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Mathematical Modelling of Non-Stationary Flow of Liquid Metal through Vertical Sprue Tubes

1. The technological process.

A mould steel casting process will be considered. The conceptual design of a casting machine, realizing the process under study, is shown in Fig. 1. The casting system consists of an enclosure for melt steel into which the lower end of the vertical sprue tube is immersed. The top end of this sprue tube is coupled with the bottom of the casting mould. The moving force is the gas pressure difference between the lower and the upper cameras, i.e. between the melt enclosure and the mould. Under the action of this gas pressure difference the melt flows counter the gravity force through the sprue tube from the enclosure to the mould. The casting process under consideration may be divided into three time stages. The first stage is sprue tube filling. This process passes in a very short time (4-7 sec) under a high gas pressure difference. That is why no essential disturbances of the temperature field in the tube wall and in the melt during this stage appear. In order for the fountain formation at the mould entry to be avoided, the controlling pressure difference drops when all the tube is filled. The second stage is mould filling. During this stage the melt flow rate through the sprue tube is much lower than during the first one. The mould filling duration is about 2 to 8 and evenmore minutes. For this reason the heat transfer in sprue system is essential during the second stage. The third stage is steel solidification in the mould. It is well known that due to the different densities of the molten and solid metals and alloys, a cavity is formed in the top part of the ingot, produced by mould casting. That is why one of the main problems in this field is to minimize such top cavities. One of the ways to solve this problem in the casting machine consists in a compensation of the volume defect, arising during metal solidification in the mould, by an extra quantity of melt from the enclosure. It means that the melt flow through the sprue tube continues during the third stage, but inlet flow rate V is much lower than in the second one. It must be noted that during both (second and third) stages inlet flow rate V is generally a time dependent quantity: $V=V(t)$. The aim of the present paper is to investigate heat transfer and melt flow in the sprue tube during the casting process as described above. The paper gives an account of the suggested mathematical model and of the numerical algorithm for solving the arising system of partial differential equations. The results from some numerical experiments are reported too.

2. The mathematical model.

The sprue tube under consideration is a multilayer vertical tube of circular cross section. Its length Z is equal to eighty times its internal diameter. The geometrical dimensions ratio of the sprue tube allows the boundary layer equations [1] to be used in order to describe the melt flow through the tube conjugate with the heat transfer in the melted steel and in the tube wall. The melted steel flowing through the tube can be considered as a *Newtonian incompressible fluid* [2]. Because the inlet flow rate V depends on the time t, $V=V(t)$, the unsteady equations are considered. The domain of interest is shown in Fig. 2.

The dimensionless convection/diffusion heat equation is:

$$\rho(T)\left[c(T)\text{-}Ste\,\frac{\partial\Psi}{\partial T}\right]\left[\frac{\partial T}{\partial t}+u\,\frac{\partial T}{\partial r}+v\,\frac{\partial T}{\partial z}\right]=\frac{1}{Pe}\,\frac{1}{r}\,\frac{\partial}{\partial r}\left[rk(T)\,\frac{\partial T}{\partial r}\right].$$

Here the temperature field $T(r,z,t)$ is referred to a cylindrical coordinate system (r,z), where r, $0<r<R$, is the radial coordinate, and z, $0<z<Z$, is the axial one. By $c(T)$, $\rho(T)$, and $k(T)$ the efficient heat capacity, density and heat conductivity, taking into account the composition and the phase state of the medium, are denoted respectively. The r- and z- velocity vector components are denoted by u and v respectively. The internal radius of the tube R_1, the inlet velocity $V_0=V(0)$, the ratio R_1/V_0 and temperature gradient $\delta T=T^0\text{-}T_a$ are used, respectively as linear, velocity, time and temperature scales. Here T^0 is the melt temperature in the enclosure, adopted to be a known constant, and T_a is the temperature of the surrounding atmosphere. The *Peclet number* is defined as $Pe = V_0R_1\rho(T^0)c(T^0)/k(T^0)$ and the *Stefan number* is given by $Ste=L/(c(T^0)\delta T)$, where L is the latent heat of phase change. The function $\Psi(T)$ is the relative part of the solid phase. The derivative $\partial\Psi/\partial T$ is non-zero in the mushy region only. At the tube axis the usually posed symmetry condition is considered. On the outside tube wall *Newton's low* (heat trasfer coefficient α) holds. Since the internal tube wall is a liquid-solid interface, or else a gas space exists there, this

surface is considered as a thermal resistence (heat transfer coefficient α_1). The thermal contact between the different tube wall layers is assumed to be ideal. As it was mentioned above, the melt is considered to be*Newtonian incompressible liquid*. The governing *Boussinesq approximation* to the *parabolized Navier-Stokes equations* describing laminar flows may be written in terms of stream function ψ (scale $R_1^2 V_0$) and vorticity ω (scale V_0/R_1) as follows:

$$\frac{\partial \omega}{\partial t} + \frac{\partial u \omega}{\partial r} + \frac{\partial v \omega}{\partial z} = \frac{1}{Re}\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial r\omega}{\partial r}\right] + \frac{Gr}{Re^2}\frac{\partial T}{\partial r},$$

$$\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial \psi}{\partial r}\right] = -\omega, \quad \text{where } u = \frac{1}{r}\frac{\partial \psi}{\partial z}, \; v = -\frac{1}{r}\frac{\partial \psi}{\partial r}, \quad \omega = \frac{\partial v}{\partial r}$$

