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Poincare's Initial Conditions in Dynamics of Walk Phase for Space Model of Legged Locomotion Robot. A Case of Internal and External Resonance

The dynamics of the antropomorphic robot model in walk phase has been investigated in the paper. Described is the case of two very close own frequences values which are equal to one of the drive control moment frequences. It is called

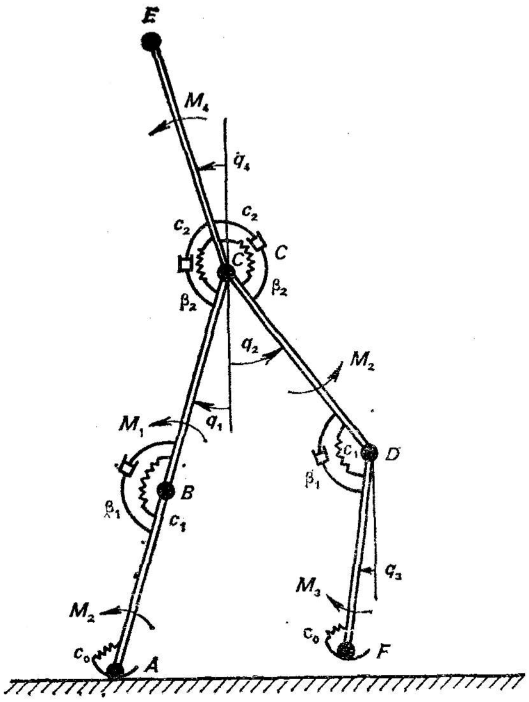

Fig.

the case of external and internal resonance. The robot model is represented in Fig. 1. It has been proved the initial coordinates and velocities values for approximate periodical solutions of the motion equations are related in a fixed way by Poincaré's method. This method is applied in the present investigation for external and internal resonance case. Obtained are necessary and sufficient conditions for the initial values of the generalised coordinates and velocities and the approximate law of the motion under chosen control drive moments.

Poincaré has proved [2] that in the case of approximate solution as that one in [1] some conditions, called initial conditions, must be satisfied after the theory of nonevident functions. Those conditions are relations between initial values of the generalised coordinates and velocities.

Two cases are possible for the periodical solutions: nonresonance and resonance (external, internal or both together). The initial conditions for those two cases could be found in different ways. Let

(1) $$q = | q_1, q_2, q_3, q_4, q_5 |^T$$

be the vector of the generalized coordinates and

(2) $$M(t) = | M_i(t) |^T = | H_{i0} + \sum_{m=1}^{\infty} H_{im} . \sin b_m(ht + h_1) |^T, \quad i = 1, 2, \dots, 5,$$

be the vector-function of the control drive moments.

The robot motion can be expressed by next quazilinear system in [1] in the vector form

(3) $$A . q^{\cdot\cdot} + C . q = \acute{e} . (F + p . M), \quad p = \frac{l_1}{l_{12}},$$

where

(4) $$F(q, q^{\cdot}) = | A_{i1} . a_s . \sin \check{r}_s + A_{i2} . a_s^3 . \cos \check{r}_s - A_{i2} . a_s^3 . \cos 3\check{r}_s |^T.$$

The members a_{ij} of matrix A, c_{ij} of the matrix C and constants A_{ik}, $i, j = 1, 2, \dots, 5$; $k = 1, 2$, depend on mass, geometrical, inertia and elastic parameters of the described robot model [1], $\acute{e}$ is a little parameter [2].

The characteric equation of [3] has the next form

(5) $$D(p_s^2) = -A . p_s^2 + C = 0, \quad s = 1, 2, \dots, 5,$$

where p_s is one of the own frequences of the described mechanical system. If two of those frequences are related by $p_j = p_l . k'$ then the problem is to be found bifrequence solution of (3).

