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MANUFACTURING FINE PITCH SPIRAL BEVEL GEARS WITH THE GENERATED DUPLEX HELICAL CUTTING METHOD. PRINCIPAL PROBLEMS

ZD. WÓJCIK

1. Introduction

Spiral bevel gears with the outer transverse module m_t less than 2 mm are now used extensively in modern technology. They are particularly recommended where the peripheral speeds are very high (as in electric or pneumatic hand tools) or where extreme smoothness and quietness are desired (e.g. in typewriters and sewing machines). At present, the theoretical basis for manufacturing this type of gears is developed, and tools and machines necessary for their production manufactured, at the following scientific centres: Gleason Works Ltd. in the U.S.A., Saratov in Russia and Klingelnberg Söhne in Germany. The Gleason and Saratov companies design and produce gears with circular curvature, while Klingelnberg Söhne specializes in bevel and hypoid gears with epicycloid longitudinal curvature.

Fine pitch spiral bevel gears cannot be cut in the same way that bevel gears are cut in the motor industry where the modules are relatively large and allow for finishing the teeth of the pinion in two cuts (one for the convex side, one for the concave side). Reinserting the blades into the gear space after one side of the space has been finished is both difficult and time consuming. Moreover, this method does not guarantee good quality of the gear since the machine settings are not fixed. The smaller the module, the greater the difficulties. Due to this, in all technologies of manufacturing fine pitch gears all the teeth in the gear and the pinion are finished at the same time (the settings remain fixed while switching from the concave to the convex side).

Gear blanks designed with Gleason's duplex helical method (the Saratov works also apply this system) will have a special DUPLEX tooth taper. It consists in cutting the cones, addenda and tooth roots of the wheel so that the bottom of the tooth space (and the top of the tooth) has a uniform width along the pitch line.

In order to do this, the sum of the root angles must satisfy the following formula:

$$\Theta_{f1} + \Theta_{f2} = \frac{\pi \cdot m_t}{2 \cdot R \cdot \cos\beta \cdot \operatorname{tg}\alpha} \cdot \left(1 - \frac{R}{r_0} \cdot \sin\beta\right) \quad [''] \tag{1}$$

With the Gleason's system the tapering depth causes changes in the pressure angles going from the middle towards the end of the teeth, a condition called a bias. If both sides of the tooth space are to be finished at the same time, the bias can only be eliminated by improving the helical motion. All gear generators which are capable of generating gear teeth with the duplex helical method, are equipped with mechanisms for obtaining the helical motion. The teeth of the wheel and the pinion are cut differently: the teeth of the large gear are finished without the helical motion while the teeth of the pinion are cut with the helical motion.

Despite the fact that the Spiral (Hypoid) Generated Duplex Helical method has been in use for more than 40 years, the Gleason company has not published its theoretical basis yet. In Poland, the method is used in producing gears for motorized tools manufactured by the CELMA company in Cieszyn, and for outboard motors (Mechanical Works in Nowa Dęba).

The production process in these two companies is based on the Gleason's technology. It seems, therefore, advisable to present the theoretical basis for technological calculations that can be used for producing fine pitch bevel gears "made in Poland".

2. Data required for technological calculations

The following data are required for technological calculations in the Spiral Generated Duplex Helical method:

z_1, z_2	– the number of teeth in the pinion and gear respectively
$m_t, \pi m_t$	– outer transverse module
$\Sigma\delta$	– shaft angle
b	– face width
R_e, R	– outer and mean cone distances
α	– normal pressure angle
β	– mean spiral angle
δ_1, δ_2	– pitch angles of the pinion and gear respectively
h_{a1}, h_{a2}	– mean addendums of the pinion and gear respectively
h_{f1}, h_{f2}	– mean dedendums of the pinion and gear respectively
C	– clearance
Θ_{f1}, Θ_{f2}	– dedendum angles of the pinion and gear respectively
δ_{f1}, δ_{f2}	– root angles of the pinion and gear respectively
d_{1ae}, d_{2ae}	– outer diameters of the pinion and gear respectively

3. Technological calculations

Determining the cutter radius

The S.G.D.H. method for gear teeth requires that the bottom of the tooth space be equivalent to the spacing width of the toolhead cutters.

In order for this condition to be satisfied, the following must occur:

$$r_0' = \frac{R \cdot \sin\beta}{1 - \operatorname{tg}\alpha\cos\beta \cdot \dfrac{2 \cdot \Theta_{f\Sigma} \cdot R_e}{10800 \cdot m_t}}, \tag{2}$$

where: $\Theta_{f\Sigma} = \Theta_{f1} + \Theta_{f2}$ – the sum of the dedendum angles.

