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Condition for Retaining Equilibrium during the Motion of a Wheel-chair

1. Formulation of the problem

As a result of the scientific and technological progress, in the recent years an ever increasing application of manipulating systems has been witnessed. According to their classification, they are automatic, biotechnical and interactive. An example of a biotechnical manipulation system, which is continuously controlled by the man-operator, is the wheel-chair.

It will be considered as a three-body system, comprising a body and two rear wheels, which are the drive. The two front steering wheels are assumed to have negligible masses as compared with the overall mass of the system. According to the structural design, they can rotate not only around their own axis, but also they can swivel around an axis rigidly fixed to the body and perpendicular to the horizontal plane within which the system moves. Since the front wheels only play the role of a support of the system during motion, it can be accepted that in these fulcrums a smooth sliding is realized.

The drive of the system is realized by two electric motors, mounted on the body, which can transfer independent torques to the two rear wheels. The electric motors are controlled by a device, which connects unambiguously a controlling lever with two degrees of freedom by their angular speeds. This paper aims at determining the conditions for retaining equilibrium during the movement of the system.

2. Equilibrium conditions during turning

One of the typical motions of the system is turning. The turning is a result of inequality between the angular speeds of the rear wheels, which are denoted by $\vec{\omega}_A$ and $\vec{\omega}_B$ with respect to the body. For the sake of definiteness, let $|\vec{\omega}_A|>|\vec{\omega}_B|>0$. The momentary motion of the body can be interpreted as a rotation with angular velocity $\vec{\omega}$ around an axis, which passes through the moment center of rotation O and is perpendicular to the plane of motion, while an adequately small part of the trajectory of the mass centre passes as an arc of a circle with centre O and radius R (Fig. 1). In the figure are also indicated: M — mass centre of the system, assumed to be in the centre of the axis of the rear wheels A and B; $2a$ — length of the rear axis; $\vec{v}_M$, $\vec{v}_A$ and $\vec{v}_B$ — speeds of the mass center and the centers of the wheels A and B.

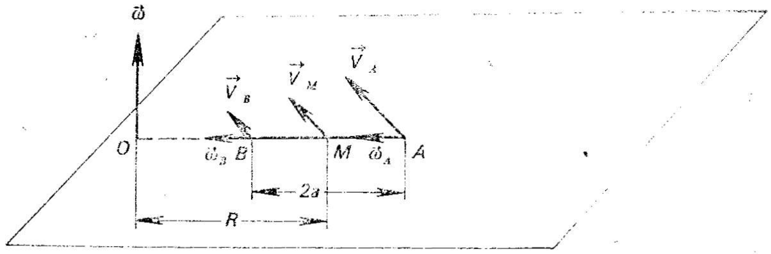

Fig. 1

At different ratios between $|\vec{\omega}_A|$ and $|\vec{\omega}_B|$, the radius of the turn is changed between zero and infinity, i. e. the body is swivelled at the site $(O \equiv M)$, it turns or moves linearly.

Let $Oxyz$ be a right inertial system, connected to the plane of motion and z be perpendicular to it. We will consider another right orthogonal coordinate system $Ox_1y_1z_1$ with origin also at point O, whose axis x_1 coincides with axis AB of the system, while axis z_1 is parallel to $\vec{\omega}$ and it coincides with z. Then, in the considered planar motion, the coordinate system $Ox_1y_1z_1$ is swivelled with respect to $Oxyz$ around the axis $z \equiv z_1$ with an angular speed $\vec{\omega}$, while the body remains immobile towards it, i. e. in equilibrium.

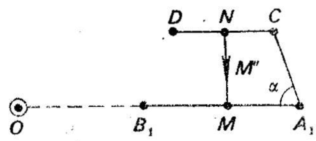

Fig. 2

The orthogonal projection of the system within the plane of motion is shown in Fig. 2. A_1 and B_1 indicate the contact points of the rear wheels with the plane, C and D of the front ones and α is the angle between A_1B_1 and A_1C.

Equilibrium will be lost when the ratio between $|\vec{\omega}_A|$ and $|\vec{\omega}_B|$ reaches a point when the wheels B, D will be detached from the plane of motion, i. e. the body wil change its position with respect to the system $Ox_1y_1z_1$. Since $Ox_1y_1z_1$ is a non-inertional system, equilibrium in it will be attained when the sum of all forces, inertional ones included, and their momentums is zero. In $Ox_1y_1z_1$ the axis A_1C is immobile and the equation for the relative equilibrium of the moments with respect to the axis is

$$\vec{\mathbf{T}}+\vec{\mathbf{T}}_e=\vec{\mathbf{O}}, \tag{1}$$

where $\vec{\mathbf{T}}$ is the moment of the weight force $|\vec{\mathbf{P}}|=mg$, m — the full mass of the system and $\vec{\mathbf{T}}_e$ — the moment of the centrifugal force $|\vec{\mathbf{S}}_e|=m\omega^2R$ [1].

