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CONTINUUM
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Double-Gradient Stabilizaton in Non-Linear Thermovisco-Plasticity*

1. Statement of the problem

We consider a thermoviscoplatic solid exhibiting only thermal softening and strain rate sensitivity, submitted to a simple shearing between two parallel planes $x'=0$ and $x'=l$. The shearing is caused by a steady shear force at $x'=l$, while the solid is fixed at $x'=0$. Denoting by $\sigma'(x',t')$, $\theta(x',t')$, $y'(x',t')$ and $v'(x',t')$ the stress, temperature, strain and velocity field respectively, the balance laws of equilibrium and energy, the compatibility conditions describing the problem take the form, for $0 \leq x' \leq l$ $0 \leq t' \leq \infty$,

(1) $$\sigma'_{x'} = \rho v'_{t'},$$

(2) $$\theta'_{t'} = (\lambda/\rho c_U)\sigma' Y'_{t'} + k\theta'_{x'x'},$$

(3) $$Y'_{t'} = v'_{x'},$$

where ρ is the referential density, λ — the Taylor-Quinney coefficient, c_U — the specific heat and k — the thermal diffusivity coefficient. The subscript indicates differentiation with respect to the variable indicated.

We assume that the elastic strain is negligible and the stress is given by a constitutive law of the following "strain gradient" form

(4) $$\sigma' = \sigma^h(\theta', Y'_{t'}) + \sigma^i(Y'_{x'x'}, Y'_{x'x'x'x'}),$$

where σ^h denotes the "homogeneous" thermoviscoplastic part of the stress and σ^i expresses the "inhomogeneous" part, which is the macroscopic manifestation of the evolution of microstructural variables [1]. We adopt the following expression for the "homogeneous" and the "inhomogeneous" part of the stress [1, 2]

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(5) $$\sigma^h = G\theta'^{\,v} \,|\, Y'_{t'} \,|^{m-1}\, Y'_{t'} ,$$

(6) $$\sigma^i = -cY_{x'x'} - dY'_{x'x'x'x'} ,$$

where G is the shear modulus, v and m are the thermal softening and strain-rate sensitivity coefficients and c, d ($c \geq 0$, $d \geq 0$) are microscaling parameters. The introduction of c and d permits to study the effect of internal inhomogeneities on the thermomechanical process.

In recent papers [3, 4], it was shown that the quasi-static situation is the asymptotic "solution" of the corresponding adiabatic dynamic shearing. In the absence of heat conduction ($k=0$), the energy balance equation takes the form

(7) $$\theta'_{t'} = (\lambda/\rho c_U)\sigma' Y'_{t'} .$$

In this case, the energy equation obtains a hyperbolic character and the thermomechanical process becomes less stable. From the physical point of view, the only dissipative mechanism is due to the thermorheological (m and v) and microscaling (c and d) parameters.

We now suppose that inhomogeneous part of stress is due only to the second order strain gradient ($d=0$).

Finally, we assume that equations (1)-(7) are complemented by the following initial conditions ($0 \leq x \leq l$)

(8) $\theta'(x', 0) = \psi(x') > 0$, $Y(x', 0) = \omega(x')$, $Y'_{t'}(x', 0) = z'(x')$, where $Y'_{0x'}(0) = Y'_{0x'}(1) = 0$

and the following boundary conditions ($0 \leq t' < 1$)

(9) $$\theta'_{x'}(0, t') = \theta'_{x'}(l, t)' = 0, \quad \sigma'(l, t') = \sigma^* > 0.$$

These conditions imply that the boundaries are thermally insulated and that the shearing is caused by a steady force σ^* at $x' = l$. Moreover, we assume that the boundary and the initial conditions are compatible.

