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P. Genova
E. Zakhariev

Quasi-Static Modelling of Elactic Poly-Contour Manipulating Systems

Mathematical modelling of manipulating systems is the basis for designing both the mechanical structure and the control systems, as well as the strategies of motion. Manipulators and robots are not static systems having strictly changing functions. They are primarily designed as universal devices with the possibility to be re-adjusted for various operating conditions, the behaviour of the mechanical structure being dependent on the character of motions and loading. The kinematic and dynamic parameters, as well as the elasticity of the links effect the functionality of the mechanical structure and conrol systems.

The modelling of the dynamics of manipulating systems is based on the methods of theoretical mechanics and the theory of machines and mechanisms [1, 2, 3]. It is comparatively recently that studies on dynamics of manipulators and robots have begun to consider the elasticity of links [4, 5, 6].

The present paper suggests a force, deformation and freguency model of the mechanical structure, used at the stage of designing. Methods are also suggested for obtaining an objective assessment according to force, energy and accuracy indices concerning the behaviour of the structure at each point of the working space. The resulting information is used for designing the motors, the transmission mechanisms, the mechanical structure, and control systems.

1. Quazi-static modelling of the dynamics of manipulating systems

The great variety of motion performed by the universal robot does not allow to assess correctly at which point of the working space and along what trajectory of motion the elements and the joints of the mechanical structure will be loaded to a maximum degree.

The initial conditions for designing the robot are the following: working space, load-carrying capacity, maximum velocity and, in particular, maximum acceleration of motion of the output link. The constraints are: forces in bearing connections, maximal forces or moments in the output mechanisms, maximal stresses in the elements of the mechanical structure, etc.

The extreme force characteristcs, taking into consideration the dynamics of the mechanical structure, are defined with the characteristics of motion as extreme. Each point of the working space is compared to vectors $V_{\max}$ and $A_{\max}$ having the size of

$|V_{max}|$ and $|A_{max}|$, equal to the maximum linear velocities and accelerations of the output link, using all possible combinations of direction of the vectors on a selected grid of axes. It is sufficient if these axes are the positive and negative directions of the axes of the absolute coordinate system of the robot (Fig. 1). In this case, the possible number of combinations of vectors V_{max} and A_{max} is 36. By using all-purpose programmes for kinematic and kinetostatic analysis, the generalized velocities, accelerations, the generalized forces, the forces in the connections (reactions), the stresses in selected cross-sections are computed (defining the maximum values and comparing them to the admissible values) for each point of the working space, and for each combination of directions of vectors V_{max} and A_{max}. The results of the analysis thus carried out according to the suggested procedure concerning robot (Fig. 1) are shown in Fig. 2 for the whole working region where Fig. 2-*a* illustrates the maximum forces (moment M_1) in the input of base 1; Figs 2*b* and 2*c* illustrate the maximum forces (F_2 and F_3) for hydrocylinders 2 and 3, respectively; Figs 2*d*, 2*e*, and 2*f* show, respectively, the maximum power, the kinetic energy, and the energy of acceleration of the mechanical system. The broken lines in the working space, shown in the figures, connect those points that have the same extreme characteristics.

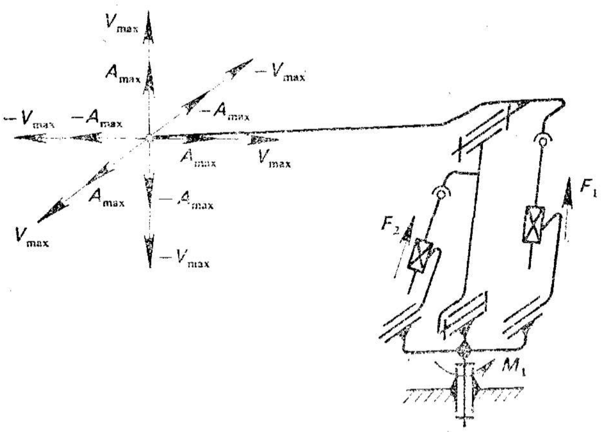

Fig. 1

2. Modelling of deformation and natural frequencies of the mechanical system

Deformations and natural frequencies of the mechanical system are the basic operating characteristics of industrial robots. In modelling poly-contour manipulating systems, it is abvisable to use the finite elements theory. Mechanical devices and structures may be considered to be a set of elements connected in a finite number of nodal points. Links of robots are constructed as thin-wall bars and beams connected by joints. The investigation in this paper uses a model consisting of spatial beams, considered to be elements with two finite nodes (Fig. 3). Deformations (vector Δ^i) and internal forces (vector R^i) at the nodes of the i-th element are connected by the following relationship

