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Delamination of Multilayered Plates in Bending under Monotone Boundary and Nonmonotone Interlayer Conditions. A Variational-Hemivariational Inequality Approach*

1. Introduction

Delamination and stick-slip interlayer phenomena due to interlaminar stresses or free edge effects and highly nonlinear boundary conditions constitute the main strength degradation mechanisms in modern composite material structures which have the form of multilayered plates. In this paper some of the aforementioned phenomena arising in multilayered plates in bending will be modelled, by means of generalized variational techniques recently proposed and studied [1-7].

In particular, delamination phenomena are phenomenologically modelled by means of nonmonotone, generally multivalued laws between interlaminar bonding forces and relative displacements. Partial and total strength degradation phenomena are included in this formulation. These laws are written in a hemivariational inequality form by means of appropriately defined nonconvex and possibly nonsmooth superpotentials and using the generalized gradient operator in the sense of Clarke [3, 5].

On the other hand monotone, generally multivalued laws are introduced in order to describe nonlinear boundary conditions and unilateral phenomena between adjacent layers of the structure. Unilateral contact mechanisms, holonomic plastic and locking hinges and Coulomb friction are among the effects which can be modelled in this way. Convex, generally nonsmooth superpotentials produce the aforementioned laws through the subdifferential operator of convex analysis in the sense of Moreau [4] and result in equivalent variational inequality relations.

Combination of the two aforementioned mechanisms with classical elasticity relations which model the bending behaviour of each separate layer (being here the bending theory of plate relations), gives as a result a variational-hemivariational inequality problem. Existence of the solution and some approximation and regularity results are obtained by means of nonclassical estimates, monotonicity arguments, compactness and average value techniques.

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2. On the mechanical model

2.1. Modelling of each layer

Let us assume for simplicity of the presentation a two-layer plate, as the one shown in Fig. 1. Each plate is referred to a right-handed Cartesian coordinate system $Ox_1x_2x_3$, the middle surface of the plate in the Ox_1x_2 plane being denoted by Ω_j, $j=1, 2$, with boundary Γ_j, $j=1, 2$. Let also the inter-layer bonding material occupy the region $\Omega' \in \Omega_1 \cap \Omega_2$ (planes Ox_1x_2 of various layers are identified for modelling purposes). Kirchhoff bending theory of plates is assumed to describe the behaviour of each separate layer:

$$K_j \Delta\Delta\zeta_j = f_j \text{ in } \Omega_j, \quad j=1, 2. \tag{1}$$

Here K_j is the bending rigidity of j-th layer, ζ_j denotes it's normal deflection and f_j is the total loading perpendicular to the plate. Positive directions are shown in Fig. 1. Moreover f_j is splitted into external loading $\bar{\bar{f}}_j \in L^2 (\Omega_j)$ and binding forces contribution $\bar{f}_j$, i. e.

$$f_j = \bar{f}_j + \bar{\bar{f}}_j, \quad j=1, 2. \tag{2}$$

Extension of the present theory for multilayered [6] and orthotropic [9] plates are possible. Large displacement effects have also been studied in the framework of von Karman plate theory [1, 6, 10]. Use of Mindlin-type plate theory would allow for more accurate modelling of stick-slip interlayer phenomena, as it will be mentioned in the sequel.

2.2. Modelling of interlayer delamination phenomena

Let $[\zeta] = \zeta_1 - \zeta_2$ denote the relative deflection of the two plates. We assume a nonmonotone phenomenological interlaminar law connecting binding interlayer forces

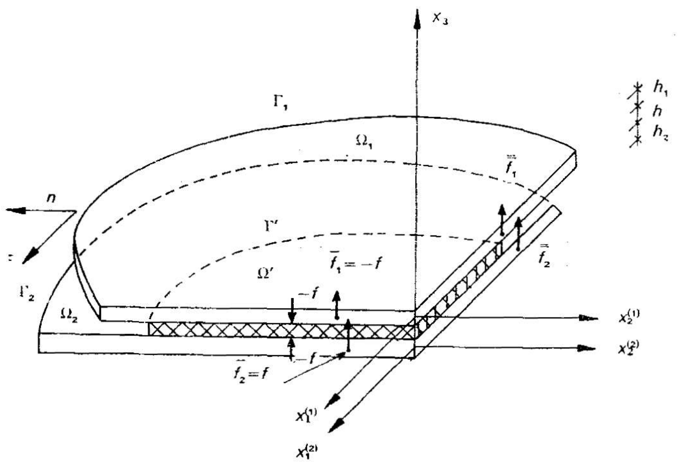

Fig. 1. Two-layer model plate: notations

with relative deflections, as the one depicted in Fig. 2. This law is written in the general form:

$$\bar{f}_{l2} = -\bar{f}_{l1} = -f \in \hat{\beta}([\zeta]) = \bar{\partial} j([\zeta]) \text{ on } \Omega', \tag{3}$$

$$\text{and } \bar{f}_{jl} = 0 \text{ on } \Omega_j - \Omega', \quad j = 1, 2.$$

Here $\bar{\partial}$ denotes the generalized gradient operator in the sense of F. H. Clarke, j(.) is the nonconvex superpotential of the interlaminar law and $\hat{\beta}(.)$ is the nonmonotone multifunction $\hat{\beta}$: $\mathbf{R} \to \rho(\mathbf{R})$, which is obtained in the following way: Let β be a locally bounded, measurable function β: $\mathbf{R} \to \mathbf{R}$, i. e. $\beta \in L^{\infty}_{\log}(\mathbf{R})$. Moreover, for any $\varepsilon > 0$ and $\xi \in \mathbf{R}$ we define the numbers

$$\bar{\beta}_\varepsilon = \operatorname*{esssup}_{|\xi_1 - \xi| < \varepsilon} \beta(\xi) \text{ and } \underline{\beta}_\varepsilon = \operatorname*{esssup}_{|\xi_1 - \xi| < \varepsilon} \beta(\xi), \tag{4}$$

which are increasing and decreasing functions of ε, respectively. Thus the limits, as $\varepsilon \to 0$ exist, $\lim \bar{\beta}_\varepsilon = \bar{\beta}$ and $\lim \underline{\beta}_\varepsilon = \underline{\beta}$, and the multivalued function $\hat{\beta}$: $\mathbf{R} \to \rho(\mathbf{R})$ is defined by:

$$\hat{\beta}(\xi) = [\underline{\beta}(\xi), \bar{\beta}(\xi)]. \tag{5}$$

If $\beta(\xi_{\pm 0})$ exists for every $\xi \in \mathbf{R}$, then the superpotential j can be determined and it is a locally Lipschitz function j: $\mathbf{R} \to \mathbf{R}$. Moreover we use the notion of the generalized directional derivative $j^0(.,.)$, due to Clarke [3], which is defined by:

$$j^0(\xi, z) = \limsup_{\lambda \to 0_+,\, h \to 0} \frac{1}{\lambda} \int_{\xi+h}^{\xi+h+\lambda z} \beta(\tau) d\tau. \tag{6}$$

Relation (3) is by definition equivalent to the local hemivariational inequality:

$$j^0([\zeta], \cdot \xi) \geq -f(\xi), \quad \forall \xi \in R. \tag{7}$$

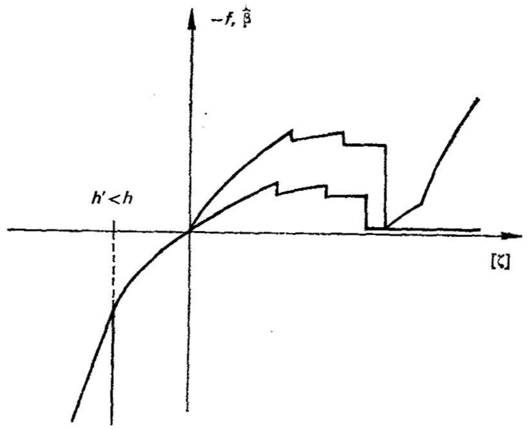

Fig. 2. Nonmonotone phenomenological laws for the delamination problem

2.3. Boundary conditions of monotone type

On parts of the boundary of each plate $\Gamma'_j \subset \Gamma_j$, $\Gamma''_j \subset \Gamma_j$, $j=1, 2$, we assume monotone boundary conditions of the type shown in Fig. 3. As usual in plate theory the boundary conditions relate generalized shearing force with deflection and bending moment with rotation normal to the boundary (see e. g. [2]). The unit normal and tangential vectors to the boundary are denoted by n and τ respectively. By means of appropriate convex. l. s. c. and proper superpotentials σ_{1j}, σ_{2j} the aforementioned laws are written in the following subdifferential form:

(8) $$M_j \in b'_j\left(\frac{d\zeta_j}{dn}\right) = \partial\varphi_{1j}\left(\frac{d\zeta_j}{dn}\right), \quad -\bar{Q}_j \in b''_j(\zeta_j) = \partial\varphi_{2j}(\zeta_j), \quad j=1, 2.$$

Here ∂ denotes the subdifferential operator of convex analysis and b'_j, b''_j, $j=1, 2$ are maximal monotone multivalued operators from $\mathbf{R}$ to $\rho(\mathbf{R})$.
In order to use (8) in the variational framework of the problem the following convex, l. s. c. and proper on $H^2(\Omega)$ (i. e. the appropriate functional framework for plate problem (1), see e. g. [2]) functionals are introduced:

(9) $$\varphi_j(\zeta_j) = \begin{cases} \displaystyle\int_{\Gamma'_j} \varphi_{1j}\left(\frac{d\zeta_j}{dn}\right) d\Gamma + \int_{\Gamma''_j} \varphi_{2j}(\zeta_j) d\Gamma, \\ \text{if } \varphi_{1j}\left(\frac{d\zeta_j}{dn}\right) \in L^1(\Gamma'_j) \text{ and } \varphi_{2j}(\zeta_j) \in L^1(\Gamma''_j), \\ \infty, \text{ otherwize, } j=1,2. \end{cases}$$

Relations (8) are now equivalent to the following variational inequality relation:

(10) $$\varphi_j(z) - \varphi_j(\zeta) \geq \langle \bar{Q}_j, \ (z_j - \zeta_j) \rangle_{\frac{3}{2}} + \langle M_j, \left(\frac{dz_j}{dn} - \frac{d\zeta_j}{dn}\right) \rangle_{\frac{1}{2}}, \quad \forall z_j \in H^2(\Omega_j).$$

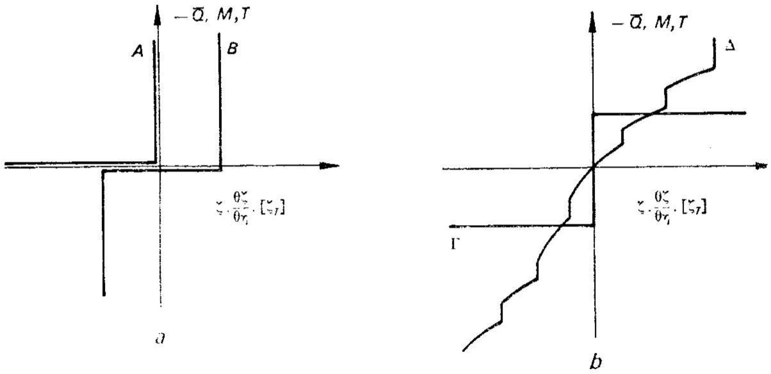

Fig. 3. Monotone boundary conditions and interlayer effects (e. g.: A: unilateral contact, B: locking, Γ: dry-friction-type law)

Here $\langle .,. \rangle_{\frac{3}{2}}$ denotes the duality pairing on $[H^{\frac{-3}{2}}(\Gamma)] X [H^{\frac{3}{2}}(\Gamma)]$ and $\langle .,. \rangle_{\frac{1}{2}}$ is the duality pairing on $[H^{\frac{-1}{2}}(\Gamma)] X [H^{\frac{1}{2}}(\Gamma)]$, according to the trace theorem for the Sobolev spaces $H^2(\Omega_j)$.
In addition we assume that classical homogeneous boundary conditions hold at certain parts of the boundaries such as to guarantee the coerciveness of the plate bending energy bilinear forms $a_j(.,.)$ (i. e. "rigid plate" displacements are excluded). Semicoercive problems are recently studied by the second author [9]. Thus the functional framework is a closed, linear subspace Z_j of the Sobolev space $H^2(\Omega_j)$. Nonhomogeneous boundary conditions are reduced through translation to the homogeneous ones.

