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AXIAL MOTION EFFECT ON DYNAMIC RESPONSE OF VIBRATING TWO-SPAN BEAM, SUPPORTED ON AXIALLY VIBRATING COLUMN

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1. Introduction

The need of establishing the effect of moving loads on the dynamic behaviour of structures first appeared 150 years ago in the case of railway bridges. The greatest difficulty in solving this problem is the large number of parameters which enter into it. Thus, in order to face such a problem, significant simpifying assumptions should be done, in particular as far as the nature of the moving load is concerned. The ralatively simpler assumption one can make is to consider a single concentrated load moving with a uniform velocity. On this assumption was based the excellent treatment of Inglis [1], followed later by numerous investigators ([2-6]).

The present study is an extension of these analyses to the case of a two-span beam supported on axially vibrating column, acted upon by a constant force, moving with constant velocity across the spans. Analysis based on the well known normal modes technique leads to closed form solutions for the evaluation of the modal amplitudes.

Numerical results in both tabular and graphical form are presented herein, which compared with the existing ones [7,8] confirm the discrepancies which may occur between the classical dynamic analysis and the one used in this investigation.

2. Mathematical formulation

The geometrically perfect two-span beam, supported on a column shown in Fig.1, is made of homogeneous, isotropic and linearly elastic material with modulus

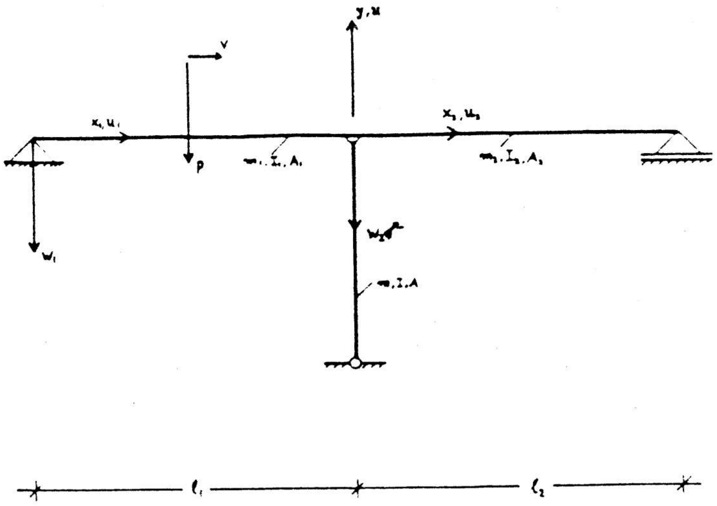

Fig. 1. Geometry and sign convention

of elasticity E. Each span is of length l_i, constant cross-sectional area A_i, constant moment of inertia I_i and mass per unit length $m_i(i = 1, 2)$, while the corresponding properties of the column are h, A, I and m respectively.

2.1. Free motion

Neglecting the effect of structural dumping, taking into account the axial inertia of the column and letting $w_i(x_i, t)$ be flexural deflection of the center line of the i-th span $(i = 1, 2)$, while $u(y, t)$ is axial displacement of the column, the differential equations governing the free motion of the beam are:

$$EI_i w_i^{(\text{iv})}(x_i, t) + m_i \ddot{w}_i(x_i, t) = 0 \quad (i = 1, 2), \tag{1}$$

$$EAu''(y, t) - m\ddot{u}(y, t) = 0, \tag{2}$$

subjected to the following boundary conditions

$$w_1(0, t) = w_2(l_2, t) = u(0, t) = w_1''(0, t) = w_2''(l_2, t) = 0,$$

$$w_1(l_1, t) = w_2(0, t) = -u(h, t), \quad -EI_1 w_1''(l, t) = -EI_2 w_2''(0, t)$$

$$EI_1 w_1'''(l_1, t) - EI_2 w_2'''(0, t) + EAu'(h, t) = 0. \tag{3}$$

For a free motion we can assume that

$$w_i(x_i,t) = \widehat{W}_i(x_i)e^{j\omega t} \quad (i=1,2)$$

(4) $$u(y,t) = \widehat{U}(y)e^{j\omega t}, \quad j^2 = -1.$$

Introducing the dimensionless quantities

$$\bar{x}_i = \frac{x_i}{l_i}, \quad \bar{y} = \frac{y}{h}, \quad \bar{w}_i = \frac{\widehat{W}_i}{l_i}, \quad \bar{u} = \frac{\widehat{U}}{h}, \quad \lambda_i^2 = \frac{A_i}{I_i}l_i^2, \quad \lambda^2 = \frac{A}{I}h^2,$$

