

## FLUID MECHANICS

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### Hydroelasticity in Cases of Waterhammer

Waterhammer at pumping stations is of a special practical interest. First of all it is necessary to establish the type of waterhammer, the hold up time of the flow in the pipe, the greatest pressure fall or rise respectively, as well as the moment of interruption of the water column and its removal. When solving these problems, the influence of hydroelasticity, is of a special importance. Numerous solutions of both partial differential equations of first order describing the phenomenon of waterhammer [1,2,6,7] are well known from the literature. Here we shall use only the equations given in [7]:

$$(1) \quad \frac{\partial}{\partial x} \left( \frac{p}{\rho} \right) + v \frac{\partial v}{\partial x} + \frac{\partial v}{\partial t} + g \left( \frac{\partial H}{\partial x} + S_f \right) = 0$$

$$(2) \quad v \frac{\partial}{\partial x} \left( \frac{p}{\rho} \right) + \frac{\partial}{\partial t} \left( \frac{p}{\rho} \right) + a^2 \frac{\partial v}{\partial x} = 0$$

where  $p$ ,  $\rho$  denote pressure, respectively density of the flow,  $v$  - flow velocity,  $H$  - pipe elevation (see Fig.1),  $S_f = \frac{v|v|}{C^2 R}$  - frictional slope,  $a$  - waterhammer wave spreading velocity,  $C = \frac{1}{n} R^{1/6}$  - Chezi coefficient,  $R = \frac{F}{X}$  - coefficient of roughness.

Both partial differential equations (1) and (2) are of a hyperbolic type and according to the general theory of hyperbolic equations [7],

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their characteristic equations and characteristic directions are determined by the conditions:

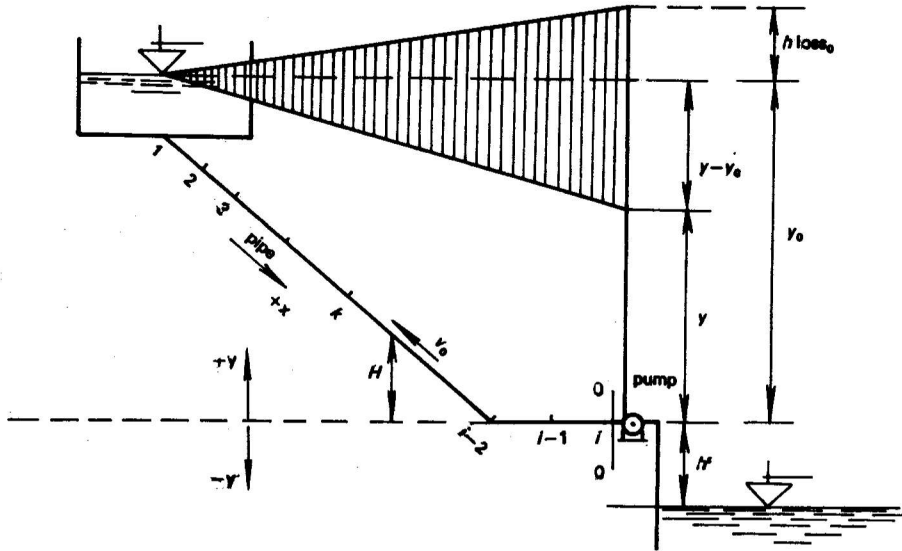


Fig. 1.

$$(3) \quad \frac{\lambda_1 B_1 + \lambda_2 B_2}{\lambda_1 A_1 + \lambda_2 A_2} = \frac{\lambda_1 D_1 + \lambda_2 D_2}{\lambda_1 C_1 + \lambda_2 C_2} = \zeta,$$

where  $\zeta = \frac{dt}{dx}$  is the characteristic direction being sought,  $A_1, A_2, \dots, D_1, D_2$  - coefficients in the starting quasilinear system which might depend on  $v, p, x, t$  - see eq.(4),(5). If we inscribe equations (1) and (2) generally as a system of quasilinear equations of first degree with dependent variables  $u, h$  and independent variables  $x, t$  we shall obtain:

$$(4) \quad A_1 \frac{\partial u}{\partial x} + B_1 \frac{\partial u}{\partial t} + C_1 \frac{\partial h}{\partial x} + D_1 \frac{\partial h}{\partial t} + E_1 = 0,$$

$$(5) \quad A_2 \frac{\partial u}{\partial x} + B_2 \frac{\partial u}{\partial t} + C_2 \frac{\partial h}{\partial x} + D_2 \frac{\partial h}{\partial t} + E_2 = 0.$$

