BULGARIAN ACADEMY OF SCIENCES
JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
Sofia, 1991, Year XXII, No 3

I. Kochev

Design of a Gear Drive for Minimal Crankshaft Reaction in Planar Linkages

1. Introduction

High speeds, heavy loads, complete force balancing and other factors may cause considerable rise of the joint loads in linkages and several technique have been developed to prevent or restrain this negative trend. The application of springs or similar devices is a well documented practice, and in high speed machines similar goals can be pursued by the attachement of balancing masses. In the latter case, the control over the reactions is generally examined as a part of multi-objective optimization strategies [1-8] and as these investigations confirmed, the reduction of a given reaction is often achieved at the expense of other dynamic factors. The effectiveness of such procedures depends largely on the imposed constraints, but within reasonable limits. The ultimate reduction of some reactions appears to be around 50% level [7] and a more realistic estimate is probably 25% level. Indirect indications for such assessment provides [9], where certain reactions have their values if the counterweights are dismantled, and similar view is supported in [11] and other sources.

For the sake of completeness we shall finally mention two papers standing aside from the main stream of studies. Conte et al. combined the control of the reactions with kinematic synthesis [10], while Kochev [12] reports multifold reduction of the joint loads in symmetrical linkages if driven by symmetrical input motions.

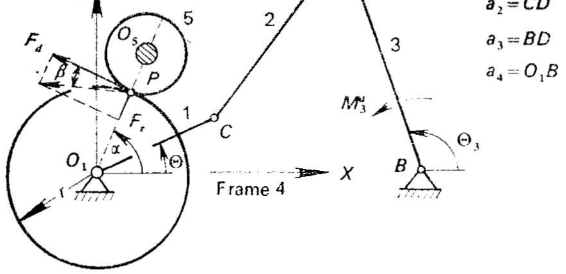

Fig. 1

Devoted to the same general topic this study offers an alternative design technique which in some situations is proved to be very rewarding. The concept presupposes planar linkages driven by circular or noncircular gearset, demands steady state motion, and formulates nontrivial optimization problem if variable forces, inertia or otherwise are dominant. Design objective is the minimization of the root mean square reaction of the crank ground bearing, which in most cases is the heavily loaded joint [13]. Design variables are the gearset centre distance $0_1 0_5$ and/or the orientation angle of the central axis (Fig. 1). The procedure does not affect other joint loads in contrast to the strategies based on mass redistribution leaving unchanged all other dynamic factors. Numerical analysis of a family of crank-rockers confirms the effectiveness of the proposed technique in a wide range of situations.

2. General considerations

Fig. 1 presents a machine driven by a gear device. The crank-rocker serves as a convenient illustration and noncircular gearset is a possible alternative. The location of the pitch point P is defined by the centre distance $d=0_1 0_5$, the orientation angle α and the prescribed gear transmission function or ratio. In term of the driving torque $T(\theta)$ which is a known function of the input angle θ, the driving force F_d is readily given by the expression

(1) $$F_d=\frac{T_g(\theta)}{d}; \quad T_g(\theta)=\frac{T(\theta)}{r(\theta)/d},$$

where the gear radius r is generally variable. Ratio $r(\theta)/d$ depends only on the specified transmission function, and the "generilized torque" $T_g(\theta)$ is introduced for further convenience.

The radial force $F_r(\theta)$ is given by $F_r(\theta)/F_d(\theta)=\tan\beta(\theta)$, and in the case of noncircular gears the pressure angle β is variable but known function of θ. From (1) readily follows

(2) $$F_r=\frac{T_g(\theta)}{d}.\tan\beta(\theta).$$

Further, the reaction components $R_x^0(\theta)$, $R_y^0(\theta)$ acting on the crank pivot O_1 and correspondent to a direct drive could be assumed as known. For a gearset drive their respective values $R_x(\theta)$ and $R_y(\theta)$ are apparently produced by the equilibrium equations

$$R_x-R_x^0+F_d\cos(\alpha+\pi/2)+F_r\cos(\alpha+\pi)=0,$$

$$R_y-R_y^0+F_d\sin(\alpha+\pi/2)+F_r\sin(\alpha+\pi)=0.$$

Replacing (1) and (2), and for solving the total reaction further follows:

(3) $$R^2(\theta)=F_0(\theta)+\frac{2}{d}[F_1(\theta)\cos\alpha+F_2(\theta)\sin\alpha]+\left(\frac{1}{d}\right)^2 F_3(\theta),$$

where

(4a) $$F_0=(R_x^0(\theta))^2+(R_y^0(\theta))^2\geq 0;$$

(4b) $$F_1=T_g(\theta)[R_x^0(\theta).\tan\beta(\theta)-R_y^0(\theta)];$$

(4c) $$F_2 = T_g(\theta)\,[R_x^0(\theta) + R_y^0(\theta) \,.\, \tan\beta(\theta)];$$

(4d) $$F_3 = [1 + \tan^2\beta(\theta)]\; T_g^2(\theta) \geq 0.$$

The integration of expression (3) and the introduction of the integrals:

$$I_0 = \frac{1}{2\pi}\int_0^{2\pi} F_0(\theta)\,.\,d\theta > 0,\quad I_1 = \frac{1}{2\pi}\int_0^{2\pi} F_1(\theta)\,.\,d\theta,$$