(5) $$k_i^4 = \frac{m_i\omega^2 l_i^4}{EI_i}, \quad V^2 = \frac{m\omega^2 h^2}{EA}, \quad \mu = \frac{I_2}{I_1}, \quad \rho = \frac{l_2}{l_1}, \quad \varphi = \frac{I}{I_1}, \quad \sigma = \frac{h}{l_1}.$$

Eqs. (1) and (2) become

(6) $$\bar{w}_i^{(\mathrm{iv})}(x_i) - k_i^4\bar{w}_i(x_i) = 0 \quad (i=1,2),$$

(7) $$\bar{u}''(\bar{y}) + V^2\bar{u}(\bar{y}) = 0.$$

while boundary conditions (3) can be written as follows:

$$\bar{w}_1(0) = \bar{w}_2(1) = \bar{u}(0) = \bar{w}_1''(0) = \bar{w}_2''(1) = 0,$$

$$\bar{w}_1(1) = \rho\bar{w}_2(0) = -\sigma\bar{u}(1), \quad \bar{w}_1'(1) = \bar{w}_2'(0), \quad \bar{w}_1''(1) = \frac{\mu}{\rho}\bar{w}_2''(0)$$

(8) $$\bar{w}'''(1) - \frac{\mu}{\rho^2}\bar{w}'''(0) + \frac{\lambda^2\varphi}{\sigma^2}\bar{u}'(1) = 0.$$

Integration of Eqs.(6) and (7) and application of boundary conditions (8) lead to a non-trivial solution of the frequency equation:

(9) $$[a_{ij}] = 0,$$

where a_{ij} are given by

$$a_{11} = \sin k_1, \quad a_{12} = \sinh k_1, \quad a_{13} = \rho\tan k_2, \quad a_{14} = \rho\tanh k_2$$

$$a_{21} = -\sin k_1, \quad a_{22} = \sinh k_1, \quad a_{23} = -\mu\frac{\lambda_2}{\lambda_1}\tan k_2, \quad a_{24} = \mu\frac{\lambda_2}{\lambda_1}\tanh k_2$$

$$a_{31} = k_1\cos k_1, \quad a_{32} = \sinh k_1, \quad a_{33} = -k_2, \quad a_{34} = -k_2$$

$$a_{41} = -k_1^3 \cos k_1 - \frac{\lambda^2 \varphi V}{\sigma^3 \tan V} \sin k_1, \quad a_{41} = k_1^3 \cosh k_1 - \frac{\lambda^2 \varphi V}{\sigma^3 \tan V} \sinh k_1$$

$$a_{43} = \frac{\mu}{\rho^2} k_2^3, \quad a_{44} = -\frac{\mu}{\rho^2} k_2^3. \tag{10}$$

Taking into account that

$$V^2 = \frac{\sigma^2}{\lambda_1^2} k_1^4, \quad k_2 = k_1 \sqrt{\rho \frac{\lambda_2}{\lambda_1}}$$

and solving numerically Eq.(9) we obtain values of the dimensionless eigenfrequencies k_{in}, corresponding to the n-th mode. Thus the flexural and axial shape functions of the n-th mode are given by:

$$\bar{w}_{1n}(\bar{x}_1) = A_{1n} \sin k_{1n}\bar{x}_1 + C_{1n} \sinh k_{1n}\bar{x}_1$$

$$\bar{w}_{2n}(\bar{x}_2) = A_{2n} \sin k_{2n}\bar{x}_2 + B_{2n} \sinh k_{2n}\bar{x}_2 + C_{2n} \sinh k_{2n}\bar{x}_2 + D_{2n} \cosh k_{2n}\bar{x}_2$$

$$\bar{u}_n(y) = R_{1n} \sin V_n \bar{y}, \tag{11}$$

where $A_{1n}, C_{1n}, A_{2n}, B_{2n}, C_{2n}, D_{2n}, R_{1n}$ are determined by properly adjusting Eq.(7) and using the Gauss-Seidel method for solving the consequent algebric system, for any mode desired.

2.2. Orthogonality condition

Using Eqs. (6), (7) and boundary conditions (8), after some manipulation, we obtain the following orthogonality condition

$$\rho \int_0^1 \bar{w}_{1n}\bar{w}_{1k} d\bar{x}_1 + \mu\rho^2 \frac{\lambda_2^2}{\lambda_1^2} \int_0^1 \bar{w}_{2n}\bar{w}_{2k} d\bar{x}_2 + \frac{\rho\lambda^2\varphi\sigma}{\lambda_1^2} \int_0^1 \bar{u}_n \bar{u}_n d\bar{y} = 0 \quad (n \neq k). \tag{12}$$

2.3. Forced motion

Equation (12) enables us to investigate the forced motion of the system, due to a given dynamic loading. For instance, if the beam is subjected to a moving load of constant magnitude P and constant velocity v, one can proceed in solving this particular problem as follows:

a. The force moves in the first (left) span $(0 \leq t \leq \frac{l_1}{v})$

In this case, one can write the following equations of motion:

$$EI_1 w_1^{(\mathrm{iv})}(x_1, t) + m\ddot{w}_1(x_1, t) = P\delta(x_1 - vt). \tag{13}$$