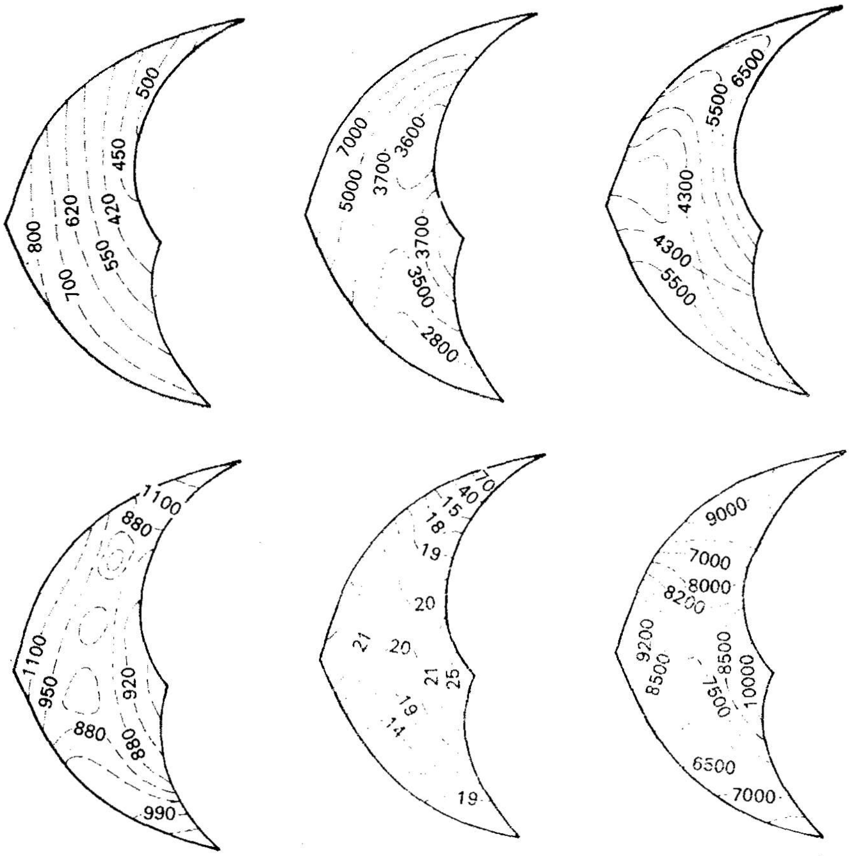

Fig. 2

$$K^i . \Delta^i = R^i, \tag{1}$$

where

$$K^i = \begin{bmatrix} K_x^i & 0 & 0 \\ 0 & K_y^i & 0 \\ 0 & 0 & K_z^i \end{bmatrix}$$

is the rigidity matrix of the *i*-th element, while

$$K_x^i = \begin{bmatrix} \frac{E.S}{L} & \frac{-E.S}{L} & 0 & 0 \\ \frac{-E.S}{L} & \frac{E.S}{L} & 0 & 0 \\ 0 & 0 & \frac{G.I_c}{L} & \frac{-G.I_c}{L} \\ 0 & 0 & \frac{-G.I_c}{L} & \frac{G.I_c}{L} \end{bmatrix},$$

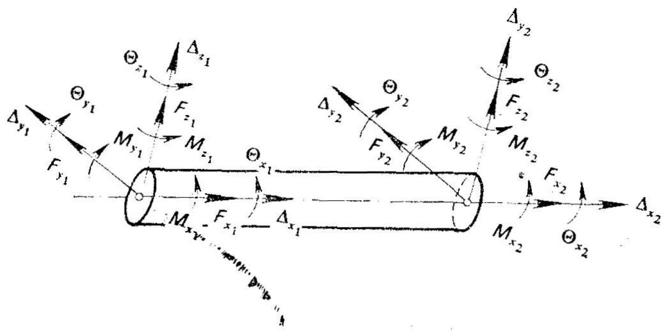

Fig. 3

$$K_y^i = \begin{bmatrix} \frac{12.E.I_z}{L^3} & \frac{-12.E.I_z}{L^3} & \frac{6.E.I_z}{L^2} & \frac{6.E.I_z}{L^2} \\ \frac{-12.E.I_z}{L^3} & \frac{12.E.I_z}{L^3} & \frac{-6.E.I_z}{L^2} & \frac{-6.E.I_z}{L^2} \\ \frac{6.E.I_z}{L^2} & \frac{-6.E.I_z}{L^2} & \frac{4.E.I_z}{L} & \frac{2.E.I_z}{L} \\ \frac{6.E.I_z}{L^2} & \frac{-6.E.I_z}{L^2} & \frac{2.E.I_z}{L} & \frac{4.E.I_z}{L} \end{bmatrix},$$

