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DYNAMIC ANALYSIS OF THREE-DEGREES-OF-FREEDOM PARALLEL TYPE MANIPULATORS

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1. Introduction

Parallel manipulators have got specific mechanical design. They are with closed kinematic chains and are considered as an alternative of the conventional robots with open kinematic chains and motors in series.

Despite the great number of publications on robot dynamics, which consider mostly serial link manipulators, little attention has been paid to parallel manipulators. Recently there has been greater interest in robots with parallel topology. In [1] was presented systematic analysis on mechanisms with parallel topology which could be used as manipulators or as parts of robots. Kinematic and dynamic analysis of a six-degrees-of-freedom parallel manipulator (Steward platform) was given in [2]. Kinematic and dynamic analysis of parallel manipulators was discussed in [4]. Also some efforts have been directed towards 3-degrees-of-freedom parallel manipulators. Lee and Shah have discussed the kinematics [5] and dynamics [6] of a 3-degrees-of-freedom parallel manipulator. The authors of this paper have performed the kinematic analysis of a type of 3-degrees-of-freedom parallel manipulator [7], [8], [9].

This paper presents the dynamic analysis of 3-degrees-of-freedom parallel manipulators of the 3-RPS type (Fig.1) – both platforms are connected with three symmetrical chains, each containing a pin joint (R), a prismatic (P) joint and a ball joint (S) and 3-RRS type (Fig.2) – two pin joints (R) and one ball joint (S). The equations of motion of the manipulator have been derived using Lagrangian approach. The dynamic analysis gives the forces required to actuate the manipulator so that the end-effector follows a predetermined trajectory.

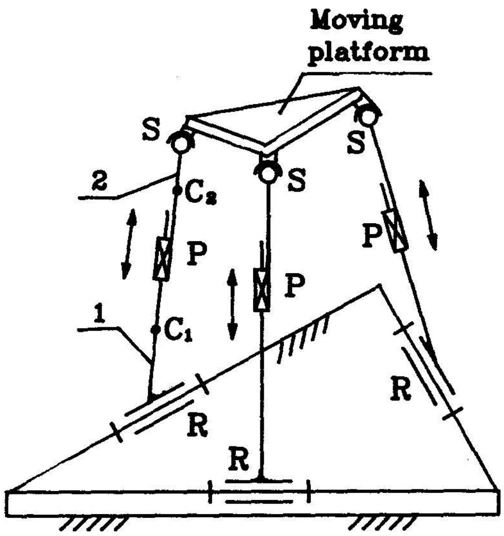

Fig. 1. Kinematic scheme of a 3-RPS parallel manipulator

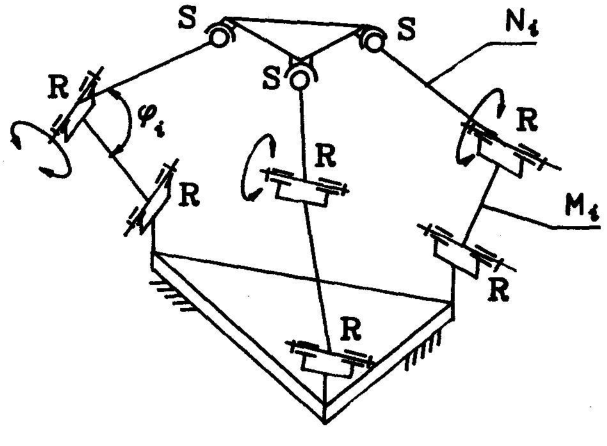

Fig. 2. Kinematic scheme of a 3-RRS parallel manipulator

Any change of the angles φ_i $(i = 1,2,3)$ results in changes of the length $L_1 = A_1B_1$ according to the formula

(1) $$L_1 = (M_i^2 + N_i^2 - 2M_iN_i\cos(\varphi_i))^{1/2}.$$

Therefore both schemes can be presented as a generalized geometrical scheme (Fig.3).

