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E. T o s h e v

Force Interaction Between Two Equal Spheres in Tandem and Uniform at Infinity Flow

The present paper deals with the numerical study of the force interaction between two equal rigid spherical particles in tandem and a viscous flow, which is uniform at infinity and parallel to the central line of the particles. Due to the axial symmetry of the geometrical configuration of the particles and the stream at infinity, the flow is axisymmetrical. By numerical integration of the 2D Navier-Stokes equations the characteristics of the flow have been obtained. Results connected with the cases of small distance between the particles (1/8 radius, 1/4 radius etc.) are found out. The present work could complement the investigations [2], [4] and [5].

The bispherical co-ordinate system (ξ,η,ϑ) is defined by the following conformal transformation:

$$z = c\,\frac{sh\xi}{ch\xi - cos\eta}\quad ; \quad \rho = c\,\frac{sin\eta}{ch\xi - cos\eta}\quad ; \quad 0\leq\vartheta\leq 2\pi$$

here (z,ρ,ϑ) is a normal cylindrical coordinate system.

The dimensionless Navier-Stokes equations in terms of bispherical co-ordinate system and (ψ,ζ) formulation have the following form:

$$\frac{\partial\,\zeta}{\partial\,t_1} + sin\eta(ch\xi - cos\eta)\text{x}$$

$$\text{(1)}\qquad \left[\frac{\partial\psi}{\partial\xi}\frac{\partial}{\partial\eta}\left[\frac{(ch\xi - cos\eta)^2}{sin^2\eta}\zeta\right] - \frac{\partial\psi}{\partial\eta}\frac{\partial}{\partial\xi}\left[\frac{(ch\xi - cos\eta)^2}{sin^2\eta}\zeta\right]\right] =$$

$$\frac{\sin\eta(ch\xi - \cos\eta)}{Re}\left[\frac{\partial}{\partial\xi}\left[\frac{ch\xi - \cos\eta}{\sin\eta}\frac{\partial\zeta}{\partial\xi}\right] - \frac{\partial}{\partial\eta}\left[\frac{ch\xi - \cos\eta}{\sin\eta}\frac{\partial\zeta}{\partial\eta}\right]\right]$$

$$\frac{\partial\varphi}{\partial t_2}\sin\eta(ch\xi - \cos\eta)\times$$

(2)

$$\left[\frac{\partial}{\partial\xi}\left[\frac{ch\xi - \cos\eta}{\sin\eta}\frac{\partial\varphi}{\partial\xi}\right] - \frac{\partial}{\partial\eta}\left[\frac{ch\xi - \cos\eta}{\sin\eta}\frac{\partial\varphi}{\partial\eta}\right]\right] = -\zeta$$

$$\varphi = \frac{\sin^2\eta}{(ch\xi - \cos\eta)^2} - \psi$$

The considered problem is strongly stationary. The terms which contain the parameters t_1 and t_2 have not any physical sense. They are introduced to make possible using of the AID numerical method mentioned below.

The dimensionless boundary conditions are:

$\zeta = 0 \quad , \quad \varphi = 0 \; ; \; \eta = 0 \; , \; \eta = \pi$ axis of symmetry

$\zeta = 0 \quad , \quad \varphi = 0 \; ; \; \xi, \eta \rightarrow 0$ infinity point

(3)

$$\varphi = \frac{\sin^2\eta}{(ch\xi - \cos\eta)^2} \; ; \; \xi = \alpha_1 \; , \; \alpha_2$$ spheres

The drag coefficients inducted on the particles are:

$$W_{1,2} = \frac{sh|\alpha_i|}{Re_c} \int_0^{\pi} \left. \frac{\sin^3\eta}{(ch\xi - \cos\eta)^3} \frac{\partial}{\partial\xi} \left[\frac{(ch\xi - \cos\eta)^2}{\sin\eta^2} \zeta \right] \right|_{\xi = \alpha_{1,}} d\eta$$

The dimensions of the variables and quantities are:

$\rho=[c]$, $z=[z]$, $\psi=[U_0c^2]$, $W=[U_0^2\rho_0F/2]$, $F=\pi a^2$, $Re_c=U_0c/\upsilon$, $Re=2U_0a/\upsilon$

where U_0 is velocity at infinity, c - focal length, a - radius of the spheres, υ - kinematic viscosity, ρ_0 - fluid density, Re_c and Re are the two Reynolds numbers, calculated by two different character lengths, W_i - drag coefficients, F - surface of the main cross-section of the particle, t_1 and t_2 - fictitious time parameters (in fact they are the iteration parameters only), $\alpha_1$and $\alpha_2$are the parameters defining the surfaces of the particles.