Here $Re=V_0R_1/\upsilon$ and $Gr=g\beta\delta TR_1^3/\upsilon^2$ are *Reynolds* and *Grashof numbers* respectively, υ- kinematic viscosity, β- volumetric coefficient of thermal expansion, g - gravity acceleration. Let $\Gamma^*(t)$ denote the boundary of the solid phase if it exists or the tube internal wall in the other case. As usual, non-slip boundary conditions are considered on $\Gamma^*(t)$. On the line $r=0$ the symmetry condition for ω is used. It can be noted that systems like the above described, but considering the steady state process without solidification in conditions of constant wall temperature, are widely investigated (see, for example, [3], [4]). The solidification of the melt and the cooling of the tube are taken into account in [5] for the case of stationary processes.

3. The numerical algorithm.

The finite difference method (see, for example, [6]) on a quasi-uniform grid [5] is used for solving the obtained system of PDE's. An implicit linearized scheme is considered. The computations are realized at each time step as follows: i) temperature field $\hat{T}$ is computed by a marching algorithm from the heat equation and the boundary conditions using the values of the velocity components and the coefficients determined at the previous level in the z-direction; ii) the new values of coefficients $c(\hat{T})$, $\rho(\hat{T})$, $k(\hat{T})$ and of function $\Gamma^*(t)$ are determined; iii) new $\hat{\omega}$ - and $\hat{\psi}$ - fields are computed consequently and at the end of the computations new velocity components are determined. It has to be noted that the marching procedure in the z-direction used here, allows a TDA - method to be used.

4. Numerical experiments.

In accordance with the description of the first stage of the casting process, the thermal changes in the tube wall and in the melt are neglected during this stage, i.e. an instant filling of the tube is assumed. During the second stage the order of *Reynolds* and *Grashof numbers* is $Re=5.7*10^4$ and

$Gr=3.8*10^5$ respectively. Therefore, forced convection has to be considered. All the more that, the flow regime is a turbulent one. Since the temperature field is of main interest for the present investigation, only the heat equation is considered at this stage. The velocity field is assumed to be known. On the other hand during the ingot solidification (the third stage) the *Reynolds number* is approximately equal to $Re=657$ and laminar, coupled, forced and natural, convection is considered.

The results from some numerical experiments are shown below. As it was noted above, the third stage of the considered technological process is most interesting for us. In Fig.3. the time dependence of the average values of the temperature in the tube for different z-cross-sections is presented. The results illustrate the unsteady behaviour of the considered process. It is shown that in every cross section the average value of this temperature decreases quickly and after that it increases slowly. This behaviour is due to the fact that during the first and second stages of the considered technological process the tube wall had not been heated enough. In Fig.4. the velocity value at the center line of the tube $v(0,z,t)$ is presented in the same z-cross-sections as a function of time. Of course, the behaviour of the velocity $v(0,z,t)$ is related to the temperature behaviour. The reported results show that for the regime considered, there does not exist a solid phase, but the mushy region is very wide at the end of the process.

All the results are obtained for a two-layerwalled tube, using the following parameters: melt temperature $T^0=1600^\circ C$, temperature *liquidus* $T_l=1469^\circ C$, temperature *solidus* $T_s=1305^\circ C$, $T_a=20^\circ C$. During the second stage $Pe=3934$. In the third stage $Re=657$, $Pe=45.2$, $Bi_1=\alpha_1 R_1/k=6.62$, $Gr=3.8*10^5$, $Bi=\alpha R_1/k=0.02$, $Ste=0.37$.

References

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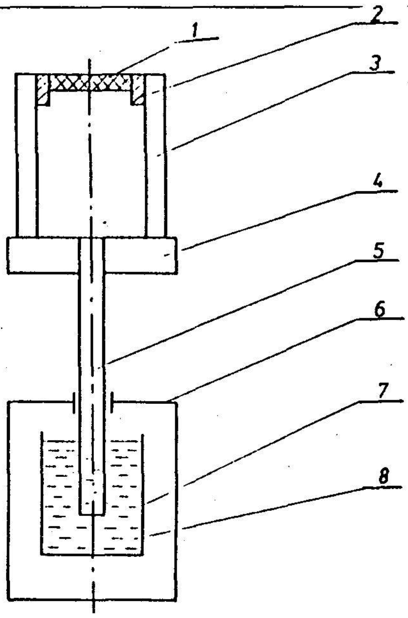

Fig. 1. The casting system:
1 - gas-permeable top cover of the mould; 2 - heat - insulating ring insertion; 3 - casting mould; 4 - mould botom; 5 - sprue tube; 6 - lower camera; .7 - enclosure; 8 - melt.