From (3) it follows that:

$$(6) \quad (A_1\zeta - B_1)\lambda_1 + (A_2\zeta - B_2)\lambda_2 = 0 ,$$

$$(7) \quad (C_1\zeta - D_1)\lambda_1 + (C_2\zeta - D_2)\lambda_2 = 0 .$$

A nonzero solution for  $\lambda_1$  and  $\lambda_2$  is only possible if:

$$\det \begin{pmatrix} A_1\zeta - B_1 & A_2\zeta - B_2 \\ C_1\zeta - D_1 & C_2\zeta - D_2 \end{pmatrix} = 0 ,$$

$$(8) \quad A\zeta^2 - B\zeta + C = 0 ,$$

where  $A = A_1C_2 - A_2C_1$ ,  $B = \left[ (A_1D_2 - A_2D_1) + (B_1C_2 - B_2C_1) \right]$ ,  $C = B_1D_2 - B_2D_1$ ,

The study of the discriminant of (8) shows that is executed the condition:

$$(9) \quad B^2 - 4AC > 0 .$$

The inequality (9) is true with the hyperbolic system (1),(2) in the to-sonic and supersonic fields. Here we shall only examine the to-sonic field ( $v/a < 1$ ) Let the characteristic direction be defined by the derivative:

$$\frac{dt}{dx} = \tan \theta = \zeta .$$

The rate of change of the dependent variables  $u(x,t), h(x,t)$  in this direction is :

$$(10) \quad \frac{\partial u}{\partial s} = \left( \frac{\partial u}{\partial x} + \zeta \frac{\partial u}{\partial t} \right) \frac{dx}{ds} ,$$

$$(11) \quad \frac{\partial h}{\partial s} = \left( \frac{\partial h}{\partial x} + \zeta \frac{\partial h}{\partial t} \right) \frac{dx}{ds} .$$

On the other hand in accordance with the definition for family characteristics of (4),(5) it follows:

$$12) \quad (\lambda_1 A_1 + \lambda_2 A_2) \frac{\partial u}{\partial x} + (\lambda_1 B_1 + \lambda_2 B_2) \frac{\partial u}{\partial t} + (\lambda_1 C_1 + \lambda_2 C_2) \frac{\partial h}{\partial x} +$$

$$(\lambda_1 D_1 + \lambda_2 D_2) \frac{\partial h}{\partial t} + (\lambda_1 E_1 + \lambda_2 E_2) = 0 .$$

After the change of the variables, according to (10), (11) we obtain:

$$(A_1 + \frac{\lambda_2}{\lambda_1}A_2)\frac{\partial u}{\partial s} + (C_1 + \frac{\lambda_2}{\lambda_1}C_2)\frac{\partial h}{\partial s} + (E_1 + \frac{\lambda_2}{\lambda_1}E_2)\frac{dx}{ds} = 0$$

With the ratio  $\lambda_2/\lambda_1$  from (6) it follows:

$$(13) \quad R\frac{\partial u}{\partial s} + (A\zeta - S)\frac{\partial h}{\partial s} + (K\zeta - L)\frac{dx}{ds} = 0 ,$$

where

$$R = A_1B_2 - A_2B_1, \quad A = A_1C_2 - A_2C_1, \quad S = B_1C_2 - B_2C_1,$$

$$K = A_1E_2 - A_2E_1, \quad L = B_1E_2 - B_2E_1.$$

Equation (13) is a characteristic equation for each real solution  $\zeta$  of (8). For the system (1),(2) the characteristic equation and the characteristic direction according to (13) and (8) are:

$$(14) \quad \frac{\partial}{\partial s} \left( \frac{p}{\rho} \right) \pm a \frac{\partial v}{\partial s} + \frac{a}{a \pm v} g \left( \frac{\partial H}{\partial x} + S_f \right) \frac{dx}{ds} = 0$$

$$\frac{1}{\zeta} = \frac{dx}{dt} = v \pm a$$

Bearing in mind that  $v \ll a$  for the characteristic equation and the characteristic direction we obtain:

$$(15) \quad \frac{\partial}{\partial s} \left( \frac{p}{\rho} + gH \pm av \right) + gS_f \frac{dx}{ds} = 0$$

$$(16) \quad \frac{dx}{dt} = \pm a$$

In all important cases in practice, waterhammer wave spreading velocity can be assumed as a constant, defined by the equations:

$$a = \sqrt{\frac{g/\gamma}{1/\epsilon + d/E/\delta}}, \quad \text{where } g \text{ denotes gravity acceleration ; } \epsilon, E - \text{ the}$$

elasticity moduls of the fluid and the pipe material;  $\delta, d$  - pipe wall thickness and pipe diameter. In the present paper we shall consider two methods of numerical integration of the initial system (1),(2). The first method is based on the direct integration of (15). Taking into account,