We obtain for the value of the moments $|\vec{\mathbf{T}}_e|$ and $|\vec{\mathbf{T}}|$ the following equation, where r is the radius of the rear wheels:

(2) $$|\vec{\mathbf{T}}_e| = mr\omega^2 R \sin\alpha\,;$$

(3) $$|\vec{\mathbf{T}}| = amg \sin\alpha.$$

Taking into consideration the signs of the moments in (1), after substitution of (2) and (3) in it, we obtain

(4) $$mr\omega^2 R \sin\alpha = amg \sin\alpha$$

when, obviously, equilibrium will not be lost if the inequality

(5) $$\omega^2 R \leq \frac{ag}{r}$$

holds.

After expressing the speeds of points A, B and M by ω, R and α, on the one hand, and by r, ω_A and ω_B, on the other, we obtain the following expressions for R and ω (further on the values of the vectors will be written without arrows and sign for the module).

(6) $$R = \frac{\omega_A + \omega_B}{\omega_A - \omega_B} \cdot a\,;$$

(7) $$\omega = \frac{r}{2a}(\omega_A - \omega_B).$$

They are substituted in (5) and after reworking the condition for equilibrium acquires the following form:

(8) $$\omega_A^2 - \omega_B^2 \leq 4g\frac{a^2}{r^3} = \text{const}.$$

The substraction term on the left side of (8) acquires maximum value when $\omega_B = 0$, so that the safest criterion for the retention of equilibrium during turning is

(9) $$\omega_A \leq 2\frac{a}{r}\sqrt{\frac{g}{r}} = \text{const},$$

where the constant on the right side of (8) and (9) totally depends on the parameters of the system design.

For comparison we can provide a numerical example: when $r = 0.36$ [m], $2a = 1{,}1$ [m], $\omega_A \leq 1$ [sec^{-1}], if the speed of the mass center does not exceed 7.2 $\left[\frac{\text{km}}{\text{h}}\right]$ the equilibrium conditions are not breached.

However, in practical applications if a man is sitting in the wheel-chair, the mass center may be changed. Let us indicate the new position with M' and the value of the component of the vector MM', perpendicular to the plane of motion with δ. If we select such a δ_{max}, that the condition $\delta < \delta_{max}$ is always fulfilled (practically $\delta_{max} \approx 0.5$ m), then in a similar way we obtain the final criterion for equilibrium:

(10) $$\omega_A \leq 2\frac{a}{r}\sqrt{\frac{g}{r + \delta_{max}}} = \text{const}.$$

Let us indicate with M'' on Fig. 2 the projection of the mass center of the plane of motion and presume that it is posed on the axis MN (N — the middle of CD). The reasoning up to now remains valid also during the new position of the mass center. If we indicate $MN=b$ and $MM''=d$, then in a similar way we reach a criterion for retaining equilibrium during linear motion during starting and stopping. They respectively are as follows:

$$\varepsilon_A \leq \frac{dg}{r\,(r+\delta_{max})} = \text{const}; \tag{11}$$

$$\varepsilon_A \leq \frac{(b-d)\,g}{r\,(r+\delta_{max})} = \text{const}, \tag{12}$$

where ε_A is the value of the angular acceleration of the wheel A with respect to the body.

The design of the system may be realized so that d remains approximately a constant parameter, as well as in the case when somebody is sitting in the wheel-chair. It may acquire a new, smaller value, with a certain safety coefficient. Then, in conformance with the restrictions (10), (11) and (12) the wheel-chair will not be out of equilibrium during its motion.

3. Device for the control of equilibrium

The radius of the turn performed by the system can be presented in the following form:

$$R=\frac{\omega_A+\omega_B}{\omega_A-\omega_B}\,.\,a=\frac{\frac{\omega_A}{k}+\frac{\omega_B}{k}}{\frac{\omega_A}{k}-\frac{\omega_A}{k}}\,.\,a, \tag{13}$$

from where follows that if ω_A and ω_B are simultaneously multiplied by one and the same coefficient, the radius of the turn is not changed — only the speed of turning is altered. The practical realization of the device is based on the criterion (8). The site, where the device is switched, is shown in the block circuit diagram (Fig. 3).
The indications in Fig. 3 are the following: *1* — two—coordinate control transducer; *2* — control block; *3* — device for preventing overturning; *4*, *5* — servoblock; *6*, *7* — motor.

