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CONTINUUM MECHANICS

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Concerning the Replacement of Steel Dies by Dies Made of Non-Ferrous Metals and Alloys

1. Introduction

The manufacturing of dies from steel is a labour-consuming process. Frequently, the complication of manufacturing its individual parts makes the die a very costly casting tool. Furthermore, because of the relatively low thermal conductivity of steel, it cannot exert influence actively on heat exchange between the cast and the die. During solidification and cooling of the cast and the die, shrinkage occurs in them. Generally, this leads to arising of gas gap between the two and considerable decrease of heat exchange. The latter is undesirable for almost every technology, because it leads to elongation of the technological cycle. These reasons determine the casters' wish to replace, where possible, the steel dies by ones made of non-ferrous metals and alloys. Their abvantages consist in the following: 1. They can be processed easily; 2. They can be casted; 3. They have a high coefficient of thermal conductivity.

There are many publications, where discussion is made on the matter of utilization of non-ferrous metals for manufacturing dies for casting of cast iron and steel. The procedure of determining the conditions under which steel cast could be obtained in aluminium dies is discussed in [1]. These conditions are connected with the initial temperature of the cast, the thickness of the insulation covering (the coating), the water coolyng regimen, the die thickness. The purpose is to realize such a heat balance that the die could not melt. The thin-walled dies with intensive water cooling and big dies without compulsory cooling are considered in order to obtain small casts. The aluminium dies are used to obtain several types of casts under production conditions.

Nomenclature:

t — time; u — temperature; x — space variable; δ — coating thickness; λ — thermal conductivity coefficient of coating; λ_d — thermal conductivity coefficient of die; C — specific heat; Q_0 — heat flow through the cast surface; T_0 — cast surface temperature; Q_d — heat flow through the inner surface of the die; T_d — temperature of the die inner surface; T_R — temperature of the steel die outer surface; f, q, α, β, φ_i — known function; R — typical dimension of the die; x_0, x_1, x_2, x_3 — coordinates of points in the die; $\tilde{Q}_d$ — heat flow through the inner surface of the new die; $\tilde{T}_d$ — temperature of the inner surface of the new die; $\tilde{T}_R$ — temperature of the outer surface of the aluminium die; A_n, B_n — constants to be determined; $U_n(t)$ — basic functions; $U_n = \frac{dU_n}{dt}$; α — nonnegative constant, regularization parameter; W_1, W_2 — constants, usually 0 or 1; $\mathbf{S}$ — functional; r — number of points, where temperature is measured; μ_2 — constants; N — number of basic functions.

2. Mathematical formulation

Let us consider a cast obtained in a steel die. For reasons explained above, in order to optimize the technology and to reduce the expenses, we want to replace that die by one, made of alloy of non-ferrous metals. This replacement, however, should be done in a way that guarantees preserving the major properties of the cast obtained by means of the previous technology, in a steel die. Therefore, we should create a new technology which must provide the same crystallization and cooling of the cast. That could be done only if provided that the temperature of the cast surface and the heat flow through it be equal for both technologies. The problem to be settled here is how to determine the temperature and the heat flow through the surface of the cast obtained in the steel form, and after that what type of cooling is necessary for the other side of the new die for obtaining the same functions of the temperature and the flow of the cast surface. For simplicity of the statement we shall consider the one-dimensional alternative of the problem, but the same idea can be also generalized in the two-dimensional and three-dimensional case.

The solution of that technological problem can be divided into two stages.

