Journal of Theoretical
and Applied Mechanics
Sofia 1993, Year XXIV No 3

REDUCE SENSITIVITY TO LOCAL MINIMA IN OPTIMAL STRUCTURAL DESIGN

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The Mathematical Programming methods have been extensively used to solve Optimal Structural Design Problem in past years. This problem is characterised by many variables (up to a few hundreds) and large number of constraints (up to a few thousands). The constraints are of different kinds and they are non-linear functions of design variables, so Nonlinear Programming methods are used to solve this problem. Because of the nonlinearity of constraints, there are many local minima, some of them too far from the global one.

In the present work the reasons for convergence to local minima are discussed and strategy to overcome these difficulties are presented.

1. Reasons for sensitivity to local minima

The optimal design problem is defined as follows:

$$\text{minimize } f_0(x),$$

$$\text{subject to } f_i(x) \leq 0, \qquad i = 1, \ldots, m, \tag{1}$$

where x is vector of design variables, $f_0(x)$ is cost function and $f_i(x)$ are constraints.

Almost all Nonlinear Programming methods solve this problem by generating a sequence $\{\mathbf{x}^k\}$ given by

$$x^{k+1} = x^k + \alpha p^k, \qquad k = 0, 1, 2, \ldots, \tag{2}$$

where α is a step size, p^k is a direction of design change, and k is an iteration index.

If current point x^k is in feasible domain then step size α is calculated by minimizing some function $\varphi(x^k + \alpha p^k)$ that is equal in some cases to cost function $f_0(x)$. The step size α is increased (decreased) until minimum of $\varphi(x^k + \alpha p^k)$ or bounds of feasible region are reached. Usually the minimum of $\varphi(x^k + \alpha p^k)$ lies outside feasible region, thus the reason for the termination of one dimensional search is the reaching of bounds of feasible region. Consequently if bounds are reached at some iteration then subsequent point will lie on bounds.

Because constraints in (1) are non-linear functions, feasible region is non convex and there are many local minima. The typical solution process for 2 variables and linear cost function is shown in Fig. 1. If starting point A is in feasible region next generated point B will lie on bound. The direction vector p must point descent direction and it is oriented to point C that is the local minimum instead to point D that is the global one.

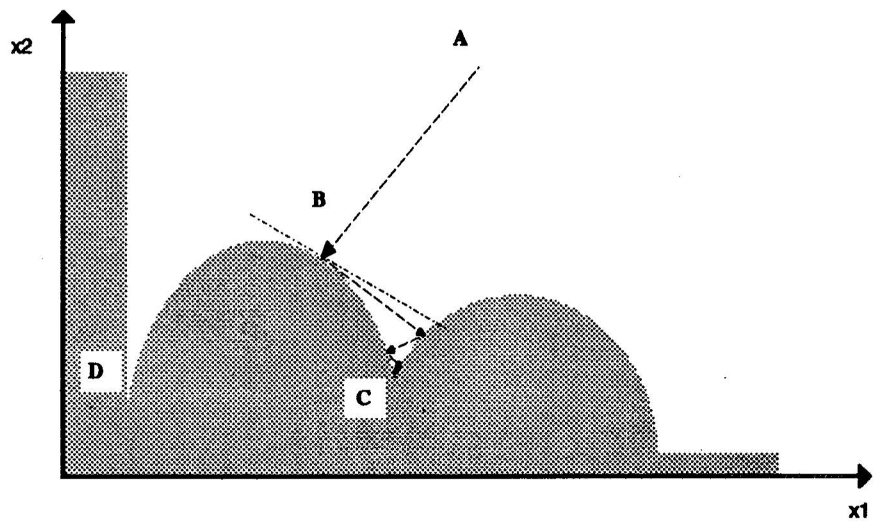

Fig. 1. Typical solution process.

In Gradient Projection Method [1] there is no one dimensional search and bounds are not always reached before reaching the global minimum. This explains its low sensitivity to local minima.

2. Strategy for reducing sensitivity to local minima

It is clear from the previous paragraph that for reducing sensitivity to local minima it must hold the sequence $\{x^k\}$ far from bounds of feasible region. To perform this, the next strategy is proposed:

1. If current point is in feasible region, then original problem (1) is substituted by

$$\min f_0(\boldsymbol{x}), \tag{3}$$

$$f_i(\boldsymbol{x}) \leq -\beta_1, \qquad i = 1, \ldots, m, \qquad \beta_1 > 0. \tag{4}$$

After calculation of direction vector $\boldsymbol{p}$, the step size determination is performed by minimising $\varphi(\boldsymbol{x}^k + \alpha \boldsymbol{p}^k)$ and constraints are

$$f_i(\boldsymbol{x}^k + \alpha \boldsymbol{p}^k) \leq -\beta_2, \qquad i = 1, \ldots, m, \qquad \beta_1 > \beta_2 > 0. \tag{5}$$

It is recommended to use the Gradient projection method or other insensitive to local minima methods for solving of problem (3)–(4). The solution of problem is $\boldsymbol{x}^*$ and there is no need to find this solution with high precision.