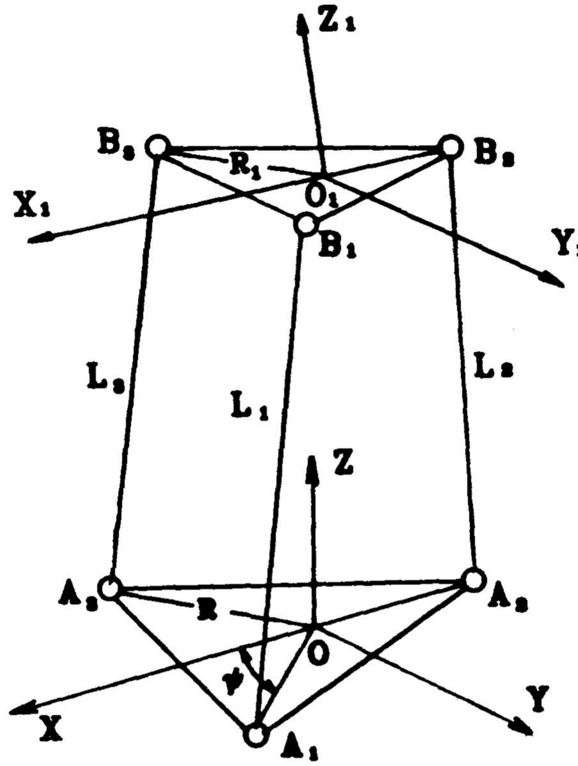

Fig. 3. Generalized geometrical scheme of the 3-RPS and 3-RRS parallel manipulators.

2. Kinematics

We define the vectors

$$
\begin{aligned}
a_1 &= O_2A_i = (R_2\cos\psi_i, R_2\sin\psi_i, 0)^{\mathsf{T}} \\
b_1 &= O_2B_i = (U_i\cos\psi_i, U_i\sin\psi_i, V_i)^{\mathsf{T}}, \quad i = 1,2,3,
\end{aligned} \tag{2}
$$

where $\psi_i = (2i-1)\pi/3$ and U_i, V_i $(i = 1,2,3)$ are unknown quantities which are to be determined.

From the conditions

$$
|a_i - b_i| = L_i, \quad |b_i - b_j| = R_1\sqrt{3}; \quad i,j = 1,2,3; \quad j \neq i \tag{3}
$$

and introducing the non-dimensional quantities $u_1 = U_i/R_1$, $v_i = V_i/R_1$ and parameters $\lambda_i = L_i/R_1$, $\rho = R/R_1$, we obtain a system of six quadratic algebraic equations for the unknown quantities u_i, v_i:

$$\begin{gathered}(u_i - \rho)^2 + v_i^2 = \lambda_i^2,\\ u_i^2 + u_j^2 + u_i u_j + (v_i - v_j)^2 = 3,\\ i, j = 1,2,3; \quad j \neq i.\end{gathered} \tag{4}$$

After calculation of u_i, v_i from (4) the solution of the direct kinematic problem is given by

$$\mathbf{r} = T\mathbf{r}_1 + \mathbf{r}_0. \tag{5}$$

In the inverse problem the orientation of the moving platform and the vector $r_0 = O_1O$ are given, while the generalized coordinates, i.e., the length L_i $(i = 1,2,3)$ for mechanism 3-RPS and angles φ_i $(i = 1,2,3)$ – for the mechanism 3-RRS are to be determined. The orientation of the moving platform is determined by the projections of the unit vector

$$k = (x, y, z)^{\mathrm{T}}, \quad x^2 + y^2 + z^2 = 1. \tag{6}$$

along axis O_1Z_1.

For the solution of the inverse problem we obtain

$$L_i = R_1\sqrt{(u_i - \rho)^2 + v_i^2}, \tag{7}$$

$$\varphi_i = \arccos\frac{M_i^2 + N_i^2 - L_i^2}{2M_iN_i}. \tag{8}$$

where u_i, v_i are given in [7] and [8].

Equation (7) gives the solution of the inverse problem for the mechanism 3-RPS and equation (8) – for the mechanism 3-RRS.

3. Dynamics

The equations of motion are derived using Lagrangian approach

$$\frac{d}{dt}\left(\frac{dL}{d\dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = F_j, \quad j = 1,2,3. \tag{9}$$

Here are given the dynamic of the 3-RPS parallel mechanism, as the derivation of the dynamic equations of the 3-RRS is analogical.