2.4. Monotone stick-slip interlayer phenomena

Analogously to relations (8) one may define monotone stick-slip interlayer mechanisms of the following form:

$$-T_\alpha \in \partial \varphi'_\alpha([\zeta_{T\alpha}]). \tag{11}$$

Here T_α denotes the tangential interlayer stress (due to shearing of the binding layer), $\alpha = 1, 2$ runs over coordinate directions and $[\zeta_{T_\alpha}]$ is the relative tangential displacement on direction α, defined by:

$$[\zeta_{T_\alpha}] = \frac{h_1}{2} g_{1.\alpha} - \frac{h_2}{2} g_{2.\alpha} \tag{12}$$

In (12) $g_{j.\alpha}$ is the rotation of the j-th plate along direction α, which in the framework of Kirchhoff plate theory is given by: $g_{j.\alpha} = \frac{d\zeta_j}{dx_\alpha}$, $j=1, 2$, $\alpha=1, 2$. We note that Mindlin-type plate theory, where $g_{j.\alpha}$' s are treated as separate variables, would be more adequate model for the description of this phenomenon. For this reason stick-slip phenomena will not be included in the analysis performed here and is left open for future investigation.

3. Variational formulation of the problem

Multiplication of (1) by appropriate test functions (variations), addition for layers $j=1, 2$, formal application of Green-Gauss theorem and use of the inequality (7) due to the interlaminar law and inequality (10) due to the subdifferential boundary conditions give rise to the following:

Problem 1. Find $\zeta_j \in Z_j \subset H^2(\Omega_j)$, $j=1, 2$, such as to satisfy the following variational-hemivariational inequality:

$$\sum_{=1}^{2} a_j(\zeta_j, z_j - \zeta_j) + \int_{\Omega'} j^0([\zeta], [z]-[\zeta]) d\Omega \tag{13}$$

$$+ \sum_{j=1}^{2} [\Phi - \text{l. c.}_j(z_j) - \Phi - \text{l. c.}_j(\zeta_j)] \geq \sum_{j=1}^{2} \int_{\Omega_j} \bar{\bar{f}}_j (z_j - \zeta_j) d\Omega, \quad \forall z_j \in Z_j, \quad j=1, 2.$$

Inequality (13) expresses the principle of virtual work in inequality form, a well known fact for unilateral problems.

4. Existence and approximation results

In the general case, where the convex superpotential φ_j is nondifferentiable, i. e. grad φ_j does not exist everywhere, we approximate it by means of a sequence of convex Gâteaux-differentiable functionals (φ_{jp}) depending on a real parameter $p \to 0$, and such as to satisfy the following assumptions:

(14a) $$\text{As } p \to 0,\ \varphi_{jp}(z) \to \varphi_j(z),\ \forall z \in Z_j,\ j=1,\ 2;$$

(14b) $$\operatorname{grad} \varphi_{jp}(0)=0,\ j=1,\ 2, \text{ and}$$

(14c) $$\text{if } z_p \to z \text{ weakly in } Z_j, \text{ for } p \to 0, \text{ and}$$
$$\text{if } \varphi_{jp}(z_p) < M_j;\ M_j \text{ is a constant, then}$$
$$\liminf_{p \to 0} \varphi_{jp}(z_p) \geq \varphi_j(z),\quad j=1,\ 2.$$

In addition we regularize the interlaminar law (3) by means a mollifier $p \in C_c^\infty(-1, +1)$, $p>0$, $\int_{-\infty}^{+\infty} p(\xi)d\xi=1$ and a real parameter $\varepsilon>0$, in the following way: $\beta_\varepsilon = p_\varepsilon * \beta$, where $p_\varepsilon(\xi) = \frac{1}{\varepsilon} p\left(\frac{\xi}{\varepsilon}\right)$ and $*$ denotes the convolution product. Moreover discretization governed by parameter n (Galerkin method) is performed and let Z_{jn} denote the corresponding n-dimensional subspace of Z_j. Thus the following regularized and discretized problem is written:

Problem $1_{\varepsilon pn}$: Find ${}_{j\varepsilon pn} \in z_{jn}$, $j=1,\ 2$, such as to satisfy the following variational equality

(15) $$\sum_{j=1}^{2} \alpha_j(\zeta_{j\varepsilon pn}, z_j) + \int_{\Omega'} \beta_\varepsilon([\zeta_{\varepsilon pn}], [z])d\Omega$$
$$+\sum_{j=1}^{2} \langle \operatorname{grad} \varphi_{jp}(\zeta_{j\varepsilon pn}), z_j\rangle = \sum_{j=1}^{2} (\bar{\bar{f}}_j, z_j)\ \forall z_j \in Z_{jn},\quad j=1,\ 2.$$

Proposition 1. On the following "ultimate increase" assumption,

(16) $$\operatorname*{ess\,sup}_{(-\infty,\,-\zeta)} \beta(\xi) \leq \operatorname*{ess\,sup}_{(+\infty,\,+\zeta)} \beta(\xi),\quad \xi \in \mathbf{R},$$

and for $\bar{\bar{f}}_j \in L^2(\Omega_j)$, problem 1 has a solution.