(14) $$EI_2 w_2^{(\mathrm{iv})}(x_2,t) + m\ddot{w}_2(x_2,t) = 0.$$

(15) $$EAu''(y,t) - m\ddot{u}(y,t) = 0.$$

Using relations (5) and introducing the dimensionless velocity $\bar{v}$, time τ, load position $\bar{y}_p$ and modal amplitude $\bar{T}_n$

(16) $$\bar{v} = v\sqrt{\frac{m_2 l_2^2}{EI_2}},\quad \tau = \frac{t}{T_1} = 2\pi\omega_1,\quad \bar{y}_p = \frac{vt}{l_2} = \frac{\bar{v}\tau}{k_{21}^2},\quad \bar{T}_n = \frac{EI_2}{Pl_2^3}T_n,$$

one can express the lateral and axial components in dimensionless form as follows:

(17) $$\widehat{w}_{in}(\bar{x}_i,\tau) = \sum_{n=1}^{\infty} \bar{w}_{in}(\bar{x}_i)\bar{T}_n(\tau)$$

(18) $$\widehat{u}_n(\bar{x}_i,\tau) = \sum_{n=1}^{\infty} \bar{u}_n(\bar{x}_i)\bar{T}_n(\tau)$$

where $\bar{w}_{in}, \bar{u}_n$ are given in relations (11).

Using boundary conditions (8) and orthogonality condition (12) we reach the following differential equation for dimensionless modal amplitude $\bar{T}_n(\tau)$:

(19) $$\ddot{\bar{T}}_n(\tau) + Y_n^2\bar{T}_n(\tau) = \frac{\mu}{\rho^2}G_n\frac{\bar{w}_{1n}(\rho\bar{y}_p)}{k_{11}^4}$$

where

(20) $$Y_n^2 = k_{1n}^4/k_{11}^4$$

and

(21) $$G_n^{-1} = \rho\int_0^1 \bar{w}_{1n}^2 d\bar{x}_1 + \mu\rho^2\frac{\lambda_2^2}{\lambda_1^2}\int_0^1 \bar{w}_{2n}^2 d\bar{x}_2 + \frac{\rho\lambda^2\varphi\sigma}{\lambda_1^2}\int_0^1 \bar{u}_n^2 d\bar{y}.$$

If the beam is initially at rest, the general integral of Eq.(19) for the time period $0 \le \bar{y}_p \le \frac{1}{\rho}$ i.e. $0 \le \tau \le \frac{k_{21}^2}{\rho\bar{v}}$ is given by:

(22) $$\bar{T}_n(\tau) = \frac{\mu G_n}{\rho^2 k_{11}^4}\left[\frac{A_{1n}}{Y_n^2 - \Omega_n^2}\left(\sin k_{1n}\rho\bar{y}_p - \frac{\Omega_n}{Y_n}\sin\frac{Y_n k_{21}^2}{\bar{v}}\bar{y}_p\right) + \frac{C_{1n}}{Y_n^2 + \Omega_n^2}\left(\sinh k_{1n}\bar{y}_p - \frac{\Omega_n}{Y_n}\sin\frac{Y_n k_{21}^2}{\bar{v}}\bar{y}_p\right)\right]$$

where

(23) $$\Omega_n = \frac{k_{1n}\rho\bar{v}}{k_{21}^2}.$$

b. The load moves in the second (right) span $(\frac{l_1}{v} \leq t \leq \frac{l_1 + l_2}{v})$.

In this case $\frac{1}{\rho} \leq \bar{y}_p \leq 1 + \frac{1}{\rho}$ or $\frac{k_{21}^2}{\rho\bar{v}} \leq \tau \leq (1 + \frac{1}{\rho})\frac{k_{21}^2}{\rho\bar{v}}$ and the equations of motion are:

(24) $$EI_1 w_1^{(iv)}(x_1, t) + m\ddot{w}_1(x_1, t) = 0$$

(25) $$EI_2 w_2^{(iv)}(x_2, t) + m\ddot{w}_2(x_2, t) = P\delta(x_2 - vt + l_1)$$

(26) $$EAu''(y, t) - m\ddot{u}(y, t) = 0$$

Nondimensionalizing and following a procedure similar to the one used in case *a* we obtain the solution for the modal amplitude

$$\bar{T}_n(\tau) = \bar{T}_n^{\mathbf{H}}(\tau) + \bar{T}_n^{\mathbf{P}}(\tau)$$

with **H** standing for homogeneous and **P** for particular.