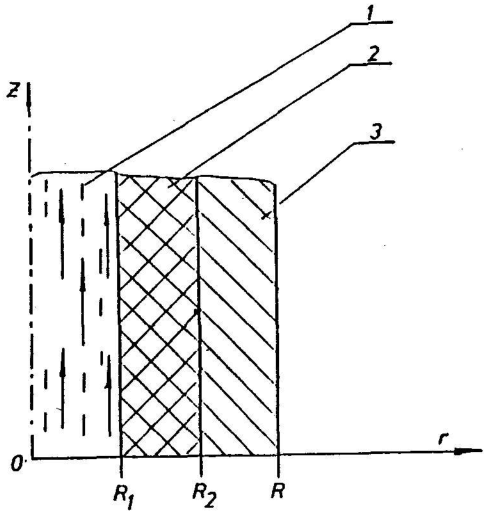

Fig. 2. The computational domain: 1 - melt flow; 2 - inner layer of the tube wall; 3 - outer layer of the tube wall.

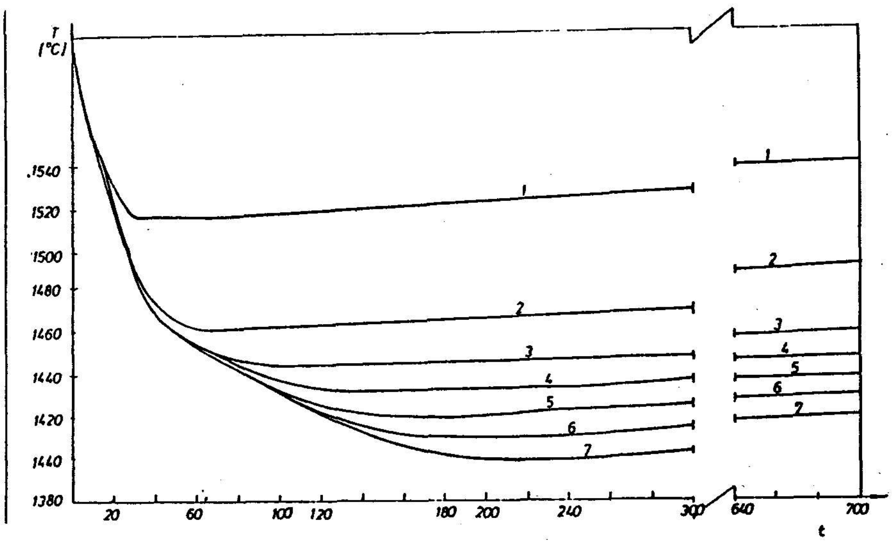

Fig. 3. Time dependence of the average temperature in the tube for different *z* - cross sections:
1-z=0.5m, 2-z=1.0m, 3-z=1.5m, 4-z=2.0m, 5-z=2.5m, 6-z=3.0m, 7-z=3.5m.

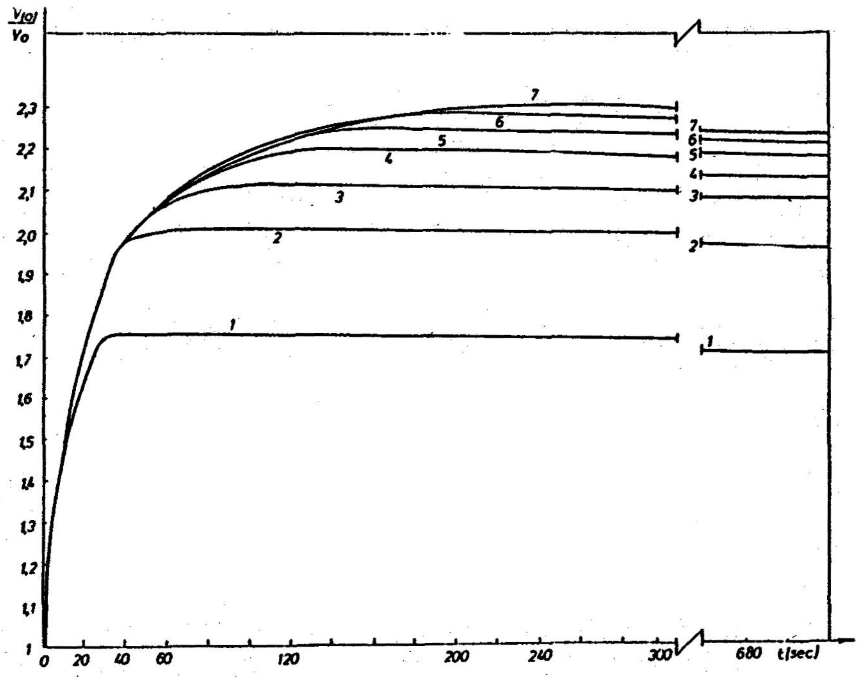

Fig. 4. Time dependence of the velocity at the center line of the tube for different z - cross sections: 1-z=0.5m, 2-z=1.0m, 3-z=1.5m, 4-z=2.0m, 5-z=2.5m, 6-z=3.0m, 7-z=3.5m.