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Thermocapillary Convection in a Rectungular Cavity at Minimum of Surface Tension

1. Introduction

A steady thermocapillary convection in a viscous liquid filling a differentially heated rectangular cavity is explored in the case when the surface tension of the liquid is a non-linear temperature function. Such a variation of the surface tension is established for binary metallic alloys as *Al–Sn* or *Ag–Pb,* and solutions like aqueous n-heptanol solutions, etc. [1]. The unusual behaviour of the surface tension, during the processing of such alloys from a liquid phase, results in specific hydrodynamic phenomena due to thermocapillary forces. As the forces dominate under microgravity conditions, the capillary flows are important for metallurgy and crystal growth in Space.

The main features of the flow pattern due to the non-linear dependence of the surface tension are demonstrated on the base of a numerical solution of the two-dimensional Navier-Stokes equations. The solution is compared to the self-similar one of these equations for an infinite liquid layer with a free surface along which the temperature changes linearly.

2. Formulation of the problem

Let us consider a rectangular cavity filled with a viscous incompressible liquid of constant properties, except for the surface tension

$$\sigma = \sigma_0 + \delta(T - T_0)^2, \tag{1}$$

where δ is a positive constant, T_0 and σ_0 are surface temperature and tension at the center of the cavity. The left and right endwalls are imposed at temperatures $T_0+\Delta T$ and $T_0-\Delta T$, respectively ($2\Delta T$ being the temperature difference). The surface tension decreases from the left wall to the center and increases from the center to the right wall. The free surface is assumed flat, undeformable, and adiabatic. The cavity bottom is also insulated. The heat flux is directed from the left wall to the right one.

The liquid motion inside the cavity is described by the following dimensionless equations, written in variables - stream function ψ $(u=\psi_y,\ v=-\psi_x)$, vorticity $\omega = -u_y + v_x$ and temperature $\theta = (T - T_o)/\Delta T$,

$$\mathrm{Re}\,(u\omega_x + v\omega_y) = \omega_{xx} + \omega_{yy},$$

(2) $$\psi_{xx} + \psi_{yy} = -\omega,$$

$$\mathrm{Ma}\,(u\theta_x + v\theta_y) = \theta_{xx} + \theta_{yy}.$$

Here the subcripts denote differentiation, x and y are Cartesian coordinates, u and v the corresponding velocity components, $\mathrm{Re} = \delta(\Delta T)^2 H/\mu\upsilon$ the Reynolds number, $\mathrm{Ma} = \mathrm{RePr}$ the Marangoni number, $\mathrm{Pr} = \upsilon/k$ the Prandtl number, μ and υ dynamic and kinematic viscosities, k thermal diffusivity. The scales for length and velocity are the layer thickness H and quantity $\delta(\Delta T)^2/\mu$ resulting from the tangential force balance at the free surface.

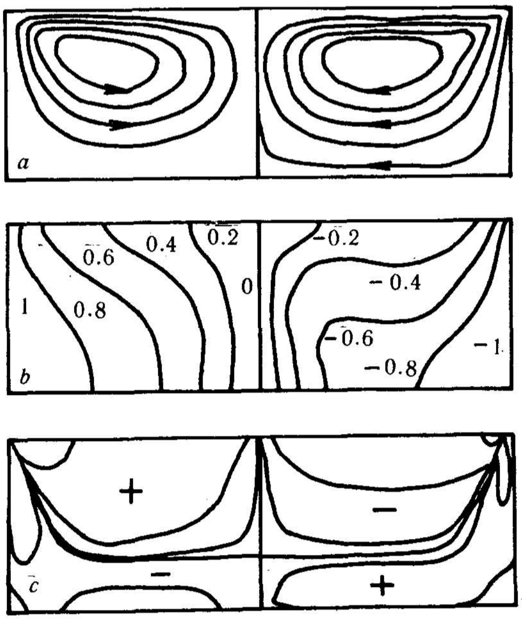

Fig.1. Flow pattern for Pr=7 and Re=100 (left) and Re=1000 (right)

The boundary conditions are as follows

$$
\begin{aligned}
&\psi = \psi_y = 0, \quad \theta = \pm 1 && \text{at } x = \mp d, \\
(3) \qquad &\psi = \psi_y = 0, \quad \theta_y = 0 && \text{at } y = 0, \\
&\psi_y = 0, \quad \omega = -\theta\,\theta_x, \quad \theta_y = 0 && \text{at } y = 1,
\end{aligned}
$$

where d=L/2H and L is the cavity length. The expression of the vorticity at the free surface follows from the tangential force balance and the other conditions are assumed as usual. The problem (2–3) is solved numerically by the method discussed elsewhere [2]. A uniform 65 x 65 grid is used. All calculations are performed for d=1,5, e.g. the cavity height is chosen to be three times smaller than its length.