The distribution of the stream function and vorticity in the flow areobtained by numerical integration of the Navier-Stokes boundary problem (1)-(3). The so called Alternating Direction Implicit (ADI) numerical method for the integration of both vorticity and stream function equations is used. The spatial approximations of the second order derivatives are made as recommended in [3]. The convective terms are approximated by using the central difference scheme. The integration is made over the uniform grid :

$$(\xi_i, \eta_j, t^n) \;;\; \xi_i = i.h,\ \eta_j = j.l,\ t^n = n.\tau$$

here h, l and τ are the steps in ξ, η and t directions respectively.

The problem of the missing boundary condition for vorticity over the solid surfaces is solved by using Tom's condition, associated with relaxation procedure:

$$\zeta_{w+1}^{n+1} = \left[- \frac{\sin\eta(ch\alpha_i - \cos\eta)}{h^2} \psi_{w+1}^{n} \right] \omega + (1 - \omega)\,\zeta_w^n \;;\; i = 1,2$$

here ω is the relaxation coefficient and $\omega = \omega(h)$, subscript n marks the number of the iterations and the subscript w marks the solid surface of the particles.

Systems of algebraic equations with tridiagonal structure are found after the discretization of the boundary-value problem. They have been solved using the Gauss successive procedure.

The main characteristics of the considered problem are the drag forces inducted over the particles from the stream.

The drag coefficients dependance on the distance between the particles for $Re = 40$ is shown in Fig. 1. The experimental value for a single sphere based on several experimental data [1] for comparison is given.

In Fig. 2 is given analogical graphical illustration of the same dependancefor $Re = 100$. In this case the upper limit of the distance between the particles is only 5 radii because of the convergence of the numerical method.

The streamlines patterns of the flow around the particles for $Re=40$ and $Re=100$ and a distance between them equal to 1/8 radius are shown in Fig. 3 and Fig. 4 respectively.

R e f e r e n c e s

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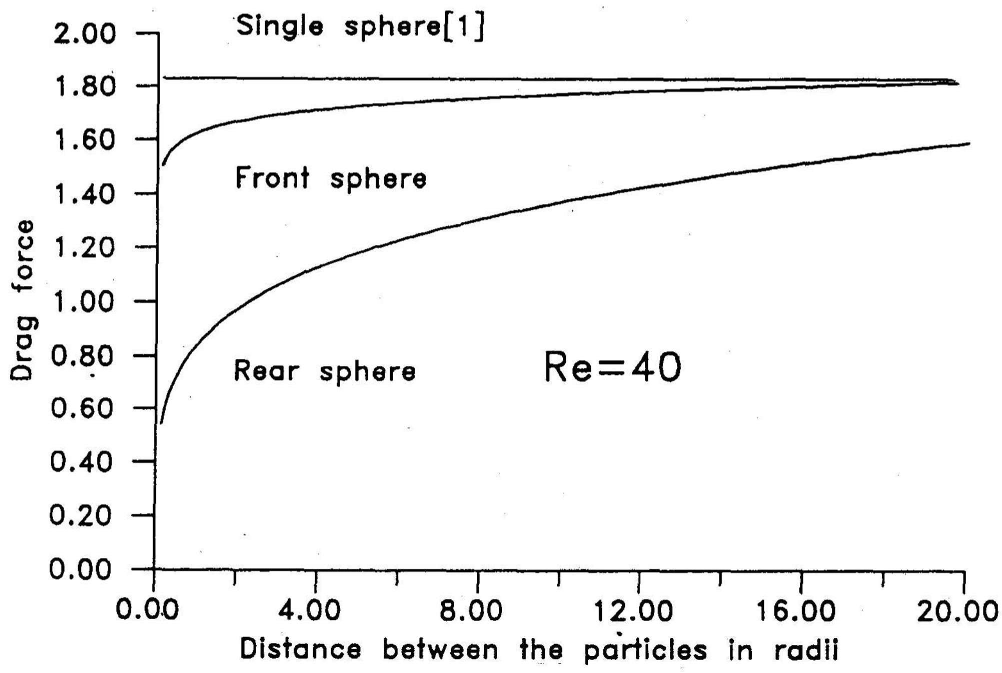

