\partial y}-\frac{E_kF_x^k}{2(1+\mu^k)}\left(\frac{\partial w}{\partial x}+\alpha^k\right)$$

$$+h^k\frac{E_kf_x^k}{2(1+\mu_k)}\left(\frac{\partial w}{\partial x}+\alpha^k\right)-h^{k-1}\frac{E_{k-1}f_x^{k-1}}{2(1+\mu_{k-1})}\left(\frac{\partial w}{\partial x}+\alpha^{k-1}\right)=I^k\rho^k\frac{\partial^2\alpha^k}{\partial t^2},$$

$$(7)\qquad D_k\frac{\partial^2\beta}{\partial y^2}+D_k\frac{1-\mu_k}{2}\frac{\partial^2\beta^k}{\partial x^2}+D_k\frac{1+\mu_k}{2}\frac{\partial^2\alpha^k}{\partial x\partial y}-\frac{E_kF_y^k}{2(1+\mu^k)}\left(\frac{\partial w}{\partial y}+\beta^k\right)$$

$$+h^k\frac{E_kf_y^k}{2(1+\mu^k)}\left(\frac{\partial w}{\partial y}+\beta^k\right)-h^{k-1}\frac{E_{k-1}f_y^{k-1}}{2(1+\mu_{k-1})}\left(\frac{\partial w}{\partial y}+\beta^{k-1}\right)=I^k\rho^k\frac{\partial^2\beta^k}{\partial t^2},$$

$$\frac{E_kF_x^k}{2(1+\mu_0)}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial\alpha^k}{\partial x}\right)+\frac{E_0F_y^k}{2(1+\mu_0)}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial\beta^k}{\partial y}\right)+p^k-p^{k-1}=h^k\rho^k\frac{\partial^2 w}{\partial t^2}.$$

Let us assume that the considered plate consists of a viscoelastic layer on the impact surface. All layers deform without slipping and the shear deformations γ_{xz}^k and γ_{yz}^k are proportional to the shear module G^k,

$$\gamma^k_{xz}=G^0/G^k\rho^0_{xz},\quad \gamma^k_{yz}=G^0/G^k\gamma^0_{yz},$$

where the layer in which the midplane of the plate is located, is denoted by 0. In this case Eq. (4) could be summed over the number of the layers. The viscoelastic properties of the covering layer are described by Kelvin's model. The governing equations of a multilayered plate with covering viscous layer are:

$$D_{11}\frac{\partial^2\alpha}{\partial x^2}+D_{12}\frac{\partial^2\alpha}{\partial y^2}+D_{13}\frac{\partial^2\beta}{\partial x\partial y}+D_{14}\frac{\partial}{\partial t}\left(\frac{\partial^2\alpha}{\partial x^2}\right)+D_{15}\frac{\partial}{\partial t}\left(\frac{\partial^2\alpha}{\partial y^2}\right)$$

$$+D_{16}\frac{\partial}{\partial t}\left(\frac{\partial^2\beta}{\partial x\partial y}\right)-D_{17}\left(\frac{\partial w}{\partial x}+\alpha\right)-D_{18}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial x}+\alpha\right)=I\frac{\partial^2\alpha}{\partial t^2},$$

(8) $$D_{12}\frac{\partial^2\beta}{\partial x^2}+D_{11}\frac{\partial^2\beta}{\partial y^2}+D_{13}\frac{\partial^2\alpha}{\partial x\partial y}+D_{14}\frac{\partial}{\partial t}\left(\frac{\partial^2\beta}{\partial y^2}\right)+D_{15}\frac{\partial}{\partial t}\left(\frac{\partial^2\beta}{\partial x^2}\right)$$

$$+D_{16}\frac{\partial}{\partial t}\left(\frac{\partial^2\alpha}{\partial x\partial y}\right)-D_{17}\left(\frac{\partial w}{\partial y}+\beta\right)-D_{18}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial y}+\beta\right)=I\frac{\partial^2\beta}{\partial t^2},$$

$$D_{31}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial\alpha}{\partial x}\right)+D_{32}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial\beta}{\partial y}\right)+D_{33}\frac{\partial}{\partial t}\left[\left(\frac{\partial^2 w}{\partial x^2}\frac{\partial\alpha}{\partial x}\right)+\left(\frac{\partial^2 w}{\partial y^2}-\frac{\partial\beta}{\partial y}\right)\right]-q=H\rho\frac{\partial^2 w}{\partial t^2}.$$