Thus, the resulting expressions for the two parts of $\bar{T}_n(\tau)$ are:

(27) $$\bar{T}_n^{\mathbf{H}}(\tau) = \frac{\dot{\bar{T}}_n\left(\frac{k_{21}^2}{\rho\bar{v}}\right)}{Y_n}\sin\left[\frac{Y_n k_{21}^2}{\bar{v}}(\bar{y}_p - 1)\right] + \bar{T}_n\left(\frac{k_{21}^2}{\rho\bar{v}}\right)\cos\left[\frac{Y_n k_{21}^2}{\bar{v}}(\bar{y}_p - 1)\right]$$

while

(28) $$\bar{T}_n^{\mathbf{P}} = \frac{\mu G_n}{\rho k_{11}^4}\left\{\frac{A_{2n}}{Y_n^2 - \Omega_n^2}\left[\sin k_{2n}\left(\bar{y}_p - \frac{1}{\rho}\right) - \frac{O_n}{Y_n}\sin\frac{Y_n k_{21}^2}{\bar{v}}\left(\bar{y}_p - \frac{1}{\rho}\right)\right] + \right.$$
$$+\frac{B_{2n}}{Y_n^2 - \Omega_n^2}\left[\cos k_{2n}\left(\bar{y}_p - \frac{1}{\rho}\right) - \cos\frac{Y_n k_{21}^2}{\bar{v}}\left(\bar{y}_p - \frac{1}{\rho}\right)\right] +$$
$$+\frac{C_{2n}}{Y_n^2 + \Omega_n^2}\left[\sinh k_{2n}\left(\bar{y}_p - \frac{1}{\rho}\right) - \frac{O_n}{Y_n}\sin\frac{Y_n k_{21}^2}{\bar{v}}\left(\bar{y}_p - \frac{1}{\rho}\right)\right] +$$
$$\left.+\frac{D_{2n}}{Y_n^2 + \Omega_n^2}\left[\cosh k_{2n}\left(\bar{y}_p - \frac{1}{\rho}\right) - \cos\frac{Y_n k_{21}^2}{\bar{v}}\left(\bar{y}_p - \frac{1}{\rho}\right)\right]\right\},$$

where $O_n = \frac{k_{1n}\bar{v}}{k_{21}^2}$ and G_n given in (21).

c. The load has left the beam $\left(t > \dfrac{l_1 + l_2}{v}\right)$.

For this case $\left(\tau > \tau_2 = (1 + \dfrac{1}{\rho})\dfrac{k_{21}^2}{\bar{v}}\right)$ of free vibrations of the beam, the following solution is valid.

$$(29)\quad \bar{T}_n(\tau) = \frac{\dot{\bar{T}}_n(\tau_2)}{Y_n} \sin \frac{Y_n k_{21}^2}{\bar{v}} \left(\bar{y}_p - \frac{1+\rho}{\rho}\right) + \bar{T}_n(\tau_2) \cos \frac{Y_n k_{21}^2}{\bar{v}} \left(\bar{y}_p - \frac{1+\rho}{\rho}\right).$$

3. Numerical results and discussion

In order to achieve linear response, the major interest of this investigation is focused on the metal structures. Thus, the beams considered have spans made of solid cross-sections (structural I), while the columns are made of latticed cross-sections (two equal U sections). The total length of the beam is kept within reasonable dimensions ($< 16m$), while in all cases $h = 0.3l_1$ and $I = 0.5I_1(\varphi = 0.30, \sigma = 0.50)$. The beams considered and their properties are shown in detail in Table 1.

3.1. Free motion

For the beams of Table 1, the three first eigenfrequencies are computed and shown in Table 2. Their values have been compared to the eigenfrequencies of the corresponding two-span continuous beams (Euler-Bernoulli theory), obtained from Timoshenko equation [9].

As expected, the $L - M$ (longitudinal motion) effect of the column, in general, decreases the values of the system's eigenfrequencies. This decrease, practically negligible for the fundamental one ($< 4\%$), can well reach 25.56% for the $2nd$ and 36.18% for the $3rd$ one (case $4a$) and cannot be omitted.