In order to prove proposition 1 we first write problem (15) in the following form: $(\Lambda(\tilde{\zeta}_{\varepsilon pn}), \tilde{z}) = 0$, $\forall \tilde{z} \in Z_{1n} X Z_{2n}$, where Λ denotes the corresponding continuous operator Λ: $Z_{1n} X Z_{2n} \to Z_{1n} X Z_{2n}$. Due to (16) one can find $p_1>0$ and $p_2>0$ such that:

$$|\beta_\varepsilon(\xi)| \leq p_2 \text{ for } |\xi| \leq p_1,\ \beta_\varepsilon(\xi) \leq 0 \text{ for } \xi > p_1$$

and $\beta_\varepsilon(\xi) \geq 0$ for $\xi \leq -p_1$.

Thus the following estimate is obtained:

(17) $$\int_{\Omega'} \beta_\varepsilon([\zeta_{\varepsilon pn}])[\zeta_{\varepsilon pn}]d\Omega = \int_{\geq|[\zeta_{\varepsilon pn}]|\geq p_1} \beta_\varepsilon([\zeta_{\varepsilon pn}])[\zeta_{\varepsilon pn}]d\Omega + \int_{|[\zeta_{\varepsilon pn}]|<p_1} \beta_\varepsilon([\zeta_{\varepsilon pn}])[\zeta_{\varepsilon pn}]d\Omega$$
$$\geq 0 - p_1 p_2 \,\text{mes}\, \Omega.$$

Coerciveness of energy bilinear forms $\alpha_j(.,.)$ and monotonicity of grad φ_j combined with assumption (14b) give rise to the following estimates:

(18) $$\alpha_j(\zeta_j, \zeta_j) \geq c_j \|\zeta_j\|^2_{H^2}, \quad \forall \zeta_j \in Z_j, \; c_j\text{: const} > 0, \text{ and}$$

(19) $$\langle \text{grad}\, \varphi_{jp}(\zeta_{j\varepsilon pn}) - \text{grad}\, \varphi_j(0), \zeta_{j\varepsilon pn} - 0\rangle \geq 0, \; \forall \zeta_{j\varepsilon pn} \in Z_{jn}, \quad j=1, 2,$$

respectively (17) (18) (19) permit us to write the following estimate

(20) $$(\Lambda(\bar{\zeta}_{\varepsilon pn}), \bar{\zeta}_{\varepsilon pn})) \geq \sum_{j=1}^{2} c_j \|\zeta_{j\varepsilon pn}\|^2_{H^2} + 0 - p_1 p_2 \,\text{mes}\, \Omega' - \sum_{j=1}^{2} c_j \|\zeta_{j\varepsilon pn}\|_{H^2}.$$

From (20) and Brouwer's fixed point theorem we prove the existence of at least one solution of (15) with $\|\zeta_{\varepsilon pn}\| \leq c$, $j=1, 2$, c constant. Note here that possible multiplicity of solutions is not excluded due to nonconvexity of debonding law.
As $(\zeta_{j\varepsilon pn})$ is bounded in Z_j we may extract a subsequence (again denoted by $(\zeta_{j\varepsilon pn})$) such that in the limit $\varepsilon \to 0$, $n \to \infty$, $p \to 0$,

$\zeta_{j\varepsilon pn} \to \zeta_j$ weakly in Z_j and strongly in $L^2(\Omega_j)$, i. e. almost everywhere in Ω_j.

Through appropriate application of Dunford-Pettis compactness criterion the weak precompactness of $\beta_\varepsilon([\zeta_{\varepsilon pn}])$ in $L^1(\Omega')$ can be shown. To this end inequality:

$\xi_0 |\beta_\varepsilon(\xi)| \leq |\beta_\varepsilon(\xi)\xi| + \xi_0 \sup_{|\xi|\leq\xi_0} |\beta(\xi)|$, estimate (17) and the following relation, due to the regularization of β are used: $\sup_{|\xi|\leq\xi_0} |\beta_\varepsilon(\xi)| \leq \operatorname{esssup}_{|\xi|\leq\xi_0+1} |\beta(\xi)|$.