We consider each link of the manipulator as a rigid body committing a spatial movement with kinetic energy, which can be written as

$$T = \frac{1}{2}(m_s v_{c_s}^2 + [\Omega_s^s]^{\mathrm{T}} J_{c_s} \Omega_s^s). \tag{10}$$

1. Kinetic energy of the moving platform.

We consider the moving platform of the mechanism as a circular plate and its mass is assumed to have the center of gravity at the origin of the $O_1X_1Y_1Z_1$ coordinate system. Therefore the kinetic energy of the moving platform can be written as

$$T_p = \frac{MV_0^2}{2} + \frac{1}{2}(J_x(\omega_x^{(1)})^2 + J_y(\omega_y^{(1)})^2 + +J_z(\omega_z^{(1)})^2), \tag{11}$$

The moments of inertia of the moving platform are

$$J_x = J_y = \frac{MR_1^2}{4}, \quad J_z = \frac{MR_1^2}{2}. \tag{12}$$

Since we are searching for the forces required to actuate the manipulator, so that the end-effector follows a predetermined trajectory, we have to express the kinetic energy in terms of the generalized coordinates and generalized velocities L_i, $\dot{L}_i$ ($i = 1,2,3$). From the system (4) are derived

$$v_i = \sqrt{\frac{L_i^2}{R_1^2} - (u_i - \rho)^2}, \quad \dot{v}_i = \frac{L_i\dot{L}_i - (u_i - \rho)R_1^2\dot{u}_i}{R_1^2\sqrt{\dfrac{L_i^2}{R_1^2} - (u_i - \rho)^2}} \tag{13}$$

After substitution of (12) and (13) in (11) the equation of the kinetic energy of the moving platform is derived in terms of the generalized coordinates and generalized velocities L_i, $\dot{L}_i$.

2. Kinetic energy of link 1.

The kinetic energy of link 1 is given by

$$T_{1_i} = \frac{1}{2}J_{A_i}\omega_{1_i}^2, \quad i = 1,2,3, \tag{14}$$

where $J_{A_i} = \frac{1}{3}m_1 l_1^2$ – moment of inertia of the link 1 with respect to the axis of rotating; m_1 – mass of link; l_1 – length of the link; ω_{1_i} – angular velocity of the link, $i = 1,2,3$.

For ω_{1_i} there can be written

$$\omega_{1_i} = \frac{\dot{u}_i L_i - (u_i - \rho)\dot{L}_i}{L_i\sqrt{\dfrac{L_i^2}{R_1^2} - (u_i - \rho)^2}}, \quad i = 1,2,3. \tag{15}$$

3. Kinetic energy of link 2.
The kinetic energy of link 2 can be written as

$$T_{2_i} = \frac{1}{2} m_2 V_{C_1}^2 + \frac{1}{2} J_{C_i} \omega_{2_i}^2, \tag{16}$$

where

$$J_{C_i} = \frac{l}{12} m_2 l_2^2; \quad \omega_{2_i} = \omega_{1_i}. \tag{17}$$

After substitution of (17) in (16) the kinetic energy of link 2 becomes

$$T_{2_i} = \frac{1}{2} m_2 [\dot{L}_i^2 + (L_i - l_c)^2 \omega_{1_i}^2] + \frac{1}{24} m_2 l_2^2 \omega_{1_i}^2. \tag{18}$$

The kinetic energy of the mechanism is given by

$$T = T_p + T_1 + T_2 \tag{19}$$

The potential energies of the moving platform, link 1 and link 2 are given by

$$P_p = Mg \frac{R_1}{3} \sum_1^3 \left[\sqrt{\frac{L_1^2}{R_1^2} - (u_i - \rho)^2} \right], \tag{20}$$

$$P_1 = m_1 g \frac{l_1}{2} \sum_1^3 \left(\frac{\sqrt{L_i^2 - R_1^2 (u_i - \rho)^2}}{L_i} \right), \tag{21}$$

$$P_2 = m_2 g \sum_1^3 \left(\frac{(L_i - l_C) \sqrt{L_i^2 - R_1^2 (u_i - \rho)^2}}{L_i} \right). \tag{22}$$

Therefore the potential energy of the mechanism is

$$P = P_p + P_1 + P_2. \tag{23}$$

In many cases of control of robots it is of interest to determine the forces required to actuate the mechanism so that the end-effector follows a predetermined trajectory. Usually the Cartesian position, the orientation, the velocity and acceleration of the end-effector are known. Therefore the generalized forces required to actuate the links of the 3-RPS parallel mechanism can be derived from the Lagrangian equations (9).