Let for nondisturbed system of (3), (nondisturbed, when $\acute{e} = 0$), be possible:

1. unique solution which corresponds to the stability position $q_0' = | q_{i0}' |^T = 0$, $i = 1, 2, \dots, 5$;

2. internal resonance exists (very near values of two own frequences) of type

(6) $$p_l \approx p_j . k', \quad j \neq l,$$

where k' is a natural number, but

$$p_n \neq p_i . k', \quad n = j . l, \quad i = 1, 2, \dots, 5, \quad n \neq i,$$

3. external resonance exists with the capital harmonic function of the row (2) of type

$$(7)\qquad \check{r}_j=p_j.t+\acute{y}_j=\frac{r}{g}.b_1.h.t+\acute{y}_j,$$

$$\check{r}_l=p_l.t+\acute{y}_l=\frac{r}{g}.k'.b_1.h^{\cdot}.t+\acute{y}_l,$$

where numbers r and g are natural and have not equal multipliers, $\check{r}_j$ and $\check{r}_l$ are angle phases corresponding to the frequences p_j and p_l, respectively $\acute{y}_j$ and $\acute{y}_l$ are the phases differences for those frequences with the resonance external frequence b_1,

4. origine befrequence oscillations (origin system, when $\acute{e}=0$) of type

$$(8)\qquad |q_{i0}|^T=|f_i^{(j)}.\acute{a}_j.\cos(p_j.t+\acute{y}_j')+f_i^{(l)}.\acute{a}_l.\cos(p_l.t+\acute{y}_l')|^T=q_0$$

where $\acute{a}_j$ and $\acute{a}_l$ are amplitude constants corresponding to p_j and p_l, $\acute{y}_j'$ and $\acute{y}_l'$ are initial phase constants, $f_i^{(j)}$ and $f_i^{(l)}$, $i=1, 2, \ldots, 5$, are natural normal forms which are nontrivial solution of

$$(9)\qquad (-A.p_k^2+C).f^{(k)}=0, \quad f^{(k)}=|f_i^{(k)}|^T, \quad k=j, l.$$

The normal forms for solving problem are as follows

$$(10)\qquad f_i^{(s)}=d_{4i}(p_s^2),$$

where $d_{4i}(p_s^2)$, $i=1, 2, \ldots, 5$, are associated matrices of the fourth row elements of the matrix (5).

Let the origin solution has a form

$$(11)\qquad q_0=|q_{01}, q_{02}, \ldots, q_{05}|^T.$$

It has been proved that the periodical solution of the origin system of the vector equation (3) not always corresponds to the solution of the found system (found means disturbed). The following cases are possible: to exist a unique periodical solution, to exist a set of solutions or to exist no solutions of the found system.

The problem to be solved in the paper is to find the conditions when unique solution exists of (3) and when $\grave{e}=0$ it turns into origin one.

Let vectors n_1 and n_2 define the initial values differences of the vectors q and $q^{\cdot}$ in the found periodical solution of (3) from the same values of q_0 and $q_0^{\cdot}$. Then the vector equation follows

$$(12)\qquad q(0, n_1, n_2, \acute{e})=q_0(0)+n_1, \quad n_k=|n_{ki}|^T, \quad k=1, 2, \ldots, 5,$$

$$q^{\cdot}(0, n_1, n_2, \acute{e})=q_0^{\cdot}(0)+n_2.$$

The necessary conditions for periodicity of a vector $q(t, n_1, n_2, \acute{e})$ as a time function with the period 2π are

$$(13)\qquad N_1(n_1, n_2, \acute{e})=q(2\pi, n_1, n_2, \acute{e})-q(0, n_1, n_2, \acute{e})=0,$$

$$N_2(n_1, n_2, \acute{e})=q^{\cdot}(2\pi, n_1, n_2, \acute{e})-q^{\cdot}(0, n_1, n_2, \acute{e})=0.$$

Fixing the period is not a restriction because the time unit could be chosen in such a way that condition to be fulfilled.

The conditions (13) guarantee the periodicity of the solution $q=|q_i(t, n_1, n_2, \acute{e})|^T$ with a period 2π and equality of its values for t and $t+2\pi$. Due to the equality of the values of q for t and $t+2\pi$ the functions $q_i(t, n_1, n_2, é)$, $i=1, 2, \ldots, 5$, depend on n_1 and n_2. Thus the vector q is equal in the intervals 10, 2π1 and 12π, 4π1. Because of the periodicity of the right sides of (3) in the moment $t+2\pi$ the motion conditions are the same as in time t. In that case the solution q is periodical and the conditions (13) will be sufficient too. After the differential equations theory and the conditions for the right sides of (3), the expressions for N_1 and N_2 are analytical functions of the arguments n_1, n_2 and $é$ if those arguments are sufficiently little. N_1 and N_2 are zero when $n_1=n_2=é=0$. Then the found solution q turns into the origin q_0 which is periodical. Thus the problem for solving the functions N_1 and N_2 after n_1 and n_2 which are zero when $é=0$ is equivalent to the problem for existing of a periodical found solution of (3).