where

$$D_{11}=\sum_k D_{0k}\qquad D_{0k}=2(h^k)^3E_0/[3(1-\mu_k)(1+\mu_k)],$$

$$D_{12}=\Sigma D_{0k}(1-\mu_k)/2,\qquad D_{13}=\sum_k D_{0k}(1+\mu_k)/2,$$

$$D_{14}=D_\eta,\qquad D_\eta=2\eta E_0h_1^3/[3E_1(1-\mu_k^2)],$$

$$D_{15}=D_{16}=D_\eta(1-\mu_k)/2,\qquad D_{17}=E_0H/[2(1+\mu_0)],$$

$$D_{18}=\eta E_0/E_1,\qquad D_{31}=D_{32}=D_{17},\quad H\rho=\sum_k h^k\rho^k,$$

$$D_{33}=\eta E_0h_1/[2(1+\mu_0)],\qquad I=2H^2H_\rho/3.$$

Conditions (5) are taken into account in Eq. (8). The extent of the deformation energy absorption could be governed by the change of η.

2. Identification problem solution

The problems of medium parameters determination on the basis of data obtained as a result of observation on the system behaviour in time are of considerable practical interest. Then, as a whole, the system response could be approximate in an appropriate way and be substituted in Eq, (8) after which these equations will be solved with respect to $\eta(x, y)$.

Eqs (8) are considered by introducing new variables: $\bar{w}=w/H$, $H=\Sigma h_k$, $\bar{x}=\pi x/l_x$, $\bar{y}=\pi y/l_y$ and $\bar{t}=tc_0/H$ (all bars are omitted below):

$$a_{11}\alpha_{xx}+a_{12}\alpha_{yy}+a_{13}\beta_{xy}-a_{14}w_x+a_{19}\alpha$$

$$+\eta(a_{15}\dot{\alpha}_{xx}+a_{16}\dot{\alpha}_{yy}+a_{17}\dot{\beta}_{xy}-a_{18}\dot{w}_x+a_{110}\dot{\alpha})=\ddot{\alpha},$$

$$a_{21}\beta_{xx}+a_{22}\beta_{yy}+a_{23}\alpha_{xy}-a_{24}w_y+a_{29}\beta$$

(9)

$$+\eta\,(a_{25}\dot{\beta}_{xx}+a_{26}\dot{\beta}_{yy}+a_{27}\dot{\alpha}_{xy}-a_{28}\dot{w}_y+a_{210}\dot{\beta})=\ddot{\beta},$$

$$a_{31}w_{xx}+a_{32}\alpha_x+a_{33}w_{yy}+a_{34}\beta_y$$

$$+\eta\,(a_{35}\dot{w}_{xx}+a_{36}\dot{\alpha}_x+a_{37}\dot{w}_{yy}+a_{38}\dot{\beta}_y)-q=\ddot{w}.$$

The boundary conditions are as follows: at $y=0,\ \pi$ the plate is clamped and at $x=0,\ \pi$ it is simply supported:

$$w(0,\ y,\ t)=w(\pi,\ y,\ t)=0,\quad w(x,\ 0,\ t)=w(x,\ \pi,\ t)=0,$$

(10)

$$\alpha_y(0,\ y,\ t)=\alpha_y(\pi,\ y,\ t)=0,\quad \alpha(x,\ 0,\ t)=\alpha(x,\ \pi,\ t)=0,$$

$$\beta(0,\ y,\ t)=\beta(\pi,\ y,\ t)=0,\quad \beta(x,\ 0,\ t)=\beta(x,\ \pi,\ t)=0.$$

The plate is subjected to blast loading with an initial velocity V_0 and duration t^* in the central zone. At $t=0$ it is not deformed.

$$w(x,\ y,\ 0)=0;\quad \alpha(x,\ y,\ 0)=\beta(x,\ y,\ 0)=0,$$

(11a)

$$\dot{\alpha}(x,\ y,\ 0)=0,\quad \dot{\beta}(x,\ y,\ 0)=0$$

$$\dot{w}(x,\ y,\ 0)=\begin{cases} V_0, & x,\ y\in[\pi/3,\ 2\pi/3] \\ 0, & x,\ y\notin(\pi/3,\ 2\pi/3), \end{cases}$$

(11b)

$$q(x,\ y,\ t)=\begin{cases} Q_0 l^{-\gamma t}, & x,\ y\in[\pi/3,\ 2\pi/3] \\ 0, & x,\ y\notin(\pi/3,\ 2\pi/3) \end{cases}\Bigg\}\ t\in[0,\ t^*]$$
$$\qquad\qquad 0,\quad x,\ y\in[0,\ \pi],\quad t>t^*.$$

2. 1. $\eta=$const

It is necessary the viscosity coefficient η to be determined in such a way that the maximum deflection of the plate centre should not be greater than a given deflection w.