Thus in the limit of $\varepsilon \to 0$, $n \to \infty$, a subsequence again denoted by $(\beta_\varepsilon([\zeta_{\varepsilon pn}]))$ exists such that $\beta_\varepsilon([\zeta_{\varepsilon pn}]) \to X_p$ weakly in $L^1(\Omega')$ and as $p \to 0$, $X_p \to X$ weakly in $L^1(\Omega')$. Moreover, from (15) we obtain that $\|\text{grad}\, \varphi_{ip}(\zeta_{\varepsilon pn}\|_Z < c$, thus a subsequence exists such that grad $\varphi_{jp}(\zeta_{\varepsilon pn}) \to \Psi_{jp}$, weakly in Z'_p, $j=1, 2$, and $\Psi_{jp} \to \psi_j$ weakly in Z'.
From (15) we get in the limit of $\varepsilon \to 0$, $n \to \infty$

(21) $$\sum_{j=1}^{2} \alpha_j(\zeta_{jp}, z_j) + \int_{\Omega'} X_p[z]d\Omega + \sum_{j=1}^{2} \langle \Psi_{jp}(\zeta_{jp}), z_j\rangle = \sum_{j=1}^{2} (\bar{\bar{f}}_j, z_j), \; \forall z_j \in Z_j, \; j=1, 2.$$

In order to prove that

(22) $$\Psi_{jp} = \text{grad}\, \varphi_{jp}(\zeta_{jp}) \text{ in } Z'_j,$$

we formulate the nonnegative, due to the monotonicity, expression:

(23) $$X_n = \sum_{j=1}^{2} \langle \text{grad}\, \varphi_{jp}(\zeta_{j\varepsilon pn}) - \text{grad}\, \varphi_j(g_j), \zeta_{j\varepsilon pn} - g_j\rangle \geq 0, \quad \forall g_j \in Z_j, \quad j=1, 2.$$

Using (15), the fact that (as $\varepsilon \to 0$, $n \to \infty$, $p \to 0$)

$$\lim \int_{\Omega'} \beta_\varepsilon([\zeta_{\varepsilon pn}])[\zeta_{\varepsilon pn}]d\Omega \to \int_{\Omega'} X[\zeta]d\Omega, \text{ and (21), we get in the limit of } \varepsilon \to 0,$$

$n \to \infty$ from (23):

$$(24) \qquad \sum_{j=1}^{2} [\langle \Psi_{jp}, \zeta_{jp} - g_j \rangle - \langle \operatorname{grad} \varphi_{jp}(g_j), \zeta_{jp} - g_j \rangle] \geq 0, \quad \forall g_j \in Z_j.$$

Taking the limit $\varepsilon \to 0$, $p \to 0$, $n \to \infty$ of (30) and applying Lebesque's theorem as $\mu \to 0$ one obtains that

$$\underline{\beta}([\zeta]) \leq X \leq \overline{\beta}([\zeta]) \text{ a. e. in } \Omega' - \omega,$$

which by taking α arbitrarily small shows (27).
For details of the outlined proof we refer the reader to [8].
Moreover, the following regularity result can be proved:
Proposition 2. On the growth assumptions: There exist constants c, c_1, c_2, c_3, $c_4 > 0$ and index $q \geq 1$ such that

$$|\beta(\xi)| \leq c(1 + |\xi|^q), \quad \forall \xi \in R, \ q \geq 1, \text{ constant } c > 0,$$

$$|b_1'(\xi)| \leq c_1(1 + |\xi|), \ |b_{1p}'(\xi)| \leq c_2(1 + |\xi|), \ \forall \xi \in R, \text{ constants } c_1, c_2 > 0,$$

$$|b_2''(\xi)| \leq c_3(1 + |\xi|^{q_1}), \ |b_{2p}''(\xi)| \leq c_4(1 + |\xi|^{q_2}), \ \forall \xi \in R, \text{ constants } c_3, c_4 > 0.$$

Then in the limit $\varepsilon \to 0$, $n \to \infty$, $p \to 0$ the following holds:

$$\xi_{j\varepsilon pn} \to \xi_j \text{ strongly in } Z_j \subset H^2(\Omega_j), \quad j = 1, 2.$$

The proof of proposition 2 can be found in [2, 6—10].

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