It is easy to verify that the quasi-static situation under the boundary condition (9.b) gives

(10) $$\sigma(x', t') = \sigma^*.$$

We now introduce the following nondimensional variables:

(11) $$x = x'/l, \ t = t'/Y^*_t, \ \theta(x, t) = \theta'(x', t')/\theta^*,$$
$$\sigma(x, t) = \sigma'(x', t')/\theta^* = 1, \ Y_t(x, t) = Y'_t(x', t')/Y^*_t.$$

The Eqs (4)-(10) take the non-dimensional form

(12) $$\theta^v \,|\, Y_t \,|^{m-1}\, Y_t - pY_{xx} = 1,$$

(13) $$\theta_t = \delta\sigma Y_t ,$$

(14) $$\theta_x(0, t) = \theta_x(l, t) = 0,$$

(15) $$\theta(x, 0) = \theta_0(x) > 0, \quad Y(x, 0) = Y_0(x),$$
$$Y_t(x, 0) = z(x), \text{ where } Y_{0x}(0) = Y_{0x}(l) = 0,$$

(16) $$\delta=\lambda\sigma^*/\rho c_U, \quad p=c/(l^2\sigma^*) \text{ and } \sigma^*=G\,\theta^{*v}\,Y_t^m.$$

Integrating (12) in time and differentiating with respect to x, we find the relationship between the second order gradients of θ and Y in order to express (11) as a "parabolic" equation in X only:

(17) $$\theta^v\,|\theta_t|^{m-1}\,\theta_t-bY_{xx}=A(x),$$

(18) $$\text{where } b=p\delta^{m-1} \text{ and } A(x)=\delta^m[1+pY_{0xx}(x)-p\delta^{-1}\,\theta_{0xx}].$$

2. The stabilizing role of double gradient of strain

In [5] it was shown that the sign of $m+v$, as well as the inclusion of the microscaling coefficient c, influence the life of the process in a well determined way. According to the results of the above paper, when $m+v<0$, then the temperature "blows-up" in a finite time (critical time), depending on the initial distribution in θ, on the control parameter δ and on the microscaling coefficient c. We present here the values of the critical time corresponding to the cases $c>0$ and $c=0$:
If $c>0$ then

(19) $$t_{cr}=(m/(-m-v)\delta)\int_0^1 \theta_0(x)^{(m+v)/m}\,dx.$$

If $c=0$ then

(20) $$t^*_{cr}=(m/(-m-v)\delta)(\max_{x\in[0,1]}\theta_0(x))^{(m+v)/m}.$$

If the initial temperature is a non-uniform function of x, we conclude from (19), (20) that

(21) $$t_{cr}>t^*_{cr}.$$

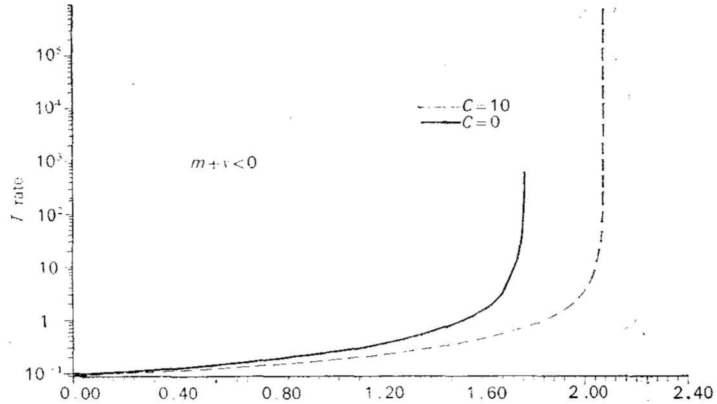

Fig. 1. The evolution of maximum temperature rate (at the point $x=(6/10)$) with the time

Indeed, since $\theta_0(x)>0$ and $m+v<0$, then $\theta_0(x)^{m+v/m} \geq [\max_{x \in [0,1]} \theta_0(x)]^{m+v/m}$ therefore

$$\int_0^1 \theta_0(x)^{(m+v)/m} dx \geq (\max \theta_0(x))^{(m+v)/m}$$

which proves (21).

Inequality (21) means that the presence of c at the constitutive relation has a retarding effect to the instability regime. Moreover, it was shown in [5] that instability regime has a qualitatively different character in these two cases ($c>0$, $c=0$). The temperature $\theta(x, t)$ in the case $c=0$ "blows-up" for the time $t=t^*_{cr}$ only at the point x where the initial temperature $\theta_0(x)$ takes the maximum value (shear banding [6], [7]), while the temperature in the case $c>0$ "blows-up" for $t=t_{cr}$ in a "weak" way, namely

$$\int_0^1 \theta^{m+v/m}(x, t_{cr})dx \to 0.$$

In order to verify these results we consider a real thermoviscoplastic material with the following mechanical, thermomecanical and geometrical parameters

