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ON AUTOMATED DERIVING AND SOLVING DIFFERENTIAL EQUATIONS OF DYNAMICS BY REDUCE AND FORTRAN

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1. Introduction

The potentials of the REDUCE system for deriving differential equations of dynamics and writing them in FORTRAN programmes are known [1,2,3,4]. There are two methods of approach for designing of these programmes in a form convenient for their numerical integration:

1. Preliminary analytical solving of the differential equation system (DES) against generalized accelerations (GA) $\ddot{q}$ by the REDUCE system:

$$A\{\ddot{q}\} = \{b\}; \Longrightarrow \{\ddot{q}\} = A^{-1} \cdot \{b\}$$

2. Solving the DES against GA is made later – during numerical computations for every current step of integration. One can use some of the standart subroutines for the solution of linear algebraic equation system with symmetric coefficient matrix.

There are two criteria for the estimation of these two methods of approach:

1. Critical potentials of the REDUCE system for analitical solving of the DES against GA and for the generation of the FORTRAN programmes.
2. The speed of the numerical solving of the DES by means of the obtained FORTRAN programmes.

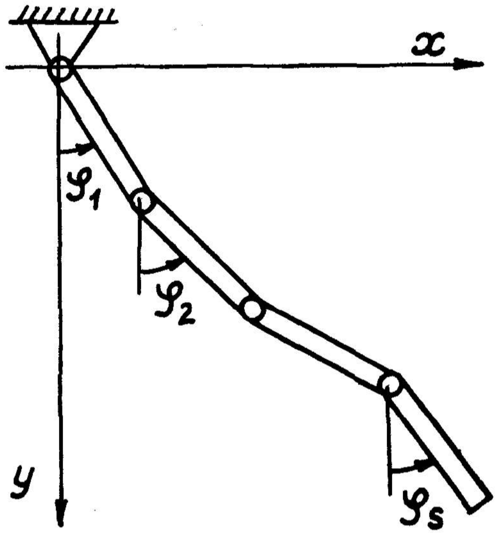

Fig. 1.

2. Object of research

Two REDUCE programmes LAGFOR1 and LAGFOR2 were created for determination of the optimal approach. They were tested on two types of problems:

1. Mechanical system with dense coefficient matrix A (Fig.1). The system consists of s homogeneous rods, each of lenght l and mass m. The rods are binded by cylindrical hinges and are disposed in a vertical plane.

   For the Lagrangian of the system we obtained the formula:

$$L = \frac{1}{24} ml^2 \sum_{j=1}^{s} \dot{\varphi}_j^2 + \frac{1}{2} ml^2 \sum_{j=1}^{s} \Big[\Big(\sum_{i=1}^{j-1} \cos\varphi_i \dot{\varphi}_i + \frac{1}{2} \cos\varphi_j \dot{\varphi}_j \Big)^2 + \tag{1}$$
$$+ \Big(\sum_{i=1}^{j-1} \sin\varphi_i \dot{\varphi}_i + \frac{1}{2} \sin\varphi_j \dot{\varphi}_j \Big)^2 \Big] + mgl \sum_{j=1}^{s} \Big(\sum_{i=1}^{j-1} \cos\varphi_i + \frac{1}{2} \cos\varphi_j \Big)$$

   In accordance with formula (1) the REDUCE programme LGN1M was created, which passes the input information to the basic programmes LAGFOR1 and LAGFOR2. The listing of this programme is shown below.

```
% =========================================
%       Programme LGN1M - dense matrix A
% =========================================
%
 WRITE " Input the degrees of freedom S !!";
 PAUSE;
 OPERATOR Q, QT, PAR;
 M := 1 $
 SUM1 := FOR J := 1:S SUM QT(J)**2 $
 SUM2 := FOR J := 1:S SUM
  (( FOR I := 1:J-1 SUM COS(Q(I))*QT(I) ) +
                1/2*COS(Q(J))*QT(J) )**2 +
  (( FOR I := 1:J-1 SUM SIN(Q(I))*QT(I) ) +
                1/2*SIN(Q(J))*QT(J) )**2 $
 SUM3 := FOR J := 1:S SUM
   ( FOR I := 1:J-1 SUM COS (Q(I)) ) + 1/2*COS(Q(J)) $
 LGN := 1/24*M*L**2*SUM1 + 1/2*M*L**2*SUM2 + M*G*L*SUM3 $
 FOR ALL X,Y LET COS(X)*COS(Y) + SIN(X)*SIN(Y) = COS(X-Y),
                SIN(X)**2 + COS(X)**2 = 1 ;
 LGN := LGN $
 CLEAR SUM1, SUM2, SUM3;
 FOR ALL X,Y CLEAR COS(X)*COS(Y) + SIN(X)*SIN(Y) ,
                  SIN(X)**2 + COS(X)**2 ;
 L := PAR(1) $
;END;
```

Two of the basic trigonometric properties are added to the programme LGN1M by means of the FOR ALL ... LET rule to simplify the expression of the Lagrangian and the obtained from it Lagrange equations.