Under the given boundary conditions the functions w, α, β, q and V_0 could be expanded to Fourier series in the interval $[0,\ \pi]$:

$$w(x,\ y,\ t)=\sum_{n,\,k} W^{kn}(t)\sin(nx)\,.\sin(ky),$$

$$\alpha(x,\ y,\ t)=\sum_{n,\,k} A^{kn}(t)\cos(nx)\,.\sin(ky),$$

(12)

$$\beta(x,\ y,\ t)=\sum_{n,\,k} B^{kn}(t)\sin(nx)\,.\sin(ky),$$

$$q(x,\ y,\ t)=\sum_{n,\,k} Q^{kn}(t)\sin(nx)\,.\sin(ky),\quad V_0(x,\ y)=\sum_{n,\,k} V^{kn}\sin(nx)\,.\sin(ky).$$

Substituting these equations in (9) the following system of ordinary differential equations with respect to t is obtained:

(13a) $$p_{2,1}^{kn} A^{kn}(t)+\eta p_{1,1}^{kn} \dot{A}^{kn}(t)-n\, a_{14}\ W^{kn}(t)+\eta a_{18}\dot{W}^{kn}(t))=\ddot{A}^{kn}(t);$$

(13b) $$p_{2,2}^{kn} B^{kn}(t)+\eta p_{1,2}^{kn} \dot{B}^{kn}(t)=\ddot{B}^{kn}(t);$$

(13c) $$p_{2,3}^{kn} W^{kn}(t)+\eta p_{1,3}^{kn} \dot{W}^{kn}(t) - n a_{32} A^{kn}(t)+\eta a_{36}\dot{A}^{kn}(t)-Q^{kn}(t)=\ddot{W}^{kn}(t);$$

for $i=1,\ 2$ $p_{1,i}^{kn}=-(a_{i5}n^2+a_{i6}k^2+a_{i10})$, $p_{2,i}^{kn}=-(a_{i1}n^2+a_{i2}k^2+a_{i9})$, and for $i=3$ $p_{1,3}^{kn}=-(a_{35}n^2+a_{37}k^2)$, $p_{2,3}=-(a_{31}n^2+a_{33}k^2)$.

These are two equations per each couple W^{kn} and A^{kn} and a separate equation for B^{kn}. The initial conditions for the coefficients in the expansion correspond to the initial conditions for $w(x, y, 0)$ and $\alpha(x, y, 0)$:

(14) $$A^{kn}(0)=0, \quad W^{kn}(0)=0, \quad \dot{A}^{kn}(0)=0, \quad \dot{W}^{kn}(0)=V_0^{kn}.$$

Analyzing the equations

$$\lambda_i^{kn}=4p_{2,i}^{kn}-(\eta_i^{kn})^2(p_{1,i}^{kn})^2\leq 0, \quad (i=1,\ 2,\ 3),$$

where λ_i^{kn} are the determinants of the characteristic equations, η could be determined for each (kn). In the case when the system gets back into its initial equilibrium state without oscillations η is defined as $\eta_0=\max_i(\eta_i^{kn})$.

In all other cases the damping of the system will be realized by oscillations near to its equilibrium state.

2.1.1. Let $\eta\leq\min_i(\eta_i^{kn})$. It is necessary η to be defined under the provision that $w(\eta, t)$ is smaller than w_0. Solving Eq. (13) for each $\eta\in[0, \min(\eta_i^{kn})]$ a solution $w(x, y$ $t, \eta)$ is found out.