Fig. 1

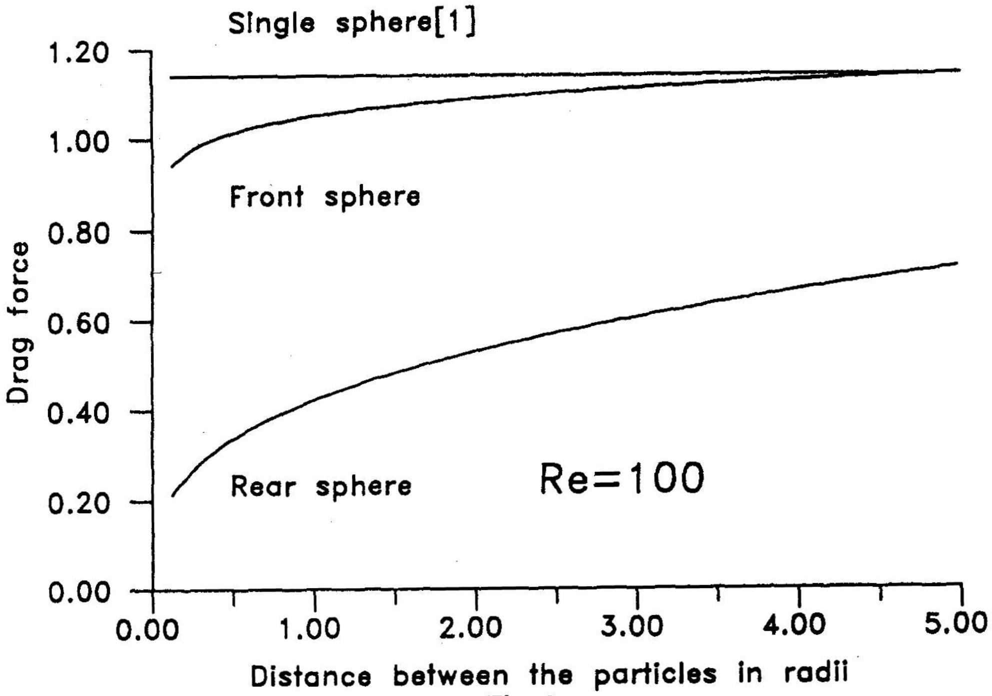

Fig. 2

Stream Function for Re=40 and D=R/8

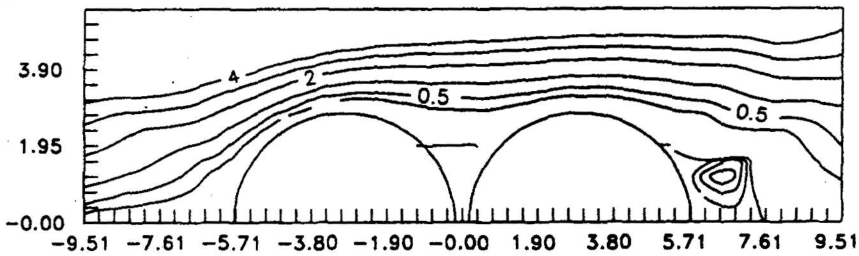

Fig. 3

Stream Function for Re=100 and D=R/8

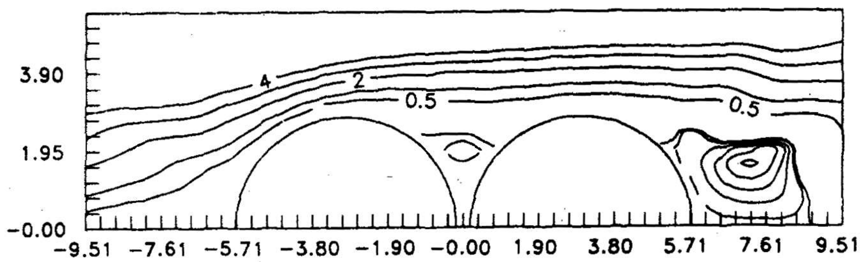

Fig. 4