2. Mechanical system with sparse coefficient matrix A (Fig.2). The system consists of n double pendulums binded to each other by identical springs of stiffness c. The sliders have identical masses $m1$ and the rods identical masses $m2$ and lenghts l.

   The Lagrangian of the system is determined by the formula:

$$
\begin{aligned}
L = {} & \frac{1}{2}(m_1+m_2)\sum_{i=1}^{n}\dot{q}_{2i-1}^2+\frac{1}{6}m_2 l\sum_{i=1}^{n}\left(l\dot{q}_{2i}^2+3\dot{q}_{2i-1}\dot{q}_{2i}\cos q_{2i}\right)- \\
& -\frac{1}{2}c\left[\,q_1^2+\sum_{i=2}^{n}\left(q_{2i-1}-q_{2i-3}\right)^2\right]+m_2 g\frac{l}{2}\sum_{i=1}^{n}\cos q_{2i}
\end{aligned} \tag{2}
$$

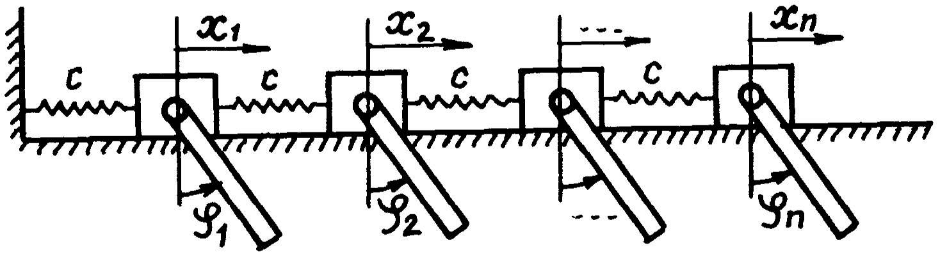

Fig. 2.

where: $q_{2i-1} = x_i$, $q_{2i} = \varphi_i$.

The relevant to formula (2) REDUCE programme LGN2M is shown below.

```
% ========================================
%       Programme LGN2M - sparse matrix A
% ========================================
%
 WRITE " Input the number of pendulums N !!";
 PAUSE $
 S := 2*N $
 OPERATOR Q, QT, PAR ;
 SUM1 := FOR I := 1:N SUM QT(2*I-1)**2 $
 SUM2 := FOR I := 1:N SUM L*QT(2*I)**2 +
                   3*QT(2*I-1)*QT(2*I)*COS(Q(2*I)) $
 SUM3 := FOR I := 2:N SUM (Q(2*I-1) - Q(2*I-3))**2 $
 SUM4 := FOR I := 1:N SUM COS(Q(2*I)) $
 LGN := 1/2*(M1 + M2)*SUM1 + 1/6*M2*L*SUM2
      - 1/2*C*(Q(1)**2 + SUM3) + 1/2*M2*G*L*SUM4 $
 CLEAR SUM1, SUM2, SUM3, SUM4;
 M1 := PAR(1) $ M2 := PAR(2) $ L := PAR(3) $ C := PAR(4) $
;END;
```

In the upper programmes the generalized coordinates are designated with Q(J) and generalized velocities – with QT(J). In addition, the expressions of the generalized unpotential forces QN(J) must be brought into analogous programmes for unconservative systems.

Test realization and results

The test was made on PC AT486 DX/33, 4Mb RAM, 256 Cache. The language MS FORTRAN 5.0 and REDUCE system version 3.3 were used. In addition, the next standard programmes were used:

GELS – for solution of linear algebraic equation systems with symmetric coefficient matrix;

HPCG – for solution of differential equation systems of first order.

The run time of the FORTRAN programmes (by the same: initial values, desired accuracy EPS = 0.0001 and interval of integration from 0 to 6.0 s.) is shown in the table below.

Dense matrix A			Sparse matrix A		
S	Anal. sol.	Numer. sol.	S	Anal. sol.	Numer. sol.
1	0.88	1.37	2	2.14	3.19
2	2.83	3.51	4	11.10	7.90
3	18.75	6.76	6	-	14.66
4	-	11.06	8	-	23.67
5	-	16.70	10	-	35.82
6	-	23.35	12	-	50.69
7	-	30.92	16	-	86.34
8	-	40.10	20	-	137.48
9	-	49.93	30	-	350.65
10	-	61.52	40	-	787.02
11	-	73.54	50	-	1409.40
12	-	88.05	60	-	2293.20

Here S designets the degrees of freedom.