If, owing to construction requirements, the values of the angles, module pitch and the cone distances cannot be changed to obtain a normalized cutter radius, the closest normal value of the cutter radius r_0 should be selected. If it is assumed that $r_0' \neq r_0$, additionally these technological quantities must be further corrected.

The blade angles, relative to the dedendums, depend on teeth geometry, the angles Θ_{f1}, Θ_{f2}, β as well as the longitudinal tooth bearing (assuming a normal cutter radius and not the one set by the construction requirements).

By setting the values of the tooth bearing expressed as the tooth bearing length factor (B), one can find the total correction of the blade angles in the cutters used for generating the pinion and the gear:

$$\Delta\alpha_{0\Sigma} = \frac{\rho_0}{r_0} \cdot \frac{3438}{\operatorname{tg}\alpha_0} \cdot \left[1 - \left(\frac{B_0}{B}\right)^2\right] + \sin\beta \cdot \Theta_{f\Sigma}, \tag{3}$$

where: ρ_0 – the radius of curvature of the teeth line (when $r_0' \neq r_0$),

r_0 – the nominal radius of the duplex cutter being used for teeth finishing,

B_0 – factor of the tooth bearing length associated with the geometry of the gear wheels and of the tool,

β – the mean spiral angle measured in the middle of the face width,

$\Theta_{f\Sigma} = \Theta_{f1} + \Theta_{f2}$ – the sum of the dedendum angles.

4. Generating gear teeth

The generating cone of the gear is determined in relation to modifications of the tooth profile (transverse tooth bearing) The modification coefficient depends on the actual module. The Gleason company assumes the modification coefficient for a module m=1,5875 (DP=16) to have a value of $K_p = 1$. This means that the curvature deflection at the end of the teeth equals $\Delta h = 0,00025'' = 0.00635$mm. For gears working under small loads and designed for uniform motion transmission, it is assumed that $K_p = 0$.

When the geometrical relations of the gear are taken into account, the change in the generating cone angle is equivalent to:

$$\Delta\delta_2 = \frac{\dfrac{\Theta_{f\Sigma} \cdot \cos\beta}{1,6} - \Theta_2' \cdot K_p}{K}, \tag{4}$$

where: 1,6 is a coefficient pertaining to tooth height

$$\Theta_2' = \frac{2 \cdot 3438 \cdot R_M \cdot \sin\alpha \cdot \cos^2\alpha \cdot \Delta h}{h_c^2}$$

and $h_c = \dfrac{h_{f1} + h_{f2} + 2c}{2}$ – mean tooth height

$\Delta h = 0,00025''$ for DP=16

$$K = 1 - R \cdot \sin\beta \cdot \left(\frac{1}{r_0'} - \frac{1}{r_0}\right)$$

$$R_M = R + \Delta R$$

The generating cone angle of the gear is:

$$\delta_{M2} = \delta_{12} + \Delta\delta_2 \tag{5}$$

The cutter used for gear generating has a difference between the outside and inside blade angle of $\Delta\alpha_{0\Sigma}$. Because of this, the inclination angle of the cutter spindle, which compensates for the differences between the blade angle and the root pressure angle, is equal to:

$$\Delta\alpha_2 = \Delta\alpha_{0\Sigma} - \sin\beta \cdot \mathrm{tg}\,\Theta_{f2}.$$