$S_f = - \frac{dh}{dx} \frac{\text{loss}}{\text{loss}}$  (at the accepted orientation of the co-ordinate system) and substituting this in (15) we obtain:

$$(18) \quad \frac{\partial}{\partial s} \left( \frac{p}{\gamma} + H \right) ds \pm \frac{a}{g} \frac{\partial v}{\partial s} ds = \frac{dh_{\text{loss}}}{ds} ds$$

Let us introduce a co-ordinate system as shown in Fig. 1. Let us introduce the indication of the initial conditions:

$$\frac{p}{\gamma} + H = y; \quad y(t=0) = y_0; \quad v(t=0) = v_0,$$

The integrations of (18) is accomplished after Euler's method on characteristical direction with a lengthwise step of  $\Delta l = \frac{l}{n}$ , and of time  $\Delta t = \frac{\Delta l}{a}$ , i.e.  $t = i\Delta t$ . Then for head  $y_i$  cross section o-o

(Fig.1) we shall obtain

$$(19) \quad y_i = y_0 + \frac{a}{g} (v_i - v_{i-1}) + \frac{a}{g} \sum_{j=2}^{i-1} (v_j + v_{j-1}) + \frac{\Delta l}{C^2 R} \text{sign}(v_i) \frac{v_i^2 + v_{i-1}^2}{2} + \frac{\Delta l}{C^2 R} \text{sign}(v_j) \sum_{j=2}^{i-1} \frac{v_j^2 + v_{j-1}^2}{2}$$

Formula (19) remains true with  $t_i \leq T_{\text{prop}} = \frac{l}{a}$ , i. e. with the  $i \leq n$  time indeces. If the conter inequality has been fillfilled, the head  $y$  has to be calculated by the formula

$$(20) \quad y_i = y_0 + \frac{a}{g} (v_i - v_{i-1}) + \frac{a}{g} \sum_{j=r+1}^{i-1} (v_j - v_{j-1}) + \frac{\Delta l}{C^2 R} \text{sign}(v_i) \frac{v_i^2 + v_{i-1}^2}{2} + \frac{\Delta l}{C^2 R} \text{sign}(v_j) \sum_{j=r+1}^{i-1} \frac{v_j^2 + v_{j-1}^2}{2},$$

where  $r_{\text{prop}} = \frac{i\Delta t}{T}$  is counter of all the phases.

The head for an arbitrary cross section, indicated as 'k', can be defined by the same method with integration of (18) according to Euler's method:

$$(21) \quad y_i^k = y_0 + \frac{a}{g} \sum_{j=r}^k (v_j - v_{j-1}) + \frac{\Delta l}{C^2 R} \text{sign}(v_j) \sum_{j=r}^k \frac{v_j^2 + v_{j-1}^2}{2},$$

where  $1 \leq k \leq n$

Using the above mentioned method, with the introduction of counter 'r' formula (21) can also be transformed in case of  $k \geq n$ . From formulas (19)-(21) it becomes clear, that the calculation of the heads  $y_i$  and  $y_i^k$  is only possible when we know the law for the change of velocity  $v(t)$ , i.e. if the boundary condition in cross section 0-0 is given.

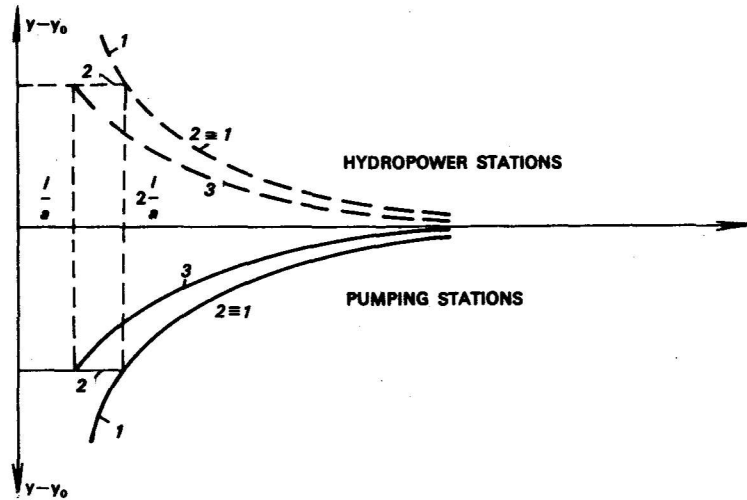


Fig. 2.