It is supposed the set of analyticity of the vector F from (3) after q and $q^{\cdot}$ includes the area of the analyticity of the origin solution q_0. In that case after the differential equations theory the solution q is analytical after $é$, n_1 and n_2 and when $é=n_1=n_2=0$ it turns into q_0.

Let the vector found solution of (3) is

(14) $$q(t, n_1, n_2, é)=q_0(t)+R^{(1)}.n_1+R^{(2)}.n_2+R^{(3)}.é,$$

where the components of the vectors $R^{(k)}=|R_i^{(k)}|^T$, $k=1, 2, 3$, are unknown time functions. For to obtain them it is necessary to replace (14) in (3) and to equalise the expressions before the equal harmonic functions.

The following vector equations are obtained for the components of $R^{(1)}$ and $R^{(2)}$

(15) $$A.R^{(1)\cdot\cdot}+C.R^{(1)}=0;$$

(16) $$A.R^{(2)\cdot\cdot}+C.R^{(2)}=0.$$

For solving (15) and (16) it is necessary to take into account the next integrate conditions

(17) $$R^{(1)}(0)=0, \quad R^{(1)\cdot}(0)=1;$$

(18) $$R^{(2)}(0)=1, \quad R^{(2)\cdot}(0)=0.$$

It could be supposed that the components of $R^{(1)}$ are of type

(19) $$R_i^{(1)}=R_{ij}^{(1)}.\cos p_j.t+R_{il}^{(1)}.\cos p_l.t,$$

and for

(20) $$R_i^{(2)}=R_{il}^{(2)}.\sin p_j.t+R_{il}^{(2)}.\sin p_l.t,$$

where $R_{in}^{(1)}$ and $R_{in}^{(2)}$, $n=j, l$, are constants, p_j and p_l are the sustained frequences from the own frequences of the mechanical system. Then those vector-constants $R_n^{(1)}$ and $R_n^{(2)}$, $n=j, l$, are solutions of the equations analogical to the equations of the normal functions,

(21) $$(-A.p_j^2+C).R_n^{(k)}=0, \quad (-A.p_l^2+C).R_n^{(k)}=0, \quad k=1, 2,$$

$$R^{(k)}=|R_{in}^{(k)}|^T, \quad n=j, l, \quad i=1, 2, \ldots, 5.$$

Consequently, they could be represented by the associated matrices of the elements of third and fourth row of a characteristic equation for the own frequences p_j and p_l. Then the components of $R_n^{(1)}$ and $R_n^{(2)}$ are

(22) $$R_{in}^{(1)}=d_{3i}(p_n^2).\cos p_n.t, \quad n=j,\ l, \quad i=1,\ 2,\dots,\ 5,$$
$$R_{in}^{(2)}=\frac{1}{p_n}.d_{4i}(p_n^2).\sin p_n.t,$$

and the conditions (13) have the form

(23) $$N_1(n_1,\ n_2,\ \acute{e})=d_3(p_j^2).\cos 2\pi p_j-1)+d_3(p_l^2).\cos 2\pi p_l-1)].n_1$$
$$+\left[d_4(p_j^2).\frac{1}{p_j}.\sin 2\pi p_j+d_4(p_l^2).\frac{1}{p_l}.\sin 2\pi p_l\right].n_2,$$
$$N_2(n_1,\ n_2,\ \acute{e})=-[p_j.d_3(p_j^2).\sin 2.\pi.p_j+p_l.d_3(p_l^2).\sin 2.\pi.p_l].n_1$$
$$+[d_4(p_j^2).(\cos 2.\pi.p_j-1)+d_4(p_l^2).(\cos 2.\pi.p_l-1).n_2$$
$$d_k(p_n^2)=|\,d_{ki}(p_n^2)\,|^T, \quad k=3,\ 4, \quad i=1,\ 2,\dots,\ 5.$$