N_0	Beam 1	$A_1(\text{cm}^2)$	$I_1(\text{cm}^4)$	$l_1(\text{m})$	λ_1	Beam 2	$A_2(\text{cm}^2)$	$I_2(\text{cm}^4)$	$l_2(\text{m})$
1a	IBP 240	106	11260	3.09	30.00	IBPv 320	312	68130	9.28
1b	I320	77.7	12610	3.81	30.00	IPE 450	98.8	33740	11.42
1c	IBP 260	118	14920	3.37	30.00	IBP 260	118	14920	10.12
2a	I 425	132	36970	5.02	30.00	IBP 600	270	171000	10.04
2b	IPBl 360	143	33090	4.56	30.00	IPBl 450	178	63720	9.13
2c	IBP 260	118	14920	3.37	30.00	IPB 260	118	14920	6.74
3a	IPBl 220	64.3	5410	3.29	35.87	IPBl 260	86.8	10450	3.29
3b	C 200	32.2	1910	2.31	30.00	C 160	24	925	2.31
4a	IBP 260	118	14920	6.74	60.00	IPB 260	118	14920	3.37
4b	2C 280	106.6	11267	8.22	80.00	IPBl 400	159	45070	4.11
5a	IBP 260	118	14920	9.00	80.00	IPB 260	118	14920	3.70
5b	IB 180	64.7	3750	6.09	80.00	IPBl 260	86.8	10450	1.83

N_0	λ_2	μ	ρ	Column	$A(\text{cm}^2)$	$I(\text{cm}^4)$	h(m)	a_1(cm)	λ
1a	62.73	6.05		$2C40 \times 20$	7.32	3378	1.55	42.96	7.20
1b	61.30	2.70	3	$2C40 \times 20$	7.32	375	1.91	45.29	8.41
1c	90.00	1.00		2C 50	14.24	4476	1.69	35.50	9.50
2a	39.89	4.63		2C 65	18.06	11091	2.51	50.00	10.13
2b	48.10	1.93	2	2C 40	12.40	9927	2.28	56.50	8.06
2c	60.00	1.00		2C 50	14.24	4476	1.69	35.50	9.50
3a	30.00	1.93		$2C40 \times 20$	7.32	1623	1.65	30.00	11.05
3b	37.21	0.48	1	$2C40 \times 20$	7.32	573	1.16	17.70	13.05
4a	30.00	1.00		2C 50	14.24	4476	3.37	35.50	13.63
4b	24.41	4.00	0.5	$2C40 \times 20$	7.32	3380	4.11	43.00	19.13
5a	24.00	1.00		2C 50	14.24	4476	4.50	35.50	25.38
5b	14.37	2.79	0.3	$2C40 \times 20$	7.32	1125	3.05	25.00	24.56

Table 1: Dimensions and properties of various metal beams. (For all cases $\Phi = 0.30$ and $\sigma = 0.50$)
$^*a_1 = 21$ cm

ρ	μ	λ_1	λ_2	λ	EIGENFREQUENCIES			difference %		
					1st(k_{11})	2nd(k_{12})	3rd(k_{13})	k_{11}	k_{12}	k_{13}
	6.05	30.00	62.73	7.20	1.3084986	2.4122365	3.0755295	1.16	4.85	15.49
					(1.3238488)	(2.5352554)	(3.6390937)			
3	2.70	30.00	61.80	8.41	1.3853477	2.5463815	3.2794702	0.26	1.56	8.18
					(1.3890142)	(2.5868018)	(3.5781077)			
	1.00	30.00	90.00	9.50	1.2233561	2.1950353	3.1415927	-0.83	0.58	13.83
					(1.2133440)	(2.2079053)	(3.6268025)			
	4.63	30.00	39.89	10.13	1.9809157	3.3957750	3.6809020	1.78	6.18	10.52
					(2.0168235)	(3.6194928)	(4.1137960)			
2	1.93	30.00	39.89	10.13	1.8814591	3.2347625	3.4218878	1.39	4.20	12.96
					(1.9079611)	(3.3764432)	(3.9313246)			
	1.00	30.00	60.00	9.50	1.7659743	3.1415927	3.3551599	0.13	15.43	31.87
					(1.7782042)	(3.7147066)	(4.9243963)			
	1.93	35.87	30.00	11.05	3.3008564	3.5358887	5.4259587	0.05	13.44	17.31
					(3.3026567)	(4.0848823)	(6.561853)			
1	0.48	30.00	37.21	13.05	2.9932825	3.5053039	5.4833310	-0.01	5.22	7.53
					(2.9929251)	(3.6983929)	(5.9298965)			
	1.00	60.00	30.00	13.63	3.4242417	5.5308092	6.2851853	3.72	25.56	36.18
					(3.5564085)	(7.4295413)	(9.8487926)			
0.5	4.00	80.00	24.41	19.13	3.6699989	6.1029888	7.0061688	3.10	9.79	17.06
					(3.7874767)	(6.7652596)	(8.4474921)			
	1.00	80.00	24.00	24.38	3.5898210	6.4155239	8.6100086	2.00	3.74	10.26
					(3.6630114)	(6.6644884)	(9.5975436)			
0.3	2.79	80.00	14.37	24.56	3.6991090	6.4626631	8.7010268	2.82	5.85	12.03
					(3.8062919)	(6.8640682)	(9.8913364)			

Table 2: Values of the three first dimensionaless eigenfrequencies for all cases of the beams shown in Table 1. (The numbers between parentheses are the dimensionless eigenfrequencies obtained by Euler-Bernoulli theory for the corresponding two-span continuous beams).