It might be interesting to note, that the method of integrating (1),(2) on a characteristic direction is not the only one possible here. In papers [1],[2] Nikolov has obtained formulas, analogous to (19)-(21) and achieved in a different way, namely by a direct integration of Bernoulli's equation, applied individually for length and for time. On the other hand in his work [3] Nikolov has likewise obtained analogous formulas (19)-(21) on the basis of Riman's general integral for the (1),(2) system, taking the function of the variables  $(t + x/a)$  of the reversed waves as a zero. According to Nikolov the physical interpretation is built up on the presumption, that by a given direction

of the velocity no accumulation of positive and negative waves exists, more over, no reflected reverse waves at the top end arise. Nikolov's paper [3] proves, that the well-known Allievi's chain equations, deduced by the accumulation of positive and negative waves, for time of stopping of the flow greater than  $2l/a$  do not take elasticity into consideration (Fig.2). The method of characteristics, deduced from the accumulation of positive and negative waves, analogously does not take elasticity into consideration. This is confirmed numerically in Tabl.1 and [5].

In order for the influence of the elastic properties of the fluid and the pipe material to be assessed numerically, we provide one more method for numerical solution of system (1),(2). We shall use the well-known method of the characteristics with the same step along length  $\Delta l=l/n$  and for time  $\Delta t=\Delta l/a$ . The differential way of writing equations (1) and (2) in finite difference by the method of characteristics is the following:

Tabl.1

Methods	$T_{prop} = \frac{l}{a}$ [s]	$T_{min}$ [s]	$k = \frac{T_{min}}{T_{prop}}$	$y_0$ [m]	$y_{min}$ [m]	$k_{elas} = \frac{\Delta y_{min}}{y_0}$ [%]
Method of two characteristic directions	0.191	1.312	6.87	61	15.43	Non-accounted elasticity
	2.130	4.024	1.90	118	-9.30	
	3.240	3.600	1.10	337	224.40	
Nikolov's method by means of eq. (19), (20)	0.191	1.264	6.65	61	16.42	1.62
	2.130	6.390	3.00	118	20.80	25.50
	3.240	3.130	0.966	337	225.70	0.38

$$(22) \quad Y_1^J = 0.5 \left[ Y_{1-1}^{J-1} + Y_{1+1}^{J-1} + \frac{a}{g} (v_{1-1}^{J-1} - v_{1+1}^{J-1}) \right]$$

$$(23) \quad v_1^j = 0.5 \left[ v_{1-1}^{j-1} + v_{1+1}^{j-1} + \frac{a}{g} \left( Y_{1-1}^{j-1} - Y_{1+1}^{j-1} \right) - \frac{a\Delta t}{C^2 R} \left( v_{1-1}^{j-1} |v_{1-1}^{j-1}| - v_{1+1}^{j-1} |v_{1+1}^{j-1}| \right) \right],$$

$$- \frac{g\Delta t}{C^2 R} \left( v_{1-1}^{j-1} |v_{1-1}^{j-1}| + v_{1+1}^{j-1} |v_{1+1}^{j-1}| \right) \right],$$

where  $Y = H + \frac{p}{\gamma}$ , the top index signifies the time  $t_j = j\Delta t$ , the lower the length  $l_i = i\Delta l$ .

The boundary condition in cross section 1 at the reservoir is  $Y = \text{const}$ . The boundary condition in cross section 0 at the pump is described most precisely by Nikolov using the method in [2], taking into account the four quadrant characteristic of the pump. In this case the method is only used for I and IV quadrant, with the characteristics  $Q-H$  and  $Q-\eta$  being approximated with cubic splines. The capacity  $N$  is calculated at random moment  $t_i = i\Delta t$  by the following formulas:

$$(24) \quad \begin{array}{ll} H_{o_1} \geq 0 & H_{o_1} < 0 \\ N_{o_1} = \frac{9.81 Q_{o_1} H_{o_1}}{\eta_1 \eta_m^{\text{mech}}} & N_{o_1} = \frac{9.81 Q_{o_1} (H_{o_1} + \Delta H_1)}{\eta_m \eta_q \eta_m^{\text{mech}}} \end{array}$$

where  $\eta_m^{\text{mech}}$  is mechanical efficiency of the motor,  $\eta_m$  - mechanical efficiency of the pump,  $\eta_q$  - efficiency of flowing water. The losses  $\Delta H_1$  are defined by the formula