(5) $$I_2 = \frac{1}{2\pi}\int_0^{2\pi} F_2(\theta)\,.\,d\theta,\quad I_3 = \frac{1}{2\pi}\int_0^{2\pi} F_3(\theta)\,.\,d\theta > 0,$$

finally results in the root mean square (rms) reaction

(6) $$\text{rms}\; R^2 = I_0 + \frac{2}{d}(I_1 \cos\alpha + I_2 \sin\alpha) + \left(\frac{1}{d}\right)^2 I_3.$$

3. Optimization

The bearing load (6) is function of two controlled parameters α and d. From the derivative rule $\partial\, rmsR^2/\partial\alpha = 0$ there follows

(7) $$\tan\hat{\alpha} = I_2/I_1$$

and consequently the couple of critical angles

(8) $$\hat{\alpha}_1,\ \hat{\alpha}_2 = \hat{\alpha}_1 + \pi.$$

The reaction has a minimum at angle $\hat{\alpha}$ which makes positive the second derivative or which satisfies the equivalent condition

(9) $$S = I_1 \cos\hat{\alpha} + I_2 \sin\hat{\alpha} < 0.$$

Equation (7) also provides the expressions

$$\cos\hat{\alpha} = \pm I_1 \,.\, (I_1^2 + I_2^2)^{-\frac{1}{2}},\quad \sin\hat{\alpha} = \pm I_2 (I_1^2 + I_2^2)^{-\frac{1}{2}},$$

whose signs are independent but related to the signs of I_1 and I_2. Simple analysis of the four alternatives confirms the following:

If $I_1 < 0$, then $rmsR$ has a minimum at $\hat{\alpha}_1$ (principal value);

If $I_1 > 0$, then $rmsR$ has a minimum at $\alpha_2 = \hat{\alpha}_1 + \pi$;

If $I_1 = 0$, then $rmsR$ has a minimum at $\hat{\alpha} = \pi/2$ if $I_2 < 0$, and at $\hat{\alpha} = -\pi/2$ if $I_2 > 0$.

In all cases expression (9) results to

(10) $$S = -\sqrt{I_1^2 + I_2^2}\,.$$

Further investigations based on equations (9), (10) and property $I_3>0$ show that the optimum angle $\hat{\alpha}$ may always nullify the partial derivative

$$\partial\, rmsR^2/\partial(1/d)=2(I_1 \cos \alpha+I_2 \sin \alpha+I_3/d),$$

producing in effect the optimum center distance

$$\hat{d}=\frac{I_3}{\sqrt{I_1^2+I_2^2}}. \tag{11}$$

In the case of circular gears we similarly have the optimum radius

$$\hat{r}=\frac{I_3}{\sqrt{I_1^2+I_2^2}}\cdot\frac{r}{d}, \tag{12}$$

which is in fact independent of r/d, since the inverse of this fraction is a multiplier in the expressions of I_1, I_2, I_3 via the generalized torque (1).

Brief inspection of the integrands (4b) and (4c) also indicates that a simultaneous nullification of the integrals I_1 and I_2 is practically impossible. Equations (11) and (12) thereby provide always feasible solutions and for many specific situations possibly quite realistic dimensions.

4. Numerical analysis

The concept is illustrated by a family of force balanced crank-rockers driven by a circular gear unit at constant speed $\dot{\theta}=1$ rad/s (Fig. 1). Original links have standard configurations [9] and centroids located at their midpoints. The analysis is based on material of conventional density $\rho=1$ kg/m^3 and step size of $\Delta\theta=4$ deg. Table 1 compiles the results for two optimization problems:

Problem A. Design parameters are α and r.

Problem B. Design parameter is α while the radius r is fixed, and without any specific motivation is assumed to be half of the crank length ($r=0.5$). Problem B pro-

Table 1

4R crank-rockers with standard mass configurations [9]

Linkage	Dimensions $a_1=1$ m a_2	a_3	a_4	Type of forces	*rms* torque Nm	Problem A opt α deg	Problem A opt r m	Problem B $r=0.5$ m I_A	Problem B $r=0.5$ m I_B
1	2	3	3	*I*	0.47	−16.2	1.03	0.52	0.49
				I&A	4.10	166.9	0.97	0.17	0.34
2	5	3	4	*I*	0.69	−69.2	1.15	0.69	0.61
				I&A	16.56	106.0	0.65	0.69	0.65
3	5	3	6	*I*	0.36	31.0	5.72	0.98	0.91
				I&A	3.13	109.3	0.97	0.14	0.33
4	5	5	2	*I*	7.23	−28.1	0.85	0.29	0.33
				I&A	14.10	131.0	0,44	0.65	0.68

vides a reasonable alternative when the optimum radius in problem A violates possible design restrictions.