2. Problem (1) is solved using fast convergence method with starting point $\boldsymbol{x}^*$. Such method is Pshenichny's recursive quadratic programming method [2].

A possible solution process with this strategy is shown in Fig. 2.

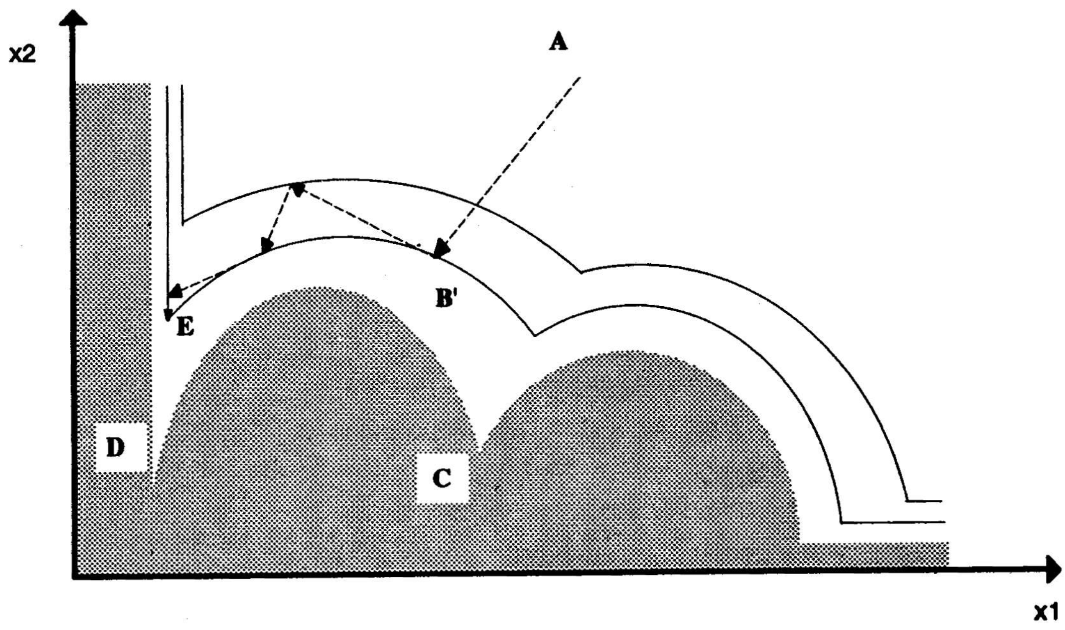

Fig. 2. Solution process with present strategy.

3. Example

The problem is to minimize the weight of truss, shown in Fig. 3. The structure is subjected to stress, displacement and member size constraints [case I of Reference 1]. Design variables are cross-sectional areas of trusses and the starting design is 6.4516 cm^2 (1 in.^{2}) for each variable.

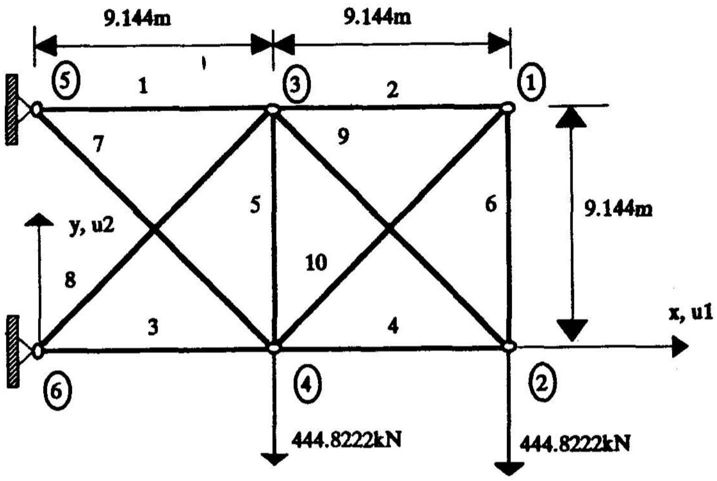

Fig. 3. 10-bar truss.

The results are given in Table 1. The solution of problem is (196.91, 0.64516, 149.68, 98.212, 0.64516, 3.5573, 48.111, 135.72, 138.89, 0.64516) cm^2 and it is different from solution reported in [1]. Therefore solution reported in [1] is only local minimum.

Table 1. Results for 10-bar truss.

	weight, kg	number of iterations	number of analysis
this work	2295.56	27	34
LINRM,[2]	3159.20	50	101
VMCON,[3]	2295.63	34	—
[1]	2295.90	15	15

4. Summary and conclusions

The new strategy that reduces sensitivity to local minima is presented in this paper. It is based on holding design point far from bounds of feasible region. The presented example and results show applicability of this strategy. The next step is to apply this strategy to a variety of static and dynamic response optimisation problems.

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Presented at the 7th Congress on Theoretical and Applied Mechanics, Sofia, August 30 – September 1, 1993
section: "Applied Mechanics"