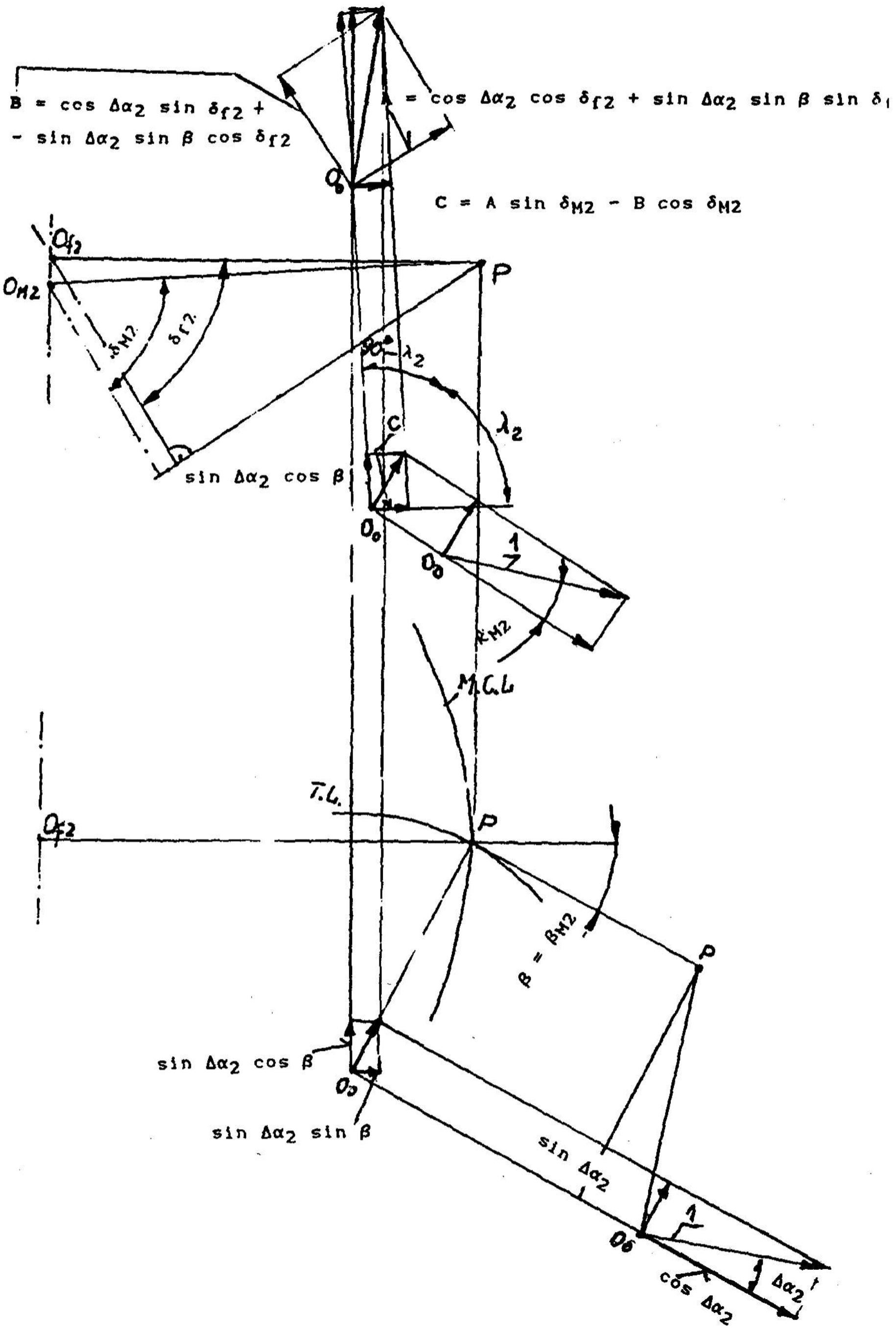
B = cos Δα2 sin δf2 +
- sin Δα2 sin β cos δf2
A = cos Δα2 cos δf2 + sin Δα2 sin β sin δf
C = A sin δM2 - B cos δM2
P
δM2
δf2
90°-λ2
λ2
C
sin Δα2 cos β
M.G.L
T.L.
P
β = βM2
P
sin Δα2 cos β
sin Δα2 sin β
sin Δα2
cos Δα2
Δα2

Fig. 1

Hence, the tool system must create a cone-shaped generating circle based on the angles $\Delta\alpha_2$ and δ_{M2}.

According to Fig. 1, the angle of the main tilt plane of the cutter axis is calculated as follows:

$$\mathrm{tg}\,\lambda_2 = \frac{\sin\Delta\alpha_2 \cdot \cos\beta}{C}, \tag{6}$$

where: $C = A \cdot \sin\delta_{M2} - B \cdot \cos\delta_{M2}$
and $A = \cos\Delta\alpha_2 \cdot \cos\delta_{f2} + \sin\Delta\alpha_2 \cdot \sin\beta \cdot \sin\delta_{f2}$
$B = \cos\Delta\alpha_2 \cdot \sin\delta_{f2} - \sin\Delta\alpha_2 \cdot \sin\beta \cdot \cos\delta_{f2}$

The tilt angle of the cutter spindle is defined as follows:

$$\sin\kappa_{M2} = \frac{\sin\Delta\alpha_2 \cdot \cos\beta}{\cos\lambda_2}. \tag{7}$$

The remaining machine setting values needed for gear generation are calculated in the same way as in the methods of teeth finishing using the cone-shaped generating circle.

6. Pinion teeth finishing

In pinion teeth finishing, the bias of the tooth bearing is eliminated by applying the helical motion.