$$K_z^i = \begin{bmatrix} \frac{12.E.I_y}{L^3} & \frac{-12.E.I_y}{L^3} & \frac{6.E.I_y}{L^2} & \frac{6.E.I_y}{L^2} \\ \frac{-12.E.I_y}{L^3} & \frac{12.E.I_y}{L^3} & \frac{-6.E.I_y}{L^2} & \frac{-6.E.I_y}{L^2} \\ \frac{6.E.I_y}{L^2} & \frac{-6.E.I_y}{L^2} & \frac{4.E.I_y}{L} & \frac{2.E.I_y}{L} \\ \frac{6.E.I_y}{L^2} & \frac{-6.E.I_y}{L^2} & \frac{2.E.I_y}{L} & \frac{4.E.I_y}{L} \end{bmatrix},$$

$$\Delta^i = [\Delta_{x1}\Delta_{x2}\theta_{x1}\theta_{x2}\Delta_{y1}\Delta_{y2}\theta_{z1}\theta_{z2}\Delta_{z1}\Delta_{z2}\theta_{y1}\theta_{y2}]^{\mathrm{T}}$$

$$R^i = [R_{x1}R_{x2}M_{x1}M_{x2}R_{y1}R_{y2}M_{z1}M_{z2}R_{z1}R_{z2}M_{y1}M_{y2}]^{\mathrm{T}}.$$

The notations of the components of vectors Δ^i, R^i are shown in Fig. 3, S is the cross section area of the element, while L is latter's length; I_c, I_y, I_z are inertia moments with respect to centre C and axes Y and Z of the element, respectively E and G are the notations for the modules of linear deformation and the moduls of angular deformation, respectively. Proceeding from matrices K^i for all elements, the mechanical system rigidity matrix K is set up, and it participates in equation

$$\Delta = K^{-1}.F, \tag{2}$$

which expresses the relation between displacements Δ and external forces F in all nodes of the mechanical structure. The mecanical system deformations are investigated with respect to the extreme characteristics of motion according to the procedure given in section 1, assuming that nodal forces are represented both by all external forces, and by the inertia forces arising from the motion of the system at maximum speed $V_{\max}$ and acceleration $A_{\max}$. In order to investigate deformations at one point of the working space for all 36 direction combinations of vectors $V_{\max}$ and $A_{\max}$, it is necessary to perform the inversion of the matrix K and its multiplication by matrix F.

The motion equation at small free oscillations of the mecanical system around its equilibrium position, without any dampening and external forces, is the following:

$$M.\frac{d^2\Delta}{dt^2}+K.\Delta=0, \tag{3}$$

where M is the mechanical structure mass matrix, consisting of the mass matrices of all elements. This, the matrix of the i-th element M^i, is:

$$M^i=\begin{bmatrix} M^i_x & 0 & 0 \\ 0 & M^i_y & 0 \\ 0 & 0 & M^i_z \end{bmatrix},$$

where

$$M^i_x=\begin{bmatrix} \frac{1}{4}.m & \frac{1}{4}.m & 0 & 0 \\ \frac{1}{4}m & \frac{1}{4}m & 0 & 0 \\ 0 & 0 & \frac{1}{4}J_x & \frac{1}{4}J_x \\ 0 & 0 & \frac{1}{4}J_x & \frac{1}{4}J_x \end{bmatrix},$$

$$M^i_y=\begin{bmatrix} \frac{1}{4}m+\frac{9}{4}J_z.L^{-2} & \frac{1}{4}m-\frac{9}{4}J_z.L^{-2} & \frac{1}{16}m.L+\frac{3}{8}J_z.L^{-1} & \frac{-1}{16}m.L+\frac{3}{8}J_z.L^{-1} \\ \frac{1}{4}m-\frac{9}{4}J_z.L^{-2} & \frac{1}{4}m+\frac{9}{4}J_z.L^{-2} & \frac{1}{16}m.L-\frac{3}{8}J_z.L^{-1} & \frac{-1}{16}m.L-\frac{3}{8}J_z.L^{-1} \\ \frac{m}{16}L+\frac{3}{8}J_z.L^{-1} & \frac{m}{16}L-\frac{3}{8}J_z.L^{-1} & \frac{m}{64}L^2+\frac{1}{16}J_z & \frac{-m}{64}L^2+\frac{1}{16}J_z \\ \frac{-m}{16}L+\frac{3}{8}J_z.L^{-1} & \frac{-m}{16}L-\frac{3}{8}J_z.L^{-1} & \frac{-m}{64}L^2+\frac{1}{16}J_z & \frac{m}{64}L^2+\frac{1}{16}J_z \end{bmatrix},$$