The behaviour of the function $w(\eta, t)$ at the given point (x_0, y_0) is sought by the condition

(15) $$\max_{\{\eta\}}\Big(\sum_{n,k}[W^{kn}(t)\sin(nx)\sin(ky)]\Big)\leq w_0$$

in the period $t\in[0, 2/\lambda_i^{kn}]$. Parameter η is numerically obtained from Eq. (15) using the algorithm for determining real zero of functions, published by Dekker [8] and improved by Brent [9]. This algorithm combines the bisection method, method regula falsi and inversed square interpolation. In this way η^* is determined in which case $w(\pi/2, \pi/2, t)\leq w_0$. Consequently, if the viscosity coefficient of the plate covering is η^*, then the deflections will not be greater than the permissible one at given loading and properties of the materials.

2.1.2. Let the loading $q(x, y, t)$ is given and the system response at $x=\pi/2$ and $y\in[0, \pi]$ as a function $w(y, t)$ is known. Then these functions may be expanded in Fourier series and substituted in Eq. (13). For each k and $n=1$ a couple of equations for $A^{k1}(t)$ and $W(t)$ are obtained. $A^{k1}(t)$ can be expressed by $W^{k1}(t)$ and η from the first equation and substituted in the second one. In the general case, a system of k ordinary differential equations is given for $W^{k1}(t)$ and $\eta^k(W^{k1}(t))$ are known). The viscosity coefficient η_0 is found out as $\eta_0=\Big(\sum_k \eta^k\Big)/k_0$, where k_0 is the number of Fourier series members, which are taken into account. This is the first approximation of η. Putting η_0 into the governing equations (8) the direct problem can be solved. The least squares error is expressed as

$$\delta(\eta, t) = \int_0^{\pi} [w(\pi/2, y, t) - w_d(\pi/2, y, t, \eta_j)]^2 \, dy, \tag{16}$$

where w is the given response, w_d is the direct problem solution and j is a number of approximation. The problem is to minimize the function δ over the sequence η_j:

$$\delta(t) = \min_{\{\eta\}} (\eta_j, t).$$

This problem is reduced to

$$\delta(t_{fix}) = \min (\eta_j, t_{fix}),$$

where t_{fix} is a time when maximum $w(\pi/2, y, t)$ is realised. For determining the minimum of δ in respect to η an algorithm similar to this one used for solving Eq. (15) is applied. This algorithm combines the method of gold section and successive square interpolation [9].

2.2. $\eta = \eta(y)$

Let it be assumed that the viscosity coefficient η changes along y. Equations (9) obtain the form

$$\begin{aligned}
&a_{11}\alpha_{xx} + a_{12}\alpha_{yy} + a_{13}\beta_{xy} - a_{14}w_x + a_{19}\alpha + \eta(a_{15}\dot{\alpha}_{xx} + a_{16}\dot{\alpha}_{yy} + a_{17}\dot{\beta}_{xy} \\
&- a_{18}\dot{w}_x + a_{110}\dot{\alpha}) + \eta_y(a_{111}\dot{\alpha}_y + a_{112}\dot{\beta}_x) = \ddot{\alpha}, \\
&a_{12}\beta_{xx} + a_{22}\beta_{yy} + a_{23}\alpha_{xy} - a_{24}\beta_y + a_{29}\beta + \eta(a_{25}\dot{\beta}_{xx} + a_{26}\dot{\beta}_{yy} + a_{27}\dot{\alpha}_{xy} \\
&- a_{28}\dot{w}_y + a_{210}\dot{\beta}) + \eta_y(a_{211}\dot{\beta}_y + a_{212}\dot{\alpha}_x) = \ddot{\beta}, \\
&a_{31}w_{xx} + a_{32}\alpha_x + a_{33}w_{yy} + a_{34}\beta_y + \eta(a_{35}\dot{w}_{xx} + a_{36}\dot{\alpha}_x + a_{37}\dot{w}_{yy} + a_{38}\dot{\beta}_y) \\
&+ \eta_y(a_{39}\dot{w}_y + a_{310}\dot{\beta}) - q = \ddot{w}.
\end{aligned} \tag{17}$$

The response of the system is given at $x = \pi/2$ as

$$w(\pi/2, y, t) = w(y, t) = \sum_k W^k(t) \sin(ky). \tag{18}$$

The function $\eta(y)$ is presented in Fourier series expansion $\{\sin(ky)\}$. The governing equations for the first mode of oscillation with respect to $x(n=1)$ are:

$$\ddot{A}^{kn} = (-a_{11} - a_{12}k^2 + a_{19})A^k(t) - a_{14}W^k(t) \tag{19a}$$
$$+ \sum_m \{\eta_m[(-a_{15} - k^2a_{16} + a_{110})(\dot{A}^{k+m} - \dot{A}^{k-m}) - a_{18}(\dot{W}^{k+m} - W^{k-m})]\};$$