Through the two-coordinate control transducer the operator's commands are transformed into signals for assigning the direction and speed of motion of the system. These electrical signals are sent to the control block, which transforms them according

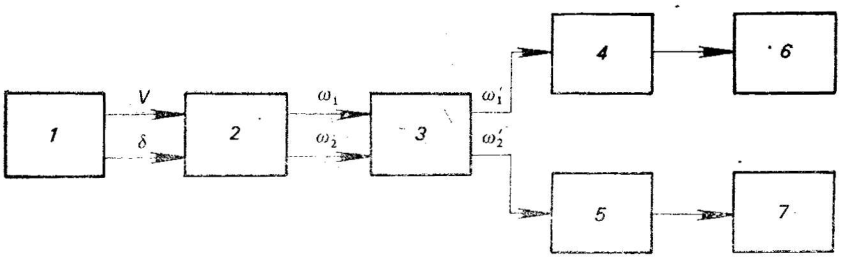

Fig. 3

to a certain algorithm and transmits them to two servoblocks, controlling the direction and rotational speed of the two DC electrical motors, coupled by reducing gears to the driving wheels of the system. Before being sent to the servoblocks, the control signals pass through a device for the control of equilibrium, where it is checked whether the assignments for the speed of the electrical motors (ω_1 and ω_2) are implemented. The equilibrium checking block changes the initial signals from the control block and instead it sends the signals ω_1' and ω_2', respectively, with values:

$$\omega_1' = \frac{\omega_1}{k}, \quad \omega_2' = \frac{\omega_2}{k}, \quad k > 1,$$

to servoblocks 1 and 2.

The device does not introduce restrictions on the control of the system, when criterion (8) is observed. If the operator assigns a command breaching this criterion, the speed of the wheels will be reduced to a value, which will again preserve equilibrium conditions (8). Meanwhile, the assigned radius of the turn will be retained. The device evaluates the expedience of the issued commands prior to their execution, and never after the equilibrium of the system has been breached.

The block diagram of the device for the control of equilibrium is shown in Fig. 4, where the following indications are used: *1*, *2* — divider; *3*, *4* — multiplier; *5* — summator; *6* — full wave rectifier; *8* — source of reference signal; *7* — amplifier; *9* — half-wave rectifier.

The output signals from the control block (ω_1, ω_2) pass through dividers *1* and *2* to the inputs of the first and second servoblocks. Each output of dividers *1* and *2* is

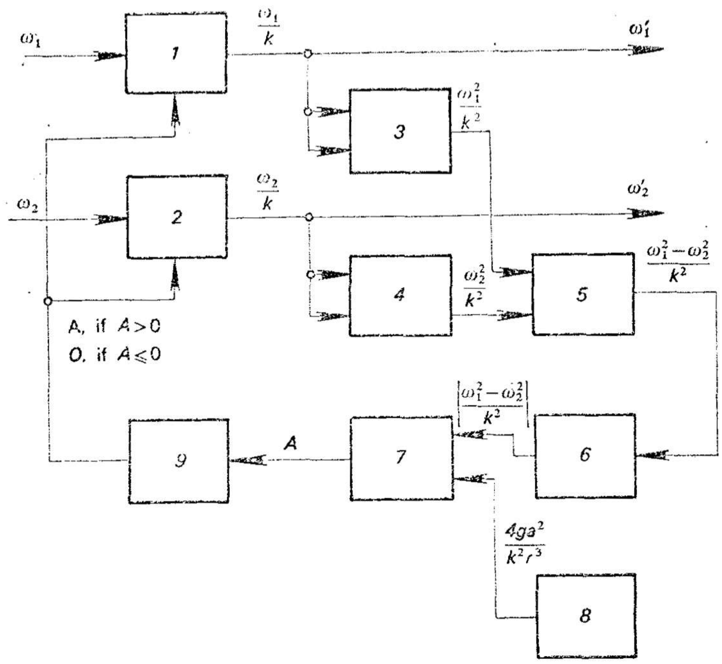

Fig. 4

connected to the two inputs of the multipliers (*3* and *4*). The signals at the output of the multipliers are $\frac{\omega_1^2}{k^2}$ and $\frac{\omega_2^2}{k^2}$, respectively. At the output of summator *5* the signal $\frac{1}{k^2}(\omega_1^2-\omega_2^2)$ is obtained. This signal is rectified by the full wave rectifier — and the signal $\left|\frac{\omega_1^2-\omega_2^2}{k^2}\right|$ is obtained at its output. The difference between this signal and the signal from reference source *8* is amplified by amplifier *7*. When this difference is positive, it is sent to dividers *1* and *2* through the half-wave rectifier.

The device operates as follows: when the input signals observe criterion (8), the output signal from block *7* is not let through rectifier *9*. Dividers *1* and *2* divide the signals sent, by a coefficient of unity, i. e.

$$\omega_1'=\omega_1,\ \omega_2'=\omega_2.$$

If the assignments for speed breach criterion (8), block 9 lets through the signal coming from block 7. Simultaneously and equally the division coefficients of blocks 1 and 2 are changed. The assignments for speed ω_1 and ω_2 are decreased. When the polarity of the signal from block 7 is changed, rectifier *9* no longer lets through the signals from amplifier *7*. The transmission coefficient of amplifier *7* is selected on the basis of stability considerations for the operation of the electronic circuitry.

References

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