Stage 1. Using the data of the temperature measurements in fixed number of points in the steel die (Fig. 1) we can restore the temperature $T_d(t)$ and the heat flow $Q_d(t)$ through the inner (operating) surface of the form. The mathematical problem, realizing this procedure, belongs to the class of the so-called boundary-value inverse heat transfer problems [2, 3]. Further, we shall state this problem and offer a common method for its solution. Let us consider the coating thickness as known, i. e. we know the thermal resistance at the cast-form boundary. Naturally, this resistance changes during the process but in the cast forming time, which is the most important stage, it is constant. In case we eliminate the specific heat of the coating, we can consider that

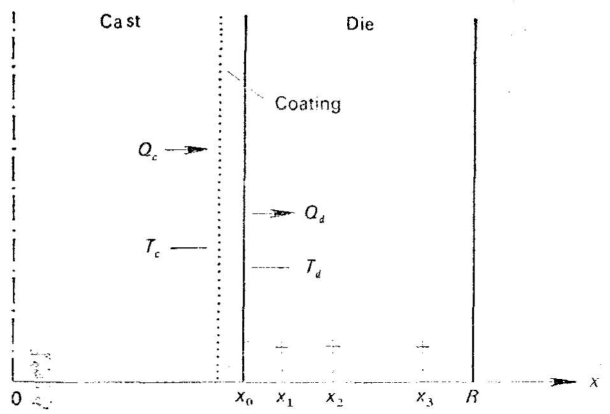

Fig. 1. The cast-die system + — points where temperature is measured; x_1, x_2, x_3 — coordinates of these points

the heat flow through the operating surface of the die and the flow through the surface of the cast is one and the same, i. e.

(1) $$Q_d(t)=Q_c(t).$$

The cast surface temperature can be determined by the formula:

(2) $$Q_c(t)=\frac{\delta}{\lambda}[T_c(t)-T_d(t)].$$

Both functions $Q_c(t)$ and $T_c(t)$ can be obtained not only by means of the above described "experimental" method. They can be also obtained by some mathematical model, which obtains Q_c and T_c according to preliminary designed properties of the cast.

Stage 2. Let us consider that we have determined the $T_c(t)$ and $Q_c(t)$ functions. Then, using again formulae (1) and (2), but now for the new die, we can obtain both the heat flow and the temperature of its operating surface. These data are already sufficient for finding the boundary condition of the outer surface of the new die. The task of determining this boundary condition after designed Q_d and T_d does not formally differ from the problem solved in stage 1.

The problem solved in stage 1 is determined by the differential equation:

(1) $$C\frac{\partial u}{\partial t}=\Delta u,\quad \Delta=\frac{1}{x^m}\frac{\partial}{\partial x}\left[x^m\lambda\frac{\partial}{\partial x}\right]\quad x\in(x_0,\ R),\quad t>0$$

and the following initial and boundary conditions:

(2) $$u(x,\ 0)=f(x);$$

(3) $$\alpha u(R,\ t)+\beta\frac{\partial u(R,\ t)}{\partial x}=q(t);$$

(4) $$u(x_i,\ t)=\varphi_i(t)\quad i=1,\ 2,\ 3,\ \dots,\ r,$$

where $$m=\begin{cases}0 & \text{Cartesian coordinates}\\ 1 & \text{cylindrical coordinates}\\ 2 & \text{spherical coordinates.}\end{cases}$$

The problem solved in stage 2 is formulated by means of (1) and (2), but instead of conditions (3) and (4) we put

(3′) $$u(x_0,\ t)=\tilde{T}_d(t);$$

(4′) $$-\lambda_d\frac{\partial u(x_0,\ t)}{\partial x}=\tilde{Q}_d(t).$$

3 Method of solution

In order to solve both problems we have used the method of regularization in an alternative, discussed in [2, 4]. Other varieties of this method can also be successfully applied, for example the algorithm described in [3].

The method here applied consists in the following: First, we make a linear parametrization of the function wanted. With the solution of the problem (1)-(4) that is the temperature of the die inner surface and instead of (4) we use

(5) $$u(x_0,\ t)=\sum_{n=1}^{N}A_nU_n(t).$$

By analogy, with solution of problem (1), (2), (3′) and (4′) we make parametrization of the outer surface temperature and instead of (4′) we use

(6) $$u(R,\ t)=\sum_{n=1}^{N} B_n U_n(t).$$

In this way the solutions of both problems depend on constants A_n and B_n. Their determination is done by minimization of a functional which, for the first problem, looks like that:

(7) $$S(A_n)=\int_0^T \sum_{i=1}^{r} [u(x_i,\ t,\ A_n)-\varphi_i(t)]\, dt$$
$$+\alpha\left[W_1\int_0^T\left(\sum_{n=1}^{N} A_n U_n\right)^2 dt+W_2\int_0^T\left(\sum_{n=1}^{N} A_n U_n'\right)^2 dt\right],$$

and for the second problem:

(8) $$S(B_n)=\int_0^T\left[\lambda_d\frac{\partial u(x_0,\ t,\ B_n)}{\partial x}+\tilde{Q}_d(t)\right]^2 dt$$
$$+\alpha\left[W_1\int_0^T\left(\sum_{n=1}^{N} A_n U_n\right)^2 dt+W_2\int_0^T\left(\sum_{n=1}^{N} A_n U_n'\right)^2 dt\right].$$

Usually, functions φ_i, $\tilde{Q}_d$ and $\tilde{T}_d$ are given as tables and the integrals in (7) and (8) are substituted by summation. In this way we obtain as follows:

(7′) $$S(A_n)=\sum_{j=1}^{M}\sum_{i=1}^{r}[u(x_i,\ t_j,\ A_n)-\varphi_i(t)]$$
$$+\alpha\left[W_1\sum_{j=1}^{M}\left(\sum_{n=1}^{N} A_n U_n(t_j)\right)^2+W_2\sum_{j=1}^{M}\left(\sum_{n=1}^{N} A_n U_n'(t_j)\right)^2\right],$$

and

(8′) $$S(B_n)=\sum_{j=1}^{M}\left[\lambda_d\frac{\partial u(x_0,\ t_j,\ B_n)}{\partial x}+\tilde{Q}_d(t_j)\right]^2$$
$$+\alpha\left[W_1\sum_{j=1}^{M}\left(\sum_{n=1}^{N} A_n U_n(t_j)\right)^2+W_2\sum_{j=1}^{M}\left(\sum_{n=1}^{N} A_n U_n'(t_j)\right)^2\right].$$

In the calculations here discussed basic functions of the type

(9) $$U_n(t)=1-\exp(-\mu_n t),$$

are used and the details regarding the particular algorithm, which realizes the above-described method and by means of which the calculations in this article are done, are described in [5].

4. Numerical experiments

In order to study the processes of casting cylindrical aluminium casts in a steel die experiments have been carried out in our experimental center. We have created