Jacobi's matrix for N_1 and N_2 after the components of n_1 and n_2 is

(24) $$\left(\frac{\partial(N_{j1},\ N_{j2})}{\partial(n_{i1},\ n_{j2})}\right)_{\acute{e}=n_{i1}=n_{i2}=0} \neq 0, \quad i,\ j=1,\ 2,\dots,\ 5.$$

After the theory of nonevident functions taking an account (24) when $\acute{e}$ is sufficiently little equations (23) have unique periodical solution $n=|\,n_1,\ n_2\,|^T$ which is analytical after $\acute{e}$.

The definition of the unknown components $R_i^{(3)}$, $i=1,\ 2,\dots,\ 5$, is done under the assumption they are of a type

(25) $$R_i^{(3)}=f_i^{(j)}.R_j^{(3)}+f_i^{(l)}.R_l^{(3)},$$

where $R_j^{(3)}$ and $R_l^{(3)}$ are periodical functions of $\acute{r}_j$, $\acute{r}_l$ and $h.t$ with the period 2π. They must be of limited amplitudes and may be of type

(26) $$R_j^{(3)}=u_1^{(j)}(a_j,\ a_l,\ \acute{y}_j,\ \acute{y}_l,\ h.t),$$
$$R_l^{(3)}=u_1^{(l)}(a_j,\ a_l,\ \acute{y}_j,\ \acute{y}_l,\ h.t).$$

The amplitudes a_j and a_l and the phase differences $\acute{y}_j$ and $\acute{y}_l$, considering (7), are defined as time functions from the next differential equations

(27) $$\frac{da_j}{dt}=\acute{e}.A_1^{(j)}(a_j,\ a_l,\ \acute{y}_j,\ \acute{y}_l),$$
$$\frac{da_l}{dt}=\acute{e}.A_1^{(l)}(a_j,\ a_l,\ \acute{y}_j,\ \acute{y}_l),$$
$$\frac{d\acute{y}_j}{dt}=\acute{e}.B_1^{(j)}(a_j,\ a_l,\ \acute{y}_j,\ \acute{y}_l)+p_j-\frac{g}{r}.b_1.h,$$
$$\frac{d\acute{y}_l}{d_t}=\acute{e}.B_1^{(l)}(a_j,\ a_l,\ \acute{y}_j,\ \acute{y}_l)+p_l-\frac{g}{r.k'}.b_1.h.$$

The expressions (26) and (27) are taken account for replacing of (14) in (3). The equations for $u_1^{(j)}$ and $u_1^{(l)}$, thus obtained, are multiplied scalarly first by $f^{(j)}$, after by $f^{(l)}$ and summarized after the index i. Because of the ortogonality of the own forms (9)

$$\sum_{s=1}^{5}\sum_{i=1}^{5} a_{is}.f_i^{(j)}.f_s^{(l)}=0,\quad j\neq l.$$

There are assumed the notes

(28) $$F^{(n)}=F.f^{(n)}=A_{11}^{(n)}.\sin \check{r}_n+A_{22}^{(n)}.a_n^3.\cos \check{r}_n-A_{22}^{(n)},\ a_n^3.\cos 3\check{r}_n,$$

$$M^{(n)}=M.f^{(n)}=H_0^{(n)}+\sum_{m=1}^{\infty}\sum_{i=1}^{5} H_{im}.f_i^{(n)}.\sin b_m.(ht+h_i),$$

$$m_n^{'}=\sum_{s=1}^{5}\sum_{i=1}^{5} a_{is}.f_i^{(n)}.f_s^{(n)},\quad c^{(n)}=\sum_{s=1}^{5}\sum_{i=1}^{5} c_{is}.f_i^{(n)}.f_s^{(n)},$$

$$H_{(n)}^0=\sum_{i=1}^{5} H_{i0}.f_i^{(u)}.$$

The resonance frequence is separated from external disturbance (2) in such a way

(29) $$H_1^{(n)}.\sin b_1.(ht+h_1)=H^{(n)}.\cos\left(\frac{g}{r}.\check{y}_n-h_1\right).\sin\left(\frac{g}{r}.\check{r}_n-H_1^{(n)}\right)$$

$$\times\sin\left(\frac{g}{r}.\check{y}_n-h_1\right).\cos\frac{g}{r}.\check{r}_n,\quad H_1^{(n)}=\sum_{i=1}^{5} H_{i1}.f_i^{(n)},\quad n=j,\ l.$$