(19*b*) $$\ddot{B}^{kn}=(-a_{21}-a_{22}k^2+a_{29})B^k(t)+\sum_m\{\eta_m[(-a_{25}-k^2a_{26}+a_{210})(\dot{B}^{k+m}-\dot{B}^{k-m})]\};$$

(19*c*) $$\ddot{W}^{kn}=(-a_{31}-a_{32}k^2)W^k(t)-a_{32}A^{kn}(t)$$
$$+\sum_m\{\eta_m[(-a_{35}-k^2a_{37}+a_{39})(\dot{W}^{k+m}-\dot{W}^{k-m})-a_{36}(\dot{A}^{k+m}-\dot{A}^{k-m})]\}-Q^k.$$

This problem may be solved by method of successive approximations under conditions (10) and (11). $\eta_0^m=\eta_0$ are accepted for the first approximation. Putting these values of η^m and Eq. (18) into (19*a*) a system of $(k+m)$ ordinary differential equations with respect to $A_0^{k1}(t)$ is obtained. After solving this system $A_0^{k1}(t)$ are substituted into (19*c*). A system with respect to η_1^m is obtained. This system may be integrated for the given time interval (e. g. duration of the period of free vibrations).The resulting system is algebraic. The determined approximation η_1^m is substituted into (19*a*) to find out $A_2^{k1}(t)$. After that from Eq. (19*c*) the next η_2^m is determined. This procedure is repeated while the obtained η_j^k do not satisfy the condition

(20) $$\left|\int_0^\pi\sum_k\eta_{j+1}^k\sin(ky)dy-\int_0^\pi\sum_k\eta_j^k\sin(ky)\,dy\right|<\rho.$$

$$\rho>0,\quad \rho\ll 1.$$

So the coefficients in the Fourier series expansion are determined and

$$\eta_j(y)=\sum_k\eta_j^k\sin(ky).$$

Substituting this function into Eq. (17) the direct problem can be solved. After that $\eta(y)$ is changed by $\Delta\eta_0$ as

$$\eta_1=\eta_0+\Delta\eta_0=\sum_k(\eta_0^k+p^k\Delta\eta_0)\sin ky,$$

where p^k are weight coefficients. The squares error is expressed as

(21) $$\delta(\{\eta\}_i,t)=\int_0^\pi[w(\pi/2,y,t)-w_d(\pi/2,y,t,\eta_i)]^2dy.$$

The problem is to minimize the function δ over the sequence of $\{\eta\}$:

(22) $$\delta(t)=\min_{\{\eta\}_i}\left(\int_0^\pi[w(\pi/2,y,t)-w_d(\pi/2,y,t,\sum_k(\eta_i^k+p^k\Delta\eta_i)\sin(ky))]^2dy\right).$$

The minimization of the several variables function $\delta(\{\eta\},t_{fix})$, is done using the finite difference modified method of Levenberg-Marqaurdt. This algorithm is numerically realised in IMSL computer codes.

3. Example

The identification problem for the viscosity coefficient is solved in the cases of $\eta = \text{const}$ and $\eta = \eta(y)$. A three-layered squared plate of two elastic (Steel-Al) layers