$$\beta=0.9,\ \rho=7800\,\text{kg/m}^3,\ \rho c_U=3.9\ 10^6\,\text{J/m}^3\text{K}, \tag{23}$$
$$Y^*_t=10^4/\text{s},\ \theta^*_0=300^\circ\,\text{K},\ l=10^{-2}\,\text{m},$$

$$G=3.28\ 10^7\,\text{SI},\ v=-.38,\ m=.05. \tag{24}$$

According to the relation (16c) the referential boundary shear stress takes the value $\delta^*=59.5$ MPa. The initial distribution of the temperature is nonuniform and is given by

$$\theta_0(x)=(25x^4/72-2x^3/3+7x^2/20+11/3000)300/92+1, \quad x \in [0, 0.6] \tag{25}$$
$$\theta_0(x)=((x-.3)^3/6-(x-.3)^2/4+21x/200-86/6000))300/92+1, \quad x \in [0.6, 1].$$

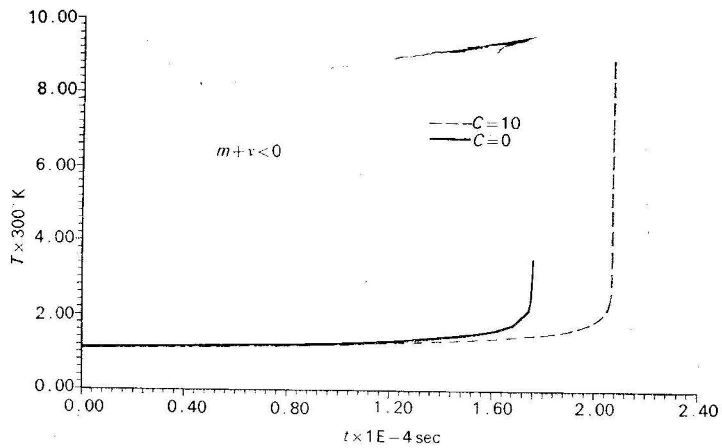

Fig. 2. The evolution of maximum temperature (at the point $x=(6/10)$) with the time

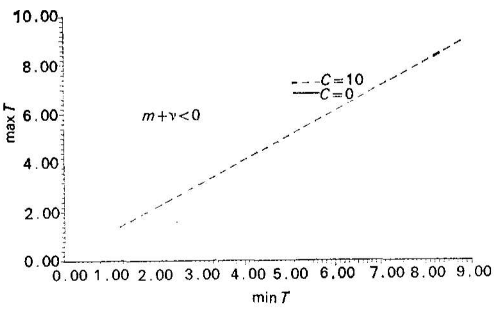

Fig. 3. The stabilizing role of $c=10\,\mathrm{N}$ at the case of $m+v<0$

We consider two cases with $c=0\mathrm{N}$ and $c=10\mathrm{N}$ and using (19) and (20) we find that $t_{cr}=2.1183$ and $t^*_{cr}=1.7646$. The dimensional critical times have the values $t'_{cr}=.2118\mathrm{m/s}$ and $t'^*_{cr}=.1765$ m/s. We can see from the numerical solution (Fig. 1, 2) that at a time very close to the theoretical critical time (t_{cr} or t^*_{cr} respectively) the temperature rate and the temperature at the point $x=6/10$ tend to infinity. Figure 3 exhibits the stabilizing role of $c=10\,\mathrm{N}$ via the curves of the maximum temperature as a function of the minimum temperature.

We now consider the case of stability regime. In [5] it was shown that when $m+v>0$ and $c>0$, then the thermomechanical process is stable and the temperature and strain evolve almost uniformly, without shear banding formation. Namely as $t\to\infty$,

(26) $$K^{-1}\,t^{m/(m+v)}\leq\theta(x,\,t)\leq Kt^{m/(m+v)}\,,$$

(27) $$K^{-1}\,t^{m/(m+v)}\leq Y(x,\,t)\leq Kt^{m/(m+v)}\,,$$

(28) $$|\,Y_x(x,\,t)\,|\leq K,$$

where K depends only on the initial distribution in $\theta_0(x)$, $Y_0(x)$ and the control parameters (σ^*, θ^*, Y^*_t). We next examine now the numerical solution for the case $m+v>0$. We define a material with the following material parameters:

(29) $$\beta=0.9,\ \rho=7800\,\mathrm{kg/m^3},\ \rho_U c=3.9\ 10^6\,\mathrm{J/m^3K},$$
$$Y^*_t=10^4/\mathrm{s},\ \theta^*_0=300^\circ\,\mathrm{K},\ l=10^{-2}\,\mathrm{m},$$

(30) $$G=1.3\ 10^7\,\mathrm{SI},\ v=-.38,\ m=.4.$$

We can conclude from Figs 4 and 5, that in the case $c=0$, the temperature gradient and strain gradient increase much more rapidly. On the contrary the inclusion of the microscaling parameter $c=1\mathrm{N}$ and $c=70\mathrm{N}$ implies that the non-uniformities are bounded, excluding shear banding. This result is in complete agreement with our analysis.

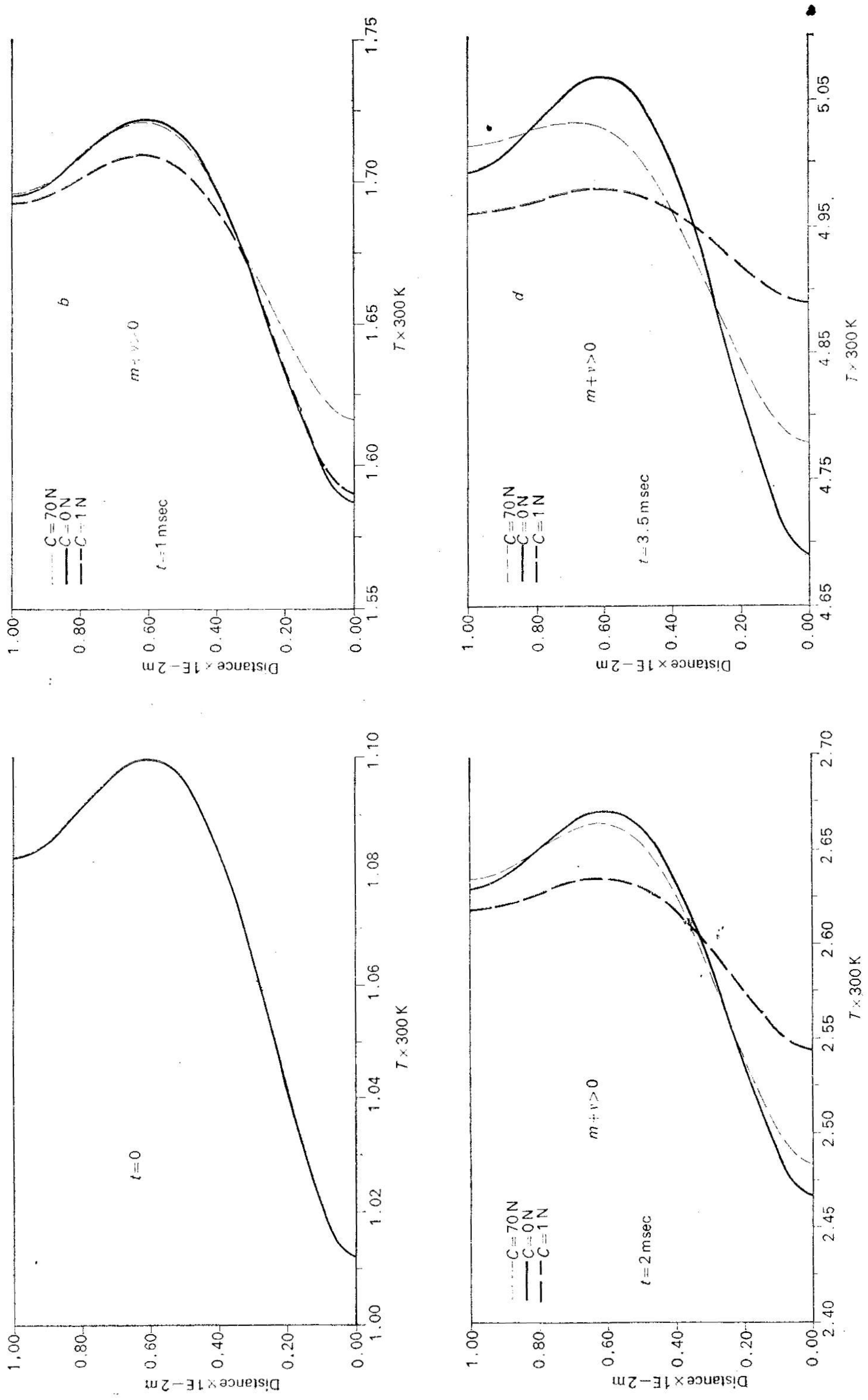

Fig. 4. The evolution of temperature profile with the time

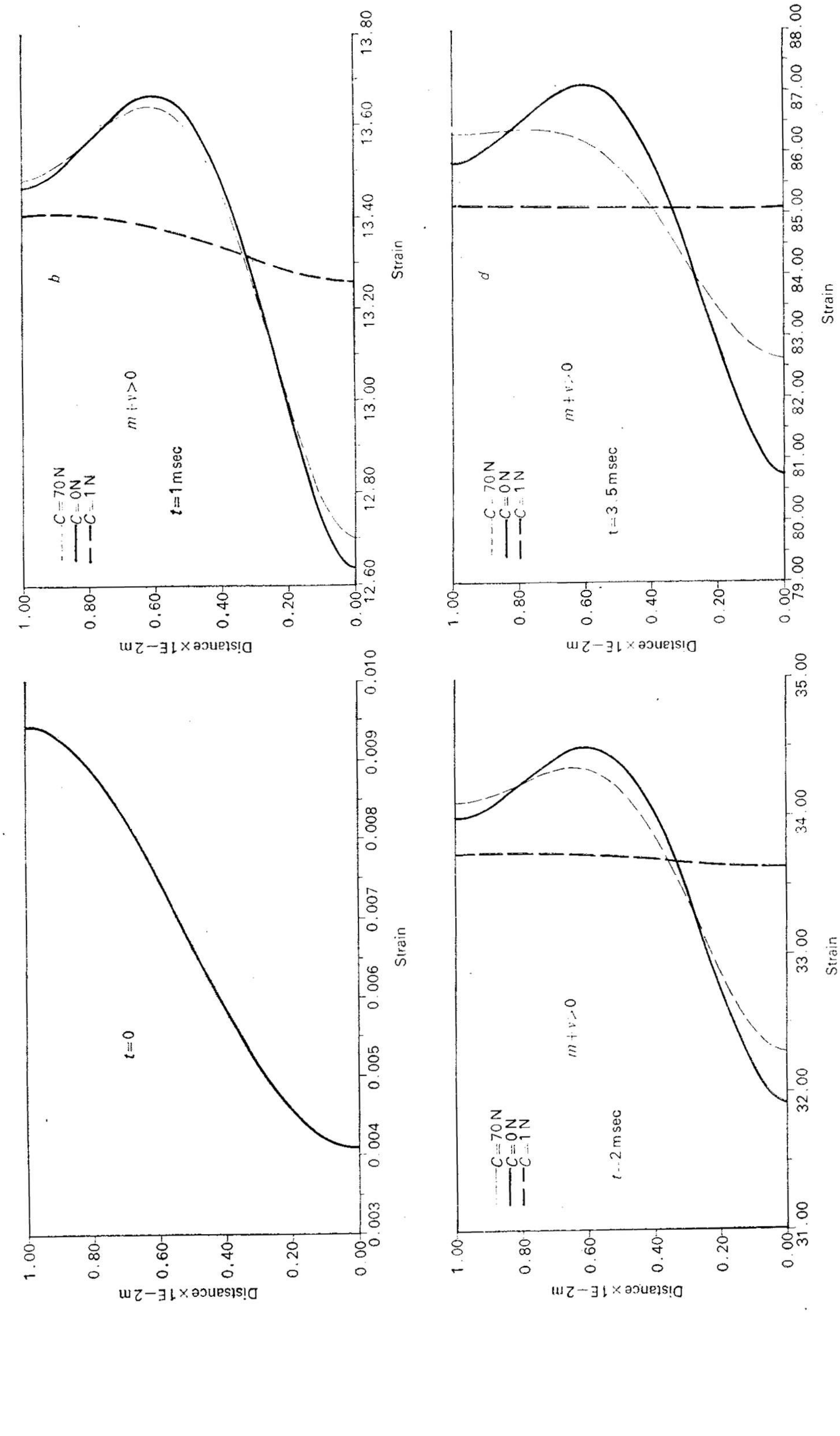

Fig. 5. The evolution of strain profile with the time

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