After doing the described algebraic operations the equations for two functions $u_1^{(j)}$ and $u_1^{(l)}$ are divided into two independent equations which could be represented by next summary form

(30) $$\frac{\partial^2 u_1^{(n)}}{\partial t^2}+\frac{c^{(n)}}{m_n^{'}}.u_1^{(n)}=F^{(n)}+\frac{I_1}{I_{12}}.M^{(n)}.$$

It is supposed the asked functions $u_1^{(n)}$, $n=j,\ l$, are represented by rows of a type

(31) $$u_1^{(n)}=v_{0l}^{(n)}+\sum_{m=2}^{\infty}(V_1^{(n)}.\sin b_m.h.t+W_1^{(n)}.\cos b_m.h.t)+v_{11}^{(n)}.\sin \check{r}_n$$

$$+v_{21}^{(n)}.\cos\check{r}_n+v_{31}^{(n)}.\cos 3.\check{r}_n,$$

where $v_{01}^{(n)}$, $v_{11}^{(n)}$, $v_{21}^{(n)}$, $v_{31}^{(n)}$, $V_m^{(n)}$ and $W_m^{(n)}$ are unknown functions which could be defined after equalizing the multipliers before the same harmonic functions in (30) after replacing (31).

Let the resonance be in the field of a chosen frequence then $k'=1$, $r=g$ and $p_j\approx p_l\approx b_1.h$. After replacing the rows of the functions $u_1^{(n)}$, $n=j,\ l$, in (30) the unknown amplitudes in (31) could be defined as solutions of algebraic systems. Those solutions for $n=j,\ l$ are

$$(32)\qquad v_{01}^{(n)}=m'_n.\frac{H_0^{(n)}}{c^{(n)}},\quad v_{31}^{(n)}=-m'_n.\frac{A_{22}^{(n)}.a_n^3}{c^{(n)}-9.p_n^2},\quad H_m^{(n)}=\sum_{i=1}^{5}H_{im}f_i^{(n)},$$

$$V_m^{(n)}=m'_n.\frac{H_m^{(n)}.\cos h_1}{c^{(n)}-b_m^2.h^2},\quad W_m^{(n)}=m'_n.\frac{H_m^{(n)}.\sin h_1}{c^{(n)}-b_m^2.h^2},\quad m=1,\ 2,\ 3,\ldots.$$

The conditions for the limit disturbed amplitudes must be fulfilled for the amplitudes $v_{11}^{(n)}$ and $v_{21}^{(n)}$. The following systems are obtained for their defining

$$(33)\qquad \frac{c^{(n)}-p_n^2.m'_n}{m'_n}.v_{11}^{(n)}=A_{11}^{(n)}.a_n+H_1^{(n)}.\cos(\acute{y}_n-h_1)$$

$$+\sum_{i=1}^{5}f_i^{(n)}.\left(2.p_n.A_1^{(n)}+a_n.(p_n-b_1.h)\frac{\partial B^{1(n)}}{\partial \acute{y}_n}\right),$$

$$\frac{c^{(n)}-p_n^2.m'_n}{m'_n}.v_{21}^{(n)}=A_{22}^{(n)}.a_n^3+H_1^{(n)}.\sin(\acute{y}_n-h_1)$$

$$+\sum_{i=1}^{5}f_i^{(n)}.\left(2.a_np_n.B_1^{(n)}-(p_n-b_1.h).\frac{\partial A_1^{(n)}}{\partial \acute{y}_n}\right).$$

Render an account of those limiting conditions $A_1^{(n)}$ and $B_1^{(n)}$, $n=j,\ l$, are defined with the expressions

$$(34)\qquad A_1^{(n)}=-\frac{A_{11}^{(n)}}{2.p_n.m_n}.a_n-\frac{H_1^{(n)}.\cos(\acute{y}_n-h_1)}{m_n.(p_n+b_1.h)},\quad m_n=\sum_{i=1}^{5}f_i^{(n)},$$

$$B_1^{(n)}=-\frac{A_{22}^{(n)}}{2.p_n.m_n}.a_n^2+\frac{H_1^{(n)}.\sin(\acute{y}_n-h_1)}{a_n.m_n.(p_n+b_1.h)}.$$