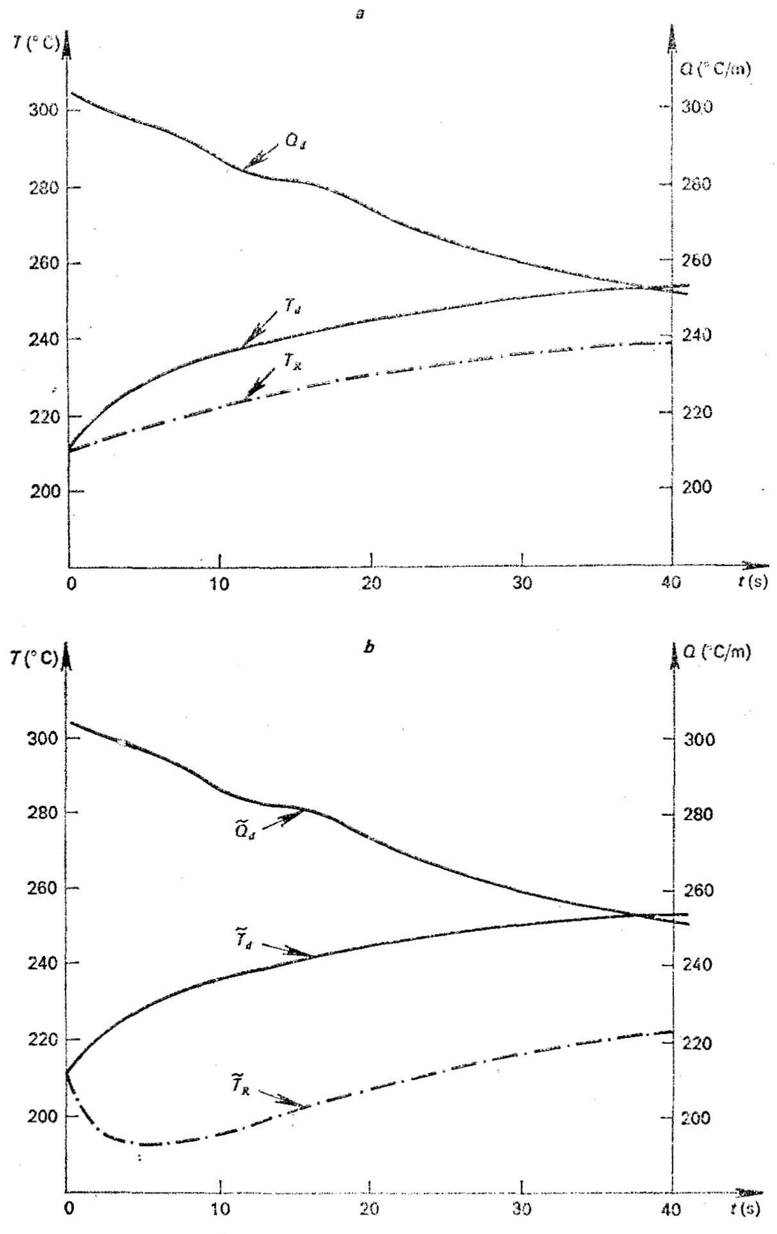

Fig. 2

also a detailed one-dimensional mathematical model describing solidification of the cast and the thermoelasic processes following the cast-die system cooling. The calculations plainly demonstrate the model adequateness [6]. That is why we have used it in order to obtain the values of T_d and Q_d. The cast is with a diametre of 150 mm and the diametre of the form is 190 mm. Fig. $2a$ shows the course of the functions T_d and Q_d, as well as the temperature of the outer side of the steel die. Fig. $2b$ shows the same functions but obtained for an aluminium form. For better comparison identical values of the parametres of the coating λ and δ are used in both cases, providing $T_d = \tilde{T}_d$ and $Q_d = \tilde{Q}_d$. One and the same is the thickness of the dies too. The obtained temperatures $\tilde{T}_R$ and $\tilde{T}_d$ provide a mean square deviation in the values of the heat flow $\tilde{Q}_d$, which is less than 4.7. The difference in the course of the curves T_R and $\tilde{T}_R$ is considerable. While the first curve increases in monotone, the second one sharply decreases at the beginning and then slowly increases.

If we have obtained the temperature $\tilde{T}_R$, this means that we have obtained also the temperature field in the whole die. Therefore, using formulae for numerical differentiation we can also determine the heat flow through its outer surface. The values of this parameter can also be obtained when solving the problem in stage 2 we make a parametrization of the flow of the right-hand boundary R of the area, but not of the temperature (ref. formula (6)).

5. Conclusion

The numerical experiment carried out demonstrates the practical possibility of realizing the above-described idea for replacement of steel dies by the dies made of non-ferrous metals. The simpler dies made of non-ferrous metals can be casted and then they can be prepared for operation by a process comparatively less labour-consuming, which decreases their cost. With a high batch production of the casts that can have a significant economic effect.

The problems (1)-(4) and (1), (2), (3′) and (4′), for determination of the boundary conditions, do not give the deformation processes in the die. The mixed effect of these processes in the cast and the die consist in a change of the thermal resistance between them. Naturally, the influence on the outer surface of the die is "filtered" by this resistance before reaching the cast, and a good model should describe these processes too. The boundary conditions of the die outer surface are realized by comparatively major error which exceeds the error admitted when the deformation effects are not taken into account. This fact makes useless the complication of the mathematical model. Naturally, these effects can be described and after that, by means of an iterative procedure, a calculation of the dynamic change of the thermal resistance can be done.

Everything mentioned above, can be also used for casts of a more complicated form. For that purpose, it is necessary to have an algorithm realizing the direct problem for the respective geometry. The generalization of the method of regularization for a two-dimensional and a three-dimensional case is trivial.

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