3.2. Forced motion

The forced motion of the system, based on the three first eigenmodes, consists of a load of constant magnitude $\bar{P}(= Pl_2^2/EI_2)$ moving across the spans with a constant velocity $\bar{v}$. The chosen values of for each beam case — presented in Table 3 — correspond to the true velocity, which the load must have, in order to cross the spans in a time $\bar{t} = 10, 5, 2$ and 1 sec respectively, while $\bar{v}_{T1}$ is the dimensionless velocity, which imposed on the load causes the passage time $\bar{t}$ to turn equal to the fundamental period T_1 of the beam considered.

N_0	$\bar{v}$				
	$\bar{t} = T_1$	$\bar{t} = 10$ sec	$\bar{t} = 5$ sec	$\bar{t} = 2$ sec	$\bar{t} = 1$ sec
1a	2.28	0.475	0.950	2.375	4.75
1b	2.51	0.576	1.151	2.878	5.76
1c	2.86	0.744	1.488	3.719	7.44
2a	2.48	0.367	0.735	1.837	3.67
2b	2.73	0.404	0.809	2.021	4.04
2c	2.98	0.371	0.742	1.856	3.71
3a	2.90	0.242	0.483	1.208	2.42
3b	3.54	0.105	0.210	0.525	0.97*
4a	1.40	0.186	0.371	0.928	1.86
4b	1.03	0.184	0.368	0.921	1.84
5a	0.80	0.215	0.430	1.074	2.15
5b	0.59	0.081	0.162	0.404	0.81

Table 3: Values of the dimensionless velocity $\bar{v}$, for four characteristic values of $\bar{t}$ represents the time needed for the load to cross the beam, while T_1 is system's fundamental period). $^*t = 1.082$sec

Furthermore, the $L - M$ effect of the column is discussed on the values of the maximum transverse deflection for certain characteristic cross-sections of both spans, in comparison with the ones obtained from the static case ($\bar{v} \simeq 0.00$) [10], as shown in Table 4, where the dynamic load factor (D.L.F) is also presented.

From this Table it is clear that the increase of the load velocity $\bar{v}$ increases the max values of $\bar{w}$, and the variation of this increase depends mainly on the relation between $\bar{v}$ and $\bar{v}_{T1}$. Thus for beams with large ρ and μ (cases $1a, b, c$) and for $\bar{v} < \bar{v}_{T1}$ the DLF varies from 1.2 to 1.85, while for $\rho \simeq 1$ and $\mu > 1$ the effect is more pronounced for cross-sections of the second span (cases 2 and 3).

This factor is larger in cases 4 and 5 ($\rho < 1$) and if $\bar{v} > \bar{v}_{T1}$ it can reach 360% having a devastating effect on the beam's load bearing capacity. So we can be lead to the conclusion that if the load travels with a $\bar{v} > \bar{v}_{T1}$ and the beam is not symmetrical

			$\bar{x}_1 = 0.50$			$\bar{x}_2 = 0.50$		
N_0	$\bar{v}_{T1}$	$\bar{v}$	$\|\max W\|$ $x10^{-2}$	DLF	load position	$\|\max W\|$ $x10^{-2}$	DLF	load position
		0.000	1.5034		0.867	4.0504		0.840
1a	2.286	0.950	1.7979	1.196	0.733	5.0277	1.241	0.706
		2.375	2.7690	1.842	1.106	7.4810	1.847	1.160
		0.000	0.8434		0.720	1.9257		0.853
1b	2.618	1.151	1.1409	1.353	0.773	2.5769	1.338	0.773
		2.878	1.4964	1.774	2.106	3.4720	1.803	1.226
		0.000	0.3446		0.746	0.9420		0.866
1c	2.163	1.488	0.4507	1.308	0.773	1.3723	1.457	0.866
		3.719	0.5545	1.609	1.609	1.5964	1.695	2.186
		0.000	2.5019		0.915	6.7384		1.005
2a	2.487	0.735	2.7964	1.118	0.945	7.3509	1.091	1.020
		1.837	4.3855	1.753	1.800	13.0805	1.941	1.170
		0.000	1.1533		0.870	3.3773		1.020
2b	2.728	0.809	1.2254	1.063	0.945	3.6944	1.094	1.020
		2.021	1.8026	1.563	1.815	6.3874	1.891	1.200
		0.000	0.6420		1.140	1.9838		1.035
2c	2.977	0.742	0.6954	1.083	1.185	2.0100	1.013	1.125
		1.856	1.0980	1.710	1.140	3.4061	1.717	1.125
		0.000	6.0799		0.500	5.0229		1.460
3a	2.901	1.208	9.8372	1.618	0.560	5.7904	1.153	1.340
		2.416	10.4890	1.725	0.900	9.4400	1.879	2.700
		0.000	1.3915		0.500	2.3568		1.540
3b	2.636	0.525	1.5325	1.101	0.540	2.5656	1.089	1.540
		0.968	1.6860	1.212	0.400	2.3008	0.976	1.660
		0.000	11.5259		0.960	8.0330		2.400
4a	1.400	0.928	20.0024	1.735	1.320	11.5005	1.432	2.460
		1.856	18.2824	1.586	4.020	7.4154	0.923	1.710
		0.000	36.3962		0.930	8.4993		2.370
4b	1.033	0.921	63.7395	1.751	1.590	11.8775	1.379	2.190
		1.842	42.2473	1.161	5.100	30.9300	3.639	2.430
		0.000	46.4590		1.560	7.8079		1.733
5a	0.300	1.074	76.2836	1.635	5.980	11.7343	1.503	6.023
		2.148	49.2055	1.055	4.463	13.7448	1.760	4.550
		0.000	116.5073		1.516	121.7876		2.860
5b	0.601	0.404	206.2311	1.770	2.166	172.0609	1.413	2.470
		0.808	171.9003	1.475	6.716	248.5963	2.041	2.946