$$\Delta H_1 = An_o^2 + Bn_o Q_{o_1} + H_{o_1}$$

The coefficients  $A$ ,  $B$  are determined through two points from  $Q-H$  of highest efficiencies  $\eta_1$ . After the calculation of the capacity  $N_1$  at the moment  $t_i = i\Delta t$  it is necessary to find the revolutions  $n_1$ . For this purpose are used the relationships

$$(25) \quad \begin{array}{ll} n_1 \geq 0.3n_o & n_1 < 0.3n_o \end{array}$$

$$n_1 = \frac{k_1}{\frac{k_1}{n_{1-1}} + \Delta t} \qquad n_1 = n_{1-1} - \frac{\Delta t}{k_1} (0.3n_0)^2$$

where

$$k_t = \frac{n_0 [GD^2]}{365000 \bar{N}_{o_1}}, \qquad \bar{N} = \frac{N_{o_{1-1}} + N_{o_1}}{2}$$

$[GD^2]$  - the rotary moment of the pumping aggregate.

By the help of similarity laws the head and the velocity at the moment  $t = t\Delta t$  are determined

$$H_1 = H_{o_1} \left( \frac{n_1}{n_0} \right)^2, \qquad v_1 = v_{o_1} \left( \frac{n_1}{n_0} \right), \qquad Q_{o_1} = v_{o_1} F_{pipe}$$

where  $H_{o_1}$  and  $Q_{o_1}$  - head and discharge of the pump for  $n_0$ .

The head in the rotodynamic pump at the moment  $t$  is calculated by formula

$$(26) \qquad h_1^{r.p.} = H_1 \mp h_1^s,$$

where  $h_1^s$  is suction height of the pump (the sign minus is valid for

unsubmerge pump, plus - for submerged pump). The head in the pump calculated by formula (26) must be equal to the head provoked by the waterhammer and calculated by formulas (19) or (20), of course with in the limits of the accepted precision. It is necessary to organize an iteration procedure related to discharge  $Q_{o_1}$  on order to achieve the

required conditions for precision. This iteration procedure is based on Newton's method of solving non-linear systems on the basis of equations (24)-(26). The numerical examples, given in the present paper are solved by the help of the formulas proposed. The prepared package of applicable programmes is intended for PC-XT (AT) microcomputers and other compatible ones. The solved examples refer to cases of simple pipes ( $d = \text{const}$ ,  $\delta = \text{const}$ ) with direct and indirect waterhammer.

At a sudden power failure if we accept that the pumps stop instantly, then we assume that the discharge in the pipe stops too. In these extreme conditions, the time of the full stopping of the flow  $T_s =$

head, accounting for elasticity, in this case is:  $y - y_0 = - \frac{av_0}{g}$  and the maximum growth of the head is  $y - y_0 = \frac{av_0}{g}$ . The maximum fall at a direct waterhammer according to the new interpretation is accepted for  $T_s \leq T_{prop} = \frac{l}{a}$ . The diagrams of the absolutely increased head in hydropower stations and the absolutely low head at pumping stations, given in the shape of  $y - y_0 = f(t)$ , with more important practical cases of direct and indirect waterhammer, are to be seen in Fig.2. The curves 1,2,3 refer to the following cases:

- a - Curve 1 gives the change of  $y - y_0 = f(t)$  without taking into account elasticity.
- b - Curve 2 refers to the method of characteristics.
- c - Curve 3 refers to Nikolov's method realized by formulas (19), (20). The results from some other important examples, which refer to waterhammer at pumping stations, are given in Tabl.1.

On the basis of the numerical examples in Tabl.1 and the diagrams in Fig.2 the following conclusions can be made:

1. The minimum head  $y_{min}$  by Nikolov's method (taking into account elasticity) is always bigger than  $y_{min}$  calculated by the the method of characteristics, i.e. Nikolov's waterhammer  $y_{max}$  is always smaller. This result is confirmed by means of laboratory measurements as well as in nature.
2. The percentage difference  $k_{elas}$  is the biggest with  $k = \frac{T_{min}}{T_{prop}} = 2 - 4$  and can go up to 15+20+30% of the geodetic head  $y_0$  ( $T_{min}$  means the time taken to reach  $y_{min}$ ).
3. With high values of  $k$  ( $k > 6$ ) and values of  $k \leq 1$  the coefficient  $k_{elas}$  decreases and tends to zero (see Tabl.1).

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