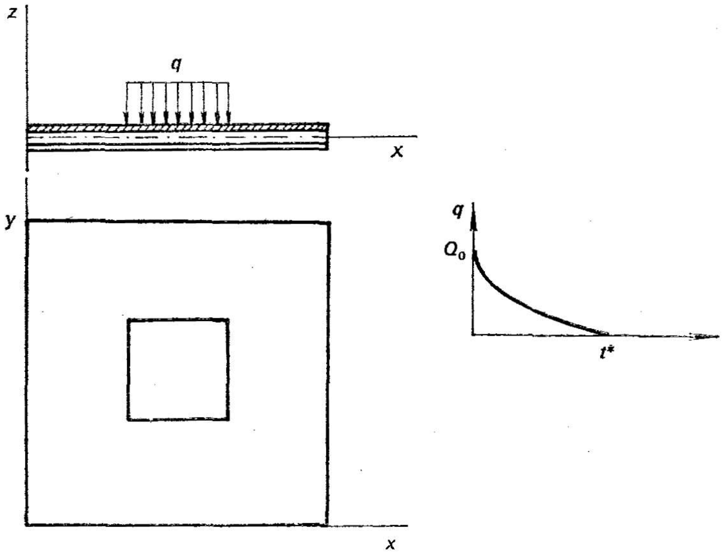

Fig. 1

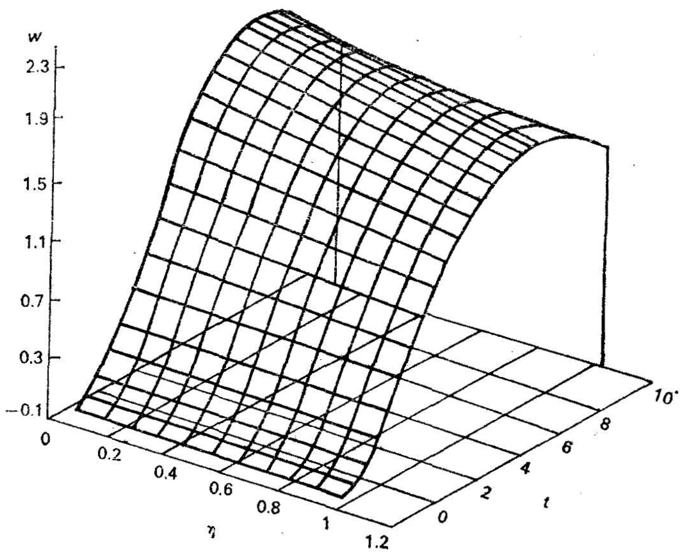

Fig. 2. The displacements $w(t, \eta)$ at the plate centre

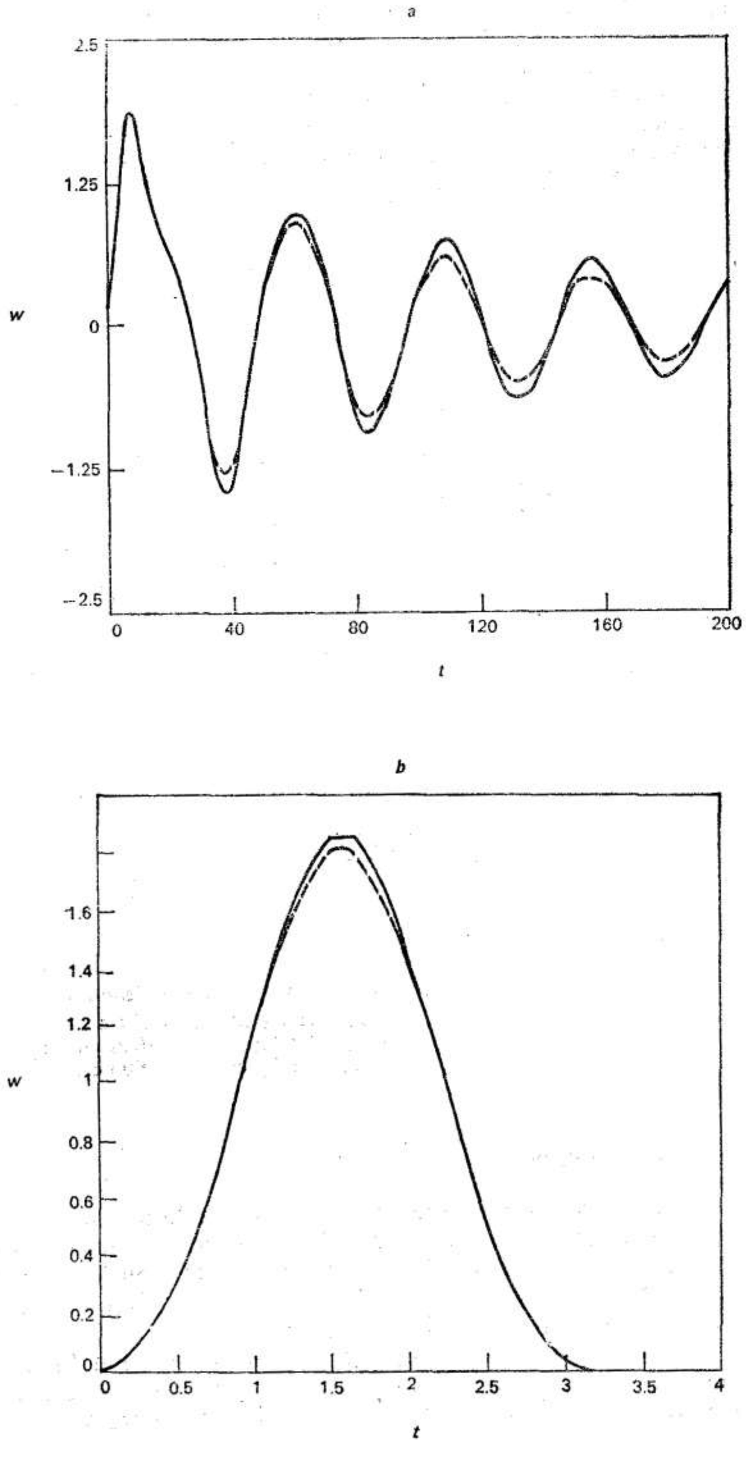

Fig. 3. *a*—the displacement $w(t)$ at the plate centre for $\eta=0,5$.
.... known response,
——— inverse solution;
b—the displacement $w(\pi/2, y)$ at $t=8, 0$.
.... known response
——— inverse solution

and a visco-elastic polymer layer is considered. The plate is subjected to a blast loading of duration t^* in its central zone (Fig. 1). The dependence of $w(\pi/2, \pi/2, t)$ on $\eta = \text{const}$ is presented in Fig. 2. It is seen that the restriction of w_0 at $(x, y) = (\pi/2, \pi/2)$ is sufficient to determine the limit value of η. A comparison between the given system response $w_0(y, t)$ and the solution $w(y, t)$ obtained by solving the inverse problem is shown in Fig. 3*a*, *b*. The solution in the case $\eta = \eta(y)$ for several values of Q_0 and V_0 is shown in Fig. 4.

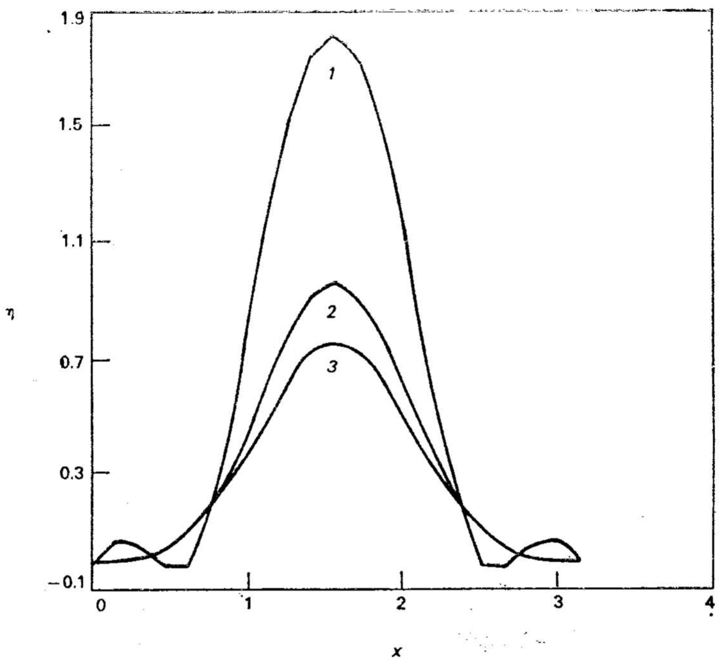

Fig. 4. Solutions for $\eta(y)$ for different values of Q_0 and V_0.
1 — Q_0=0.4 kg/cm², V_0=600 cm/s:
2 — Q_0=0.3 kg/cm², V_0=650 cm/s;
3 — Q_0=0.3 kg/cm², V_0=600 cm/s

4. Conclusions

The proposed method is very applicable to the solution of the considered class inverse problems. The bases restriction which should be observed is that functions used in the equations should permit expansion in Fourier series.

At given restriction for w only in one point of the plate the maximum values of $\eta = \text{const}$ can be easily obtained·

It is seen from the comparison made between the given system response $w_0(y, t)$ and the direct problem solution $w(y, t)$ with $\eta = \text{const}$ determined in the inverse problem, that η is obtained with sufficient accuracy.

The method permits the unknown coefficient $\eta(y)$ to be determined too, by reducing the problem to a system of algebraic equations with respect to the coefficients in Fourier series expansion.

Under the appropriate boundary conditions the proposed method can be applied to the case of $\eta=\eta(x, y)$.

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