In both cases the rate of reduction is measured by the indices

(13) $$I_A = \frac{\min \, rms \, R(\alpha, r)}{\max \, rms \, R(\alpha, r)};$$

(14) $$I_B = \frac{\min \, rms \, R(\alpha)}{\max \, rms \, R(\alpha)}, \quad r = 0.5 = \text{const}$$

listed in columns 7 and 8 of Table 1. The latter displays four representative linkages of the investigated family [9], among which the optimal radii and indices I_A and I_B reach ultimate values (boldface figures). All of these linkages experience inertia forces or an additional active resistance couple applied on the rocker 3 and defined by the expressions:

(15) $$M_3^a = 200(\max \theta_3 - \theta_3)(\theta_3 - \min \theta_3) \text{ if } \dot{\theta} \leq 0,$$

$$M_3^a = 0 \text{ if } \dot{\theta}_3 > 0.$$

The couple produces multifold increase of the input torque correspondent to the inertia forces only (Column 4). Column 3 marks the type of forces under consideration: inertia *I*, or inertia and active *I&A*.

Inspection of Table 1 allows the following observations:

(a) In most cases the proposed technique provides greater reduction of the reaction compared with the alternative methods mentioned in our introductory notes.

(b) In some cases the indices I_A and I_B become very small (linkage 3, *I&A* forces). Therefore, an ill motivated orientation of the center line may cause substantial disadvantage.

(c) Optimum radii vary considerably from moderate values (linkage 4) to apparently unacceptable figures (linkage 3, *I* forces). The resistance couple M_3^a smooths these differences and the subsequent procedure provides much more realistic results.

5. Conclusion

As simplified models of real phenomena, the optimization problems are never fully isolated from the entire design process. In this regard, the technique discussed here is essentially less restrained. Furthermore, it covers larger area of situations than it was initially stated. A gearset is certainly a prerequisite, but it could be attached to any link with rotary motion, and not necessarily as a drive unit. Therefore, geared linkages and mechanisms with gear balancers [14-16], among many others, may benefit from the same concept.

References

1. Sadler, J. P., R. W. Mayne. Balancing of mechanisms by nonlinear programming. — In: 3rd Appl. Mech. Conf., Oklahoma State Univ. 1973, 29. 1-29. 17.
2. Sadler, J. P. Balancing of six-bar linkages by nonlinear programming. — In: 4th World. Cong. on Theory of Machine and Mechanisms, Newcastle-Upon-Tyne, 1975, 139-144.
3. Tepper, F. R., G. G. Lowen. Shaking force optimization of four bar linkage with adjustable constraints on ground bearing forces. — Trans. ASME, J. Engng. Ind., 97, 1975, 643-651.
4. Tricamo, S. J., G. G. Lowen. Simultaneous optimization of dynamic reactions of a four-bar linkage with prescribed maximum shaking force. — Trans. ASME, J. Mech. Trans. Autom. Des., 105, 1983, 520-525.
5. Porter, B., D. J. Sanger. Synthesis of dynamically optimal four-bar linkages. — Proc. Conf. Mechanisms 1972, Paper, C69/72, 24-28; Institutions of Mechanical Engineers, 1972.
6. Dresig, H., S. Schönfeld. Rechnergestützte Optimierung der Antriebs- und Gestelkratgrössen ebener Koppelgetriebe — Teile I, II. Mech. Mach. Theory, 11, 1976, 363-379.
7. Walker, M. J., R. S. Haines. A study of counterweight synthesis for a 6-bar chain. — Mech. Mach. Theory, 17, 1982, No5, 327-333.
8. Lee, T. W., C. Cheng. Optimum balancing of combined shaking force, shaking moment, and torque fluctuations in high-speed linkages. — Trans. ASME, J. Mech. Trans. Autom. Des., 106, 1984, No 2, 242-251.
9. Lowen, G. G., F. R. Tepper, R. S. Berkof. The quantitative influence of complete force balancing on the forces and moments of certain families of four-bar linkages. — Mech. Mach. Theory, 9, 1974, No 3/4, 299-323.
10. Conte, F. L., G. R. George, R. W. Mayne, J. P. Sadler. Optimum mechanism design combining kinematic and dynamic-force considerations. — Trans ASME, J. Engng. Ind., 97, 1975, 662-670.
11. Kochev, I. S., G. Gurdev. Minimization of the reactions of a family of 4R crank-rockers by balancing masses. — Research Work Reports of the TU "A. Kanchev", Russe, 28, S11, 1986, 185-190 (in Bulgarian).
12. Kochev, I. S. Theory of symmetrical mechanisms. — Mech. Mach. Theory, 25, 1990, No 4, 467-478.
13. Kochev, I. S. Qualitative theory of the reactions and stresses in high speed planar linkages. — Mech. Mach. Theory (to appear).
14. Id. General method for active balancing of combined shaking moment and torque fluctuations in planar linkages. Ibid. (in press).
15. Id. Active Balancing of the Frame Shaking Moment in High Speed Planar Machines. Ibid (to appear).
16. Huang, Z., Z. D. Shi. Shaking moment balancing of link mechanisms using noncircular gearset. — J. of Heavy Machinery, 14, 1986, No 4, 40-46 (in Chinese).

Received on August 1, 1990