Table 4: Absolute maximum dimensionless flexural deflection and corresponding load position at two characteristic beam cross-sections (middle of first and second span) and three load velocities. (Note that $\bar{v}_{T1}$ is the dimensionless velocity, for which the load leaves the beam after a time equal to the beam's fundamental period).

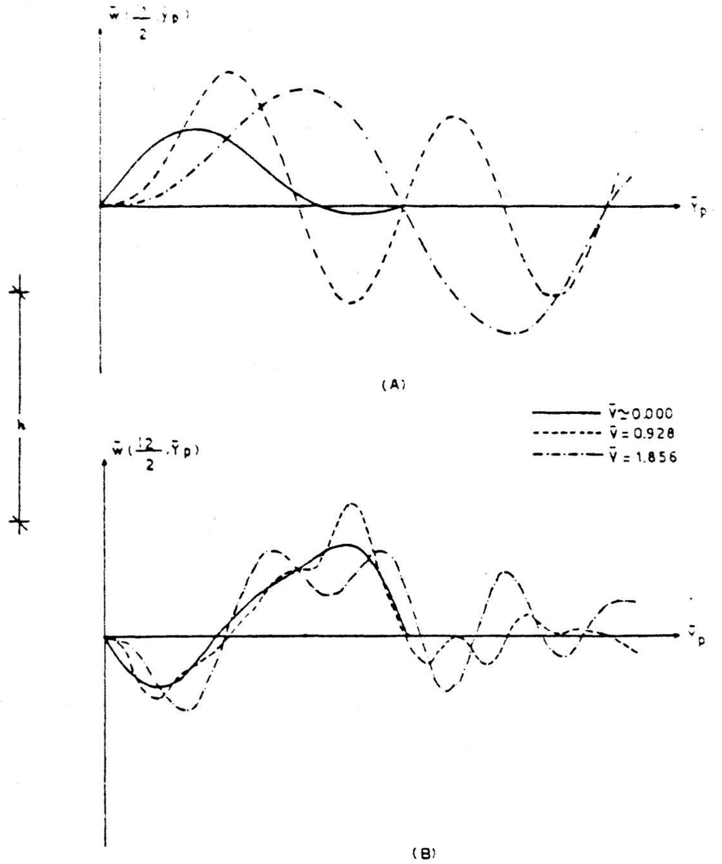

Fig. 2. Dynamical influence lines of flexural deflextion $\bar{w}$ for the beam of Case 4a at the middle of the first span (A) and second span (B).

(in geometry and properties), there is a distinct danger of resonance and the dynamic response cannot be predicted. This danger is not vissible for the classical two-span continuous beam [5,6].

Finally, in Figs.2-4 one can see the dynamic influence lines of $\bar{M}, \bar{Q}$ and $\bar{w}$ of one characteristic beam case at the middle of both spans, when column's $L - M$ effect is accounted for.

[It must be noted that in reality the moving load appears mainly as a sprung mass: the difference in the result is small, depending on numerous parameters, such as beam load bearing capacity, masses ratio etc.]