$$M^i_z=\begin{bmatrix} \frac{1}{4}m+\frac{9}{4}J_y.L^{-2} & \frac{1}{4}m-\frac{9}{4}J_y.L^{-2} & \frac{1}{16}m.L+\frac{3}{8}J_y.L^{-1} & \frac{-1}{16}m.L+\frac{3}{8}J_y.L^{-1} \\ \frac{1}{4}m-\frac{9}{4}J_y.L^{-2} & \frac{1}{4}m+\frac{9}{4}J_y.L^{-2} & \frac{1}{16}m.L-\frac{3}{8}J_y.L^{-1} & \frac{-1}{16}m.L-\frac{3}{8}J_y.L^{-1} \\ \frac{m}{16}L+\frac{3}{8}J_y.L^{-1} & \frac{m}{16}L-\frac{3}{8}J_y.L^{-1} & \frac{m}{64}L^2+\frac{1}{64}J_y & \frac{-m}{64}L^2+\frac{1}{16}J_y \\ \frac{-m}{16}L+\frac{3}{8}J_y.L^{-1} & \frac{-m}{16}L-\frac{3}{8}J_y.L^{-1} & \frac{-m}{64}L^2+\frac{1}{16}J_y & \frac{m}{64}L^2+\frac{1}{16}J_y \end{bmatrix},$$

and J_x, J_y, J_z are the mass inertia moments of the element with respect to axes X, Y, Z of its own coordinate system; m is the mass of that element.

A real-valued repetitive solution of equation (3) is existent

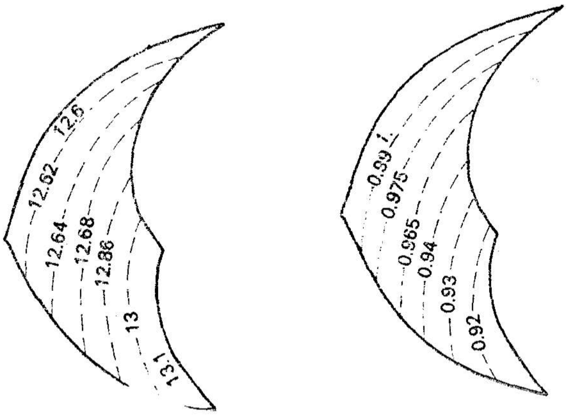

Fig. 4

$$\Delta=\Delta_o.\cos\omega t \tag{4}$$

if the following condition is satisfied:

$$(K-\omega^2.M).\Delta_o=0. \tag{5}$$

Roots ω^2 of this equation determine the natural frequencies of the mechanical system. The first, i. e. the lowest, natural frequencies are significant.

Fig. 4*a* shows the working space of robot (Fig. 1), the lowest natural frequencies being plotted for each point of the working space. Points with equal natural frequencies are connected by equilines.

The deviation of a point of the output link of the robot, due to the compliance of the mechanical structure, may be defined by means of the following procedure. Loading of the output link is simulated, by using a single force in different directions of the space (similarly to the approach described in 1), whereupon the deviations of the loading are counted off from dependence (2), and the maximum deviations are determined. Due to the proportionality of deformations, computations concerning symmetric vectors are not necessary. The elements of the compliance matrix (the inverse of the rigidity matrix K) corresponding to the degrees of freedom of the output link, give some numerical information on the compliance of the system.

Fig. 4*b* shows the working space of robot (Fig. 1), in which the standardized values of these deformations are plotted by equilines. The maximum numerical value of the deformation, defined according to the procedure suggested, is $0.12114.10^{-4}$.

3. Conclusions

The mathematical model suggested here can be applied in analysing the extreme loading and deformations of the elements at the stage of designing the mechanical structure, so that its parameters may be purposefully changed. Valuable information on designing the control system can be obtained, or vice versa: the mechanical structure may be designed according to the requirements of the control system.

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