3. Results

Some typical results – stream lines (a), isotherms (b) and lines of equal vorticity (c) are presented in Fig. 1 for Pr=7 and two values of Re. As the flow is obviously symmetrical about the axis y, the contours are plotted for Re=100 in the left part of the domain and for $Re=10^3$ in the right one. The later value of the Reynolds number corresponds to the layer of a n–heptanol solution with H=1 cm and ΔT = 2,7 K.

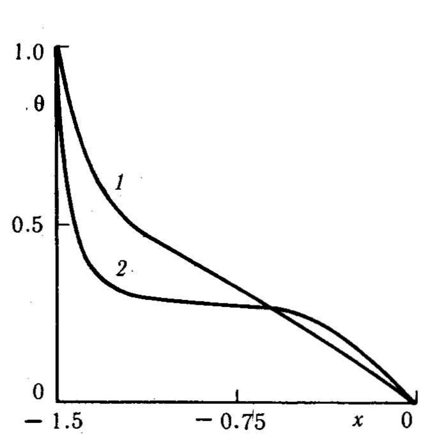

Fig.2. Surface temperature at Re=100 (curve 1) and 1000 (2)

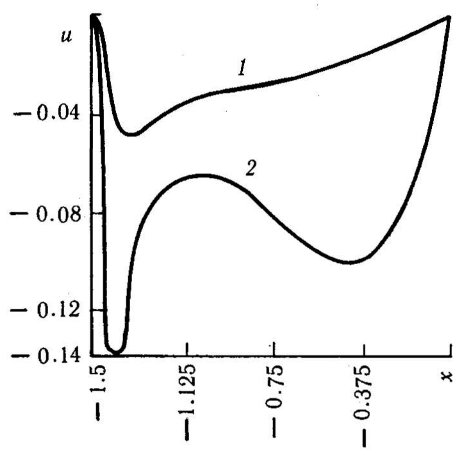

Fig.3. Surface velocity at Re=100 (curve 1) and 1000 (2)

The surface temperature and velocity are shown in Figs. 2 and 3, respectively (lines *1* for Re=100 and lines *2* for $Re=10^3$). Both quantities have a different behaviour for various Reynolds numbers. At Re=100 the temperature first decreases rapidly in the vicinity of the l.h.s. wall and then goes down linearly to zero at the cavity center. The absolute value of the velocity also changes quickly to a maximum situated close to the wall and then decreases monotonically.

At $Re=10^3$ after a rapid decrease near the wall the temperature doesn't change very much in some central part between the wall and the cavity center, where the surface velocity has two additional local extrema. The appearance of the extrema results from the weak action of the surface force in the interval between their positions due to the nearly zero temperature gradient.

As in the l.h.s. part of the cavity the surface tension decreases with temperature, the fluid in the vicinity of the free surface goes from the "cold" center to the hot wall. In the r.h.s. half σ increases with decreasing the surface temperature from the "hot" center to the cold wall and the upper liquid moves again from the center. As is shown in Fig. 1a, there exist two symmetrical vortices in the cavity, the l.h.s. vortice being counter-clockwise, while the r.h.s. one is clockwise. The flow pattern differs considerably from the one in case of linear temperature dependence of the surface tension when one clockwise vortice is only observed [3,4].

Fig. 1,b shows that the fluid motion influences the temperature field and this effect is stronger for higher Reynolds numbers when some isotherms have a horizontal portion. The explanation of the heat distribution is the following. In the l.h.s. part of the cavity a warm liquid goes from the wall to the center near the bottom, while a cold liquid in the upper part of the layer flows back to the hot wall. Thus, the heat exchange between the lower warm sublayer and the upper cold one takes place mainly in the vortice core. In the r.h.s. half of the cavity the heat flux is from the upper warm fluid to the lower cold one.