The results of the testing show the following:

1. The REDUCE system exhausts fast it's own potentials for analitical solving of the DES against GA and for generation of the FORTRAN programmes with the increase of the degrees of freedom. The programme LAGFOR1 fails (stack overflow) during deriving the Lagrangian equations in case of dense matrix at 4 degrees of freedom and in case of sparse matrix at 8 degrees of freedom.

2. In case of analitical solving the DES against GA, the size of the FORTRAN programmes and the run time of numerical computations increase sharply with

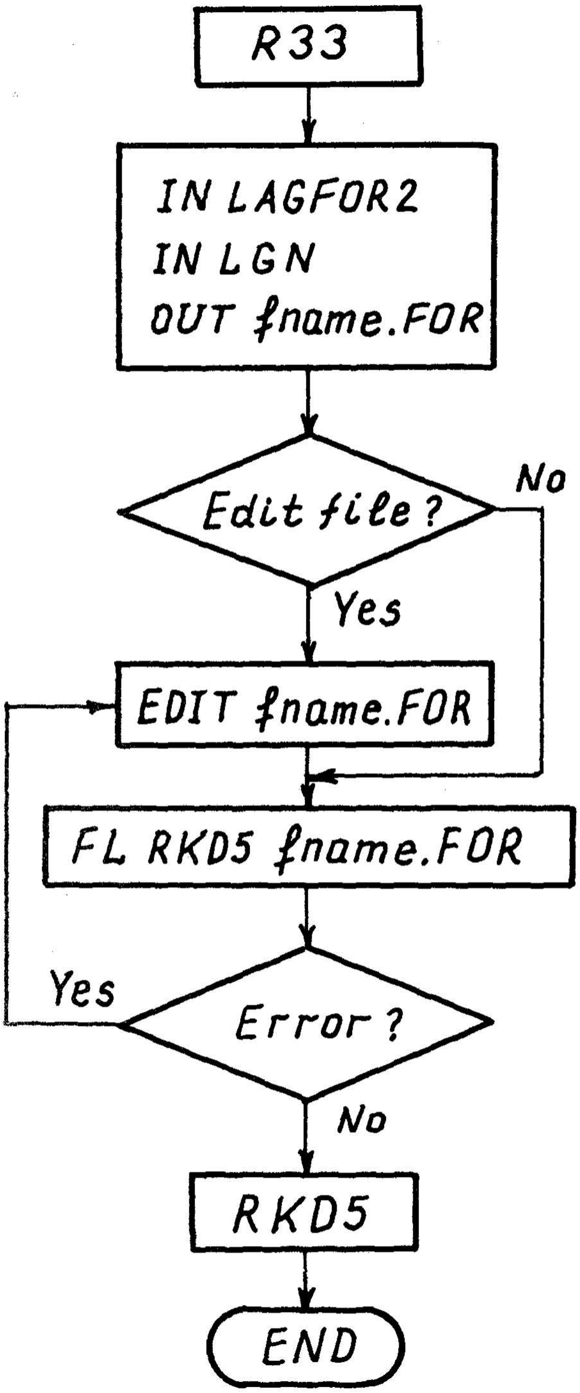
R33
IN LAGFOR2
IN LGN
OUT fname.FOR
Edit file?
No
Yes
EDIT fname.FOR
FL RKD5 fname.FOR
Yes
Error?
No
RKD5
END

Fig. 3

the increase of the degrees of freedom. Although the REDUCE system generated successfully the FORTRAN programme at 6 degrees of freedom for sparse matrix (Fig.2), this programme fails because of a stack overflow during the compilation.

3. The second approach (numerical solving of the DES against GA) demonstrates convincing advantages concerning both criteria. It allows the REDUCE system to derive the Lagrangian equations and to generate FORTRAN programmes by much greater number of degrees of freedom – for dense matrix up to 12 and for sparse matrix up to 60. In addition, we obtain shorter FORTRAN programmes which run much faster.

As a result of the experiment, an optimal version of an automated system which integrates all the stages – from deriving the differential equations to plotting graphics of the transient processes – is proposed. A special programme GRAF5 was created, wich draws the graphics for every generalized coordinate, velocity and acceleration. This programme uses the graphical potentials of MS FORTRAN 5.0 by means of the library GRAPHICS.LIB.

The block-diagram of the created BAT file is shown in Fig.3. The REDUCE system is invoked by the command R33. After that we load the basic file LAGFOR2 and the file LGN, wich assigns the degrees of freedom S, Lagrangian LGN of the system and the generalized unpotential forces QN(J). Then we input the name of the output FOR file and exit the REDUCE system.

Further we can correct the obtained FOR file by means of the available editor or can make a direct translation and linking by the command

FL RKD5 fname.FOR GRAPHICS.LIB.

In case of an error after the last command, we call the editor for appropriate corrections. At last we start the obtained EXE file by the command RKD5.

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