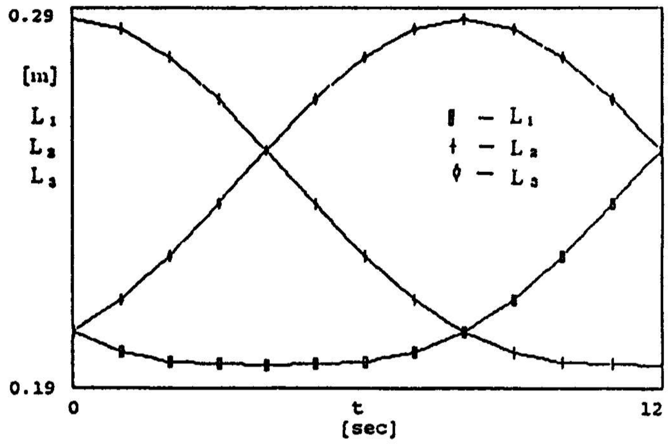

Fig. 4. Graphics of generalized coordinates

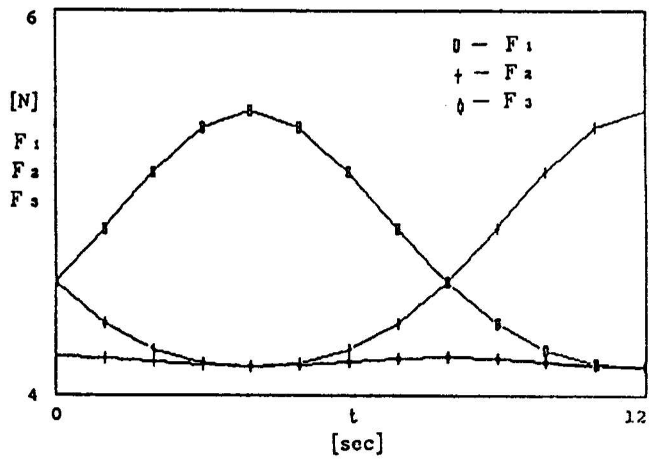

Fig. 5. Graphics of computed forces – case I

The presented dynamic analysis of the parallel manipulators discussed above has been computer simulated. Examples of dynamic simulation of the mechanism are given. The variation of the forces required to actuate the links is shown (Fig. 5, Fig. 6) in case of variation of the orientation of the moving platform which is given by the equations

$$x = 0.5\cos(\alpha), \quad y = 0.5\sin(\alpha), \quad z_0 = \text{conts}, \quad \alpha = 0 \div \pi.$$

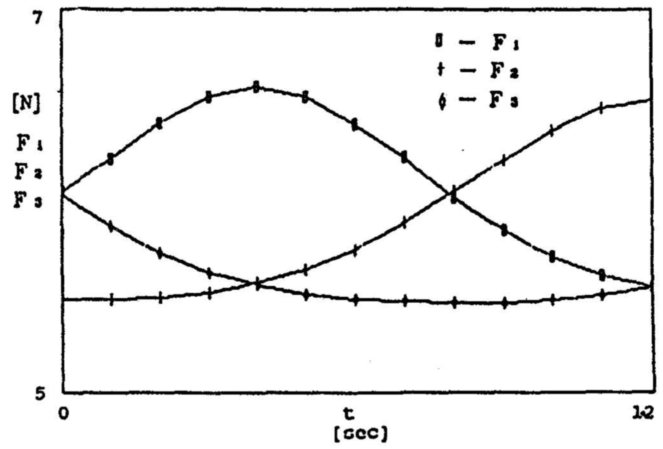

Fig. 6. Graphics of computed forces – case II

Fig. 4 shows the variation of the generalized coordinates L_i, $(i = 1,2,3)$ and Fig. 5 and Fig. 6 – the variation of the forces. The parameters used for simulation are the following:

Fig. 5 – $M = 0.8$ kg, $m_1 = 0.3$ kg, $m_2 = 0.1$ kg, $t = 12$ s;
Fig. 6 – $M = 0.1$ kg, $m_1 = 0.3$ kg, $m_2 = 0.5$ kg, $t = 1.2$ s.

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