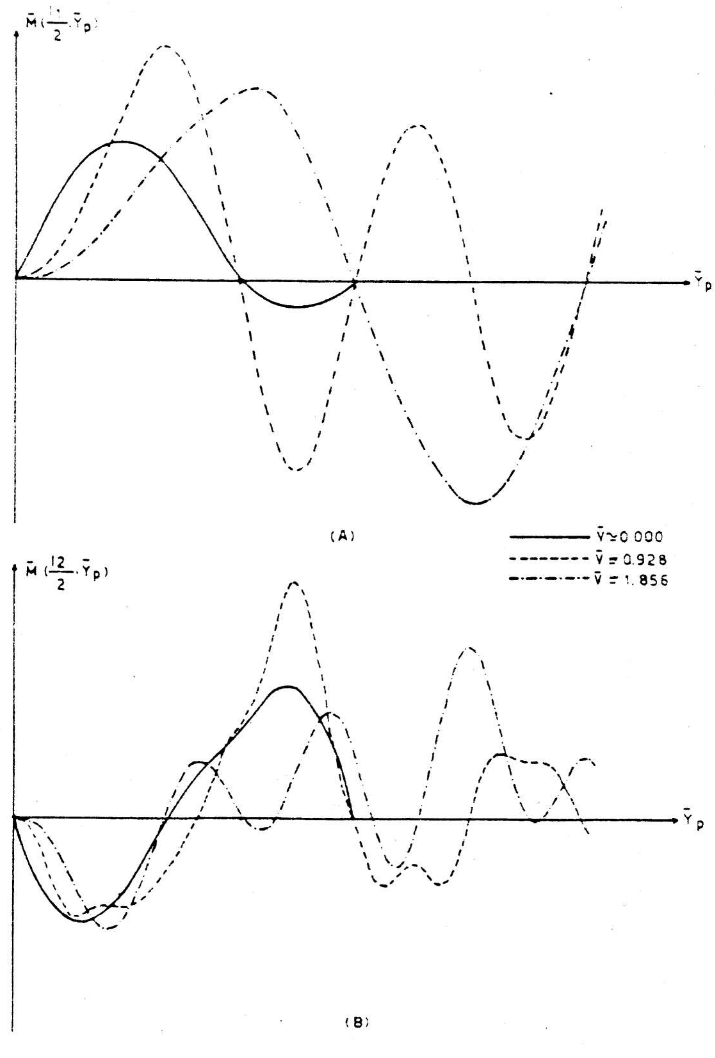

Fig. 3. Dynamical influence lines of bending moment $\bar{M}$ of Case 4a at the middle of the first span (A) and second span (B).

4. Conclusions

The most important findings, based on this model, are the following:

1) The effect of the column $L - M$ on the fundamental eigenfrequency of the system is small and can be omitted, while for higher modes this effect may be consid-

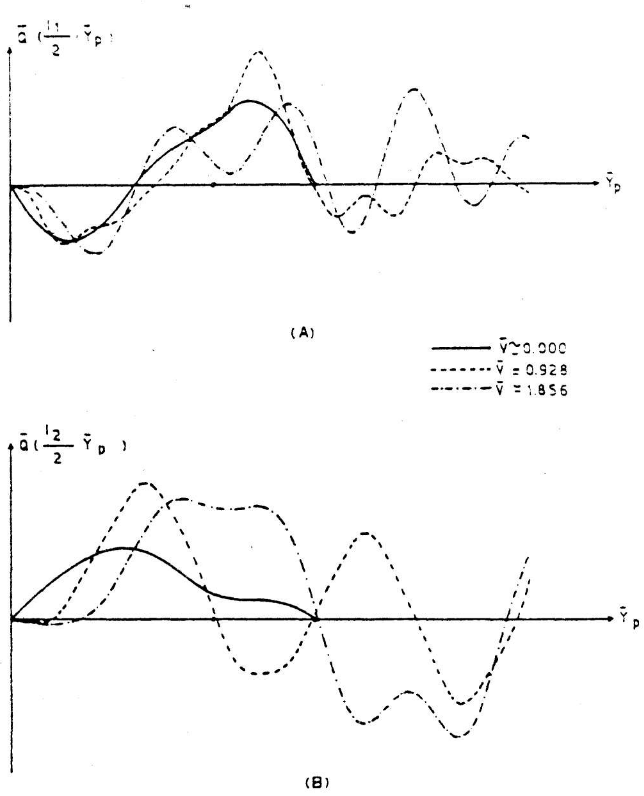

Fig. 4. Dynamical influence lines of shearing force $\bar{Q}$ of Case 4a at the middle of the first span (A) and second span (B).

erable, even for relatively slender members, under certain combinations of geometrical parameters.

2) In spite of the first conclusion, the variation of the load velocity in close relation with the system's fundamental period not only increases the max $\bar{w}, \bar{M}$, and $\bar{Q}$ but may lead to unavoidable resonance phenomena, which can be dealt with only by means of symmetrical beam design.

3) Mathematical difficulties due to multi-parametric analysis (although linear), cannot be overcome even with the aid of electronic computation and advanced numerical methods, indicating that the need of experimental data, especially in getting to know dynamic phenomena, are and will remain evident.

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