Now it is possible to obtain the evident approximate forms of the differential equations (27) render an account (34)

$$(35)\qquad \frac{da_j}{dt}=-\frac{I_{12}.A_{11}^{(j)}.a_j}{2.I_1.p_j.m_j}-\frac{I_{12}.H_1^{(j)}.\cos(\acute{y}_j-h_1)}{I_1.m_j.(p_j+b_1.h)},$$

$$\frac{da_l}{dt}=-\frac{I_{12}.A_{11}^{(l)}.a_l}{I_1.2.p_l.m_l}-\frac{I_{12}.H_1^{(l)}.\cos(\acute{y}_l-h_1)}{I_1.m_l.(p_l+b_1.h)},$$

$$\frac{d\acute{y}_j}{dt}=p_j-b_1.h-\frac{I_{12}.A_{22}^{(j)}.a_j^2}{2.I_1.p_j.m_j}+\frac{I_{12}.H_1^{(j)}.\sin(\acute{y}_j-h_1)}{I_1.a_j.m_j.(p_j+b_1.h)},$$

$$\frac{d\acute{y}_l}{dt}=p_l-b_1.h-\frac{I_{12}.A_{22}^{(l)}.a_l^2}{2.I_1.p_l.m_l}+\frac{I_{12}.H_1^{(l)}.\sin(\acute{y}_l-h_1)}{I_1.a_l.m_l.(p_l+b_1.h)}$$

and in the resonance field $\sin(\acute{y}_n-h_1)\approx\sin h_1$ and $\cos(\acute{y}_n-h_1)\approx\cos h_1$, which is seen in (35).

For the stationary regimes of the motion when the amplitudes and the phase differences are constant, i. e.

$$\frac{da_j}{dt}=0,\ \frac{da_l}{dt}=0,\ \frac{d\dot{y}_j}{dt}=0,\ \frac{d\dot{y}_l}{dt}=0,$$

a_j and a_l, with the precision of less values, could be represented as functions in the following form

(36) $$a_n=\frac{H^{(n)}}{((p_n-A_2^{(n)}.a_n)^2-b_1^2.h^2)^2+4.b_1^2.h^2.(A^{(n)})^2},\quad A_2^{(n)}=\frac{I_{12}.A_{22}^{(n)}}{2.I_1.p_n.m_n},$$

$$A^{(n)}=\frac{I_{12}.A_{11}^{(n)}}{2.I_1.p_n.m_n},\quad H^{(n)}=\frac{H_1^{(n)}}{I_1.m_n.(p_n+b_1.h)},\quad n=j,\ l,$$

and $\dot{y}_j$ and $\dot{y}_l$

(37) $$\dot{y}_n=h_1+\text{arctg}\frac{A^{(n)}.a_n^2}{p_n-A_2^{(n)}.a_n^2-b_1.h},\quad n=j,\ l.$$

The following expression represents the motion law

(38) $$q=|q_i|^T=\sum_{n=j,l}(d_{4i}(p_n^2).\left(a_n.\cos(\check{r}_n)+\frac{\sin(p_n).t}{p_n}.n_{2i}\right)+d_{3i}(p_n^2).$$

$$(\cos(p_n.t)).n_{1i}+\frac{I_{12}}{I_1}.d_{4i}(p_n^2).(v_{01}^{(n)}+\sum_{m=2}(V_m^{(n)}.\sin(b_m.h.t)+$$

$$W_m^{(n)}.\cos(b_m.h.t))+v_{31}^{(n)}.\cos(3.\check{r}_n)))^T,\quad n=j,\ l.$$

The motion law, thus obtained, can be applied in further investigations for example in defining the periodicity conditions or the conditions for the level motion of the leg joint point with the model body and so on. The application of (22) as initial values in computation procedure leads always to congruent computational procedure for the found system (3). The author's practice in the area of computations shows this procedure is not congruent for every choice of the initial values of the generalized coordinates and velocities.

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