The vorticity field shown in Fig. 1c is characterized by a change of the vorticity sign in the bulk on a line with a horizontal portion in the central part of the cavity. The appearance of this line is not specific for thermocapillary flows of liquids with surface tension depending linearly on temperature. In such flows an one-sign vorticity core is usually observed (see, for example, [4]).

The features of closed streamline flows at a non-linear temperature dependence of the surface tension can be understood if the numerical results for the cavity are compared to the self-similar solutions of equations (2) obtained in an infinite liquid layer with a linear temperature variation along the free surface. Such solutions are reported for viscous liquids with surface tension (1) filling a half space [5,6]. A self-similar solution is also obtained in the case of a liquid layer bounded by an adiabatic free surface and a rigid wall along which the temperature changes linearly [6].

As seen in Fig. 2, in the central portion of the free boundary the surface temperature could be approximately presented by the funtion

$$\theta_s = -ax, \tag{4}$$

where $(-a)$ is the slope of the temperature curve at $x=0$. The numerical analysis shows that for different values of Re the positive constant a varies from 0,5 to

1,1 (for instant, a = 0,509 at Re = 100).

To study the thermocapillary flow in an infinite layer of thickness H with dimensionless surface temperature (4), we introduce new functions by the expressions (a is included into the temperature scale)

$$\psi = x\, f(y), \quad \omega = -x\, f''(y), \quad \theta = -x\, \theta_1(y). \tag{5}$$

Substituting them into (2) yields the equations

$$f''' + \text{Re}\,(f f'' - f'^2) - \dot{\lambda} = 0, \tag{6}$$

$$\theta_1'' + \text{Ma}\,(f\, \theta_1' - f'\, \theta_1) = 0. \tag{7}$$

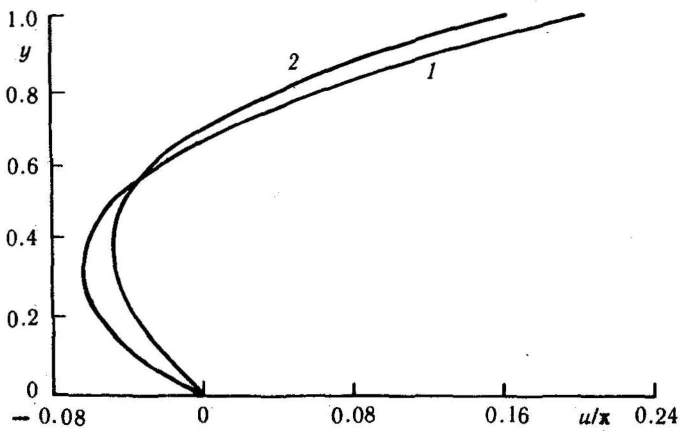

Fig. 4. Longitudinal velocity profiles u/x at Re=40 (curve 1) and Re=138 (2)

The boundary conditions are as follows

$$\begin{aligned} &f(0) = f'(0) = 0, \quad f\,(1) = 0, \\ &\theta_1'(0) = 0, \quad f''(1) = \theta_1^2(1) = 1. \end{aligned} \tag{8}$$

The prime denotes differentiation with respect to y and the arbitrary constant λ comes from the expression of the pressure

$$p = \lambda \frac{x^2}{2} + p_o(y), \tag{9}$$

where the function $p_o(y)$ is related with f through the y–momentum equation. The third-order equation (6) with four boundary conditions can be solved numerically, but separately from equation (7) using the fourth condition for determination of the unknown constant λ. The two-point boundary-value problem (6)(8) is very similar to the problem of the flow in a channel with an accelerating surface velocity studied in [7] and an analogous numerical procedure has been used.

In Fig. 4 we present some preliminary results of the numerical solution showing the existence of zero vorticity line at which the longitudinal velocity has an extremum inside the layer. The function $f'(y) = u/x$ is plotted for two values of the Reynolds number (Re = 40, λ = 0,927 and Re = 138, λ = 0,446). It is worth to note that for these, relatively small Reynolds numbers, the zero vorticity line lies at a distance of 0,35H from the wall and this value corresponds to the distance between the lowest part of the zero vorticity line and the cavity bottom shown in Fig. 1c.

A detailed analysis of the solution of the problem (6–8) will be a subject of future work.

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