With the helical motion, the angle of deflection from the pressure angle, expressed in minutes of arc, is defined by the following formula resulting from gear geometry and the assumed diameter of the cutter spindle:

$$i_x = \frac{\frac{R_M}{r_0} \cdot \Theta_{f\Sigma} - (\Theta_{f\Sigma} - \Delta\delta_2) \cdot \sin\beta}{\cos\beta} + \Delta i_x(r_0) \ [''] \tag{8}$$

In this formula the $\Delta i_x(r_0)$ value is a consequence of the assumption that the normal radius of the cutter is different from the calculated radius. The corrected angle calculated according to Euler's theory is defined as follows:

$$\Delta i_x(r_0) = \Delta R \cdot R \cdot \sin^2\beta \cdot \left(\frac{1}{r_0'} - \frac{1}{r_0}\right) \cdot \frac{1}{r_0} \cdot \frac{3438}{\mathrm{tg}\,\alpha} \ [''] \tag{9}$$

Gleason also provides for the change in the root angle of the generating circle which occurs as it turns. The change serves to obtain a desired tooth bearing.

The helical motion causes a change in the generating gear angle, making it necessary to introduce a hypoid offset for pinion finishing. The angle of the pinion generating gear (according to Fig. 2) is defined as follows:

$$\sin\delta_{M1} = \cos i_x \cdot \sin\delta_{f1} - \sin i_x \cdot \sin\beta \cdot \cos\delta_{f1} \tag{10}$$

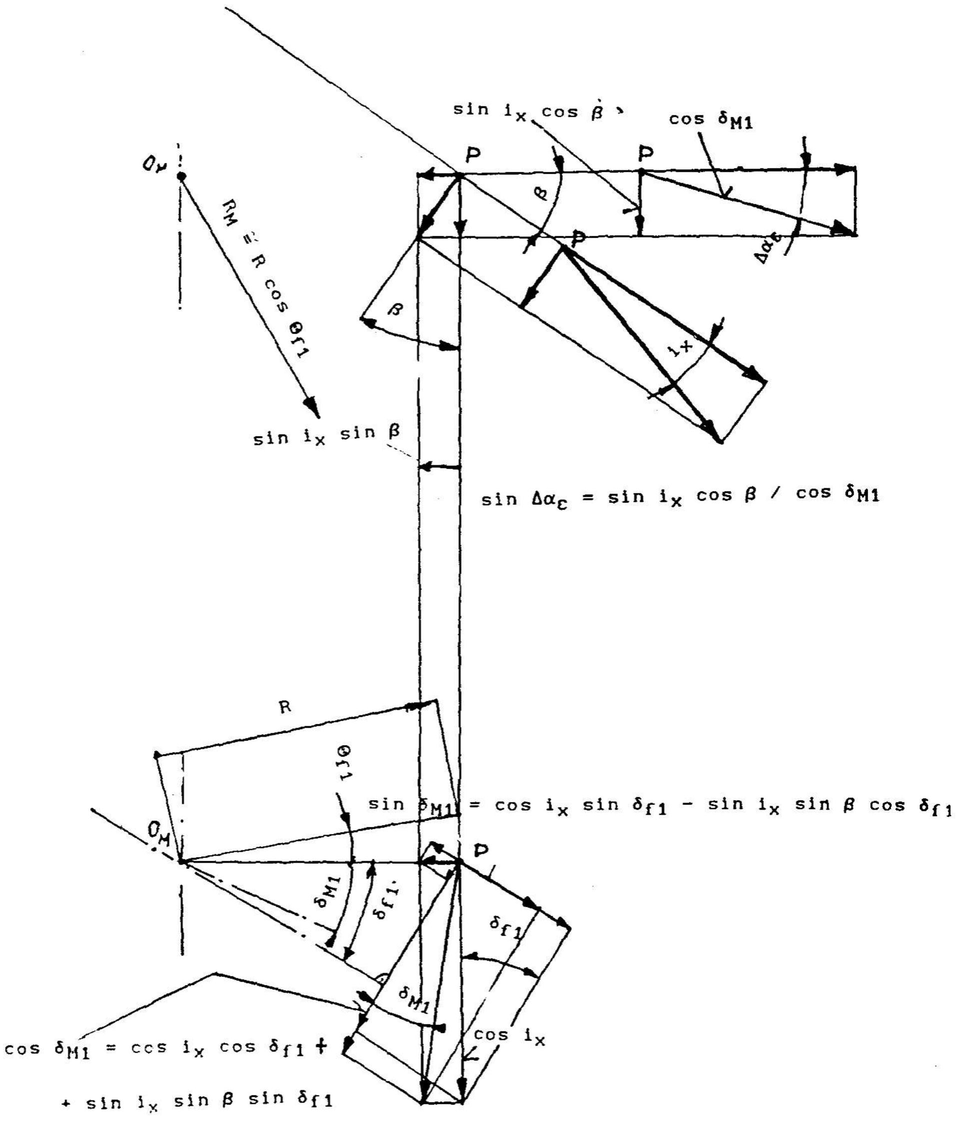
sin i_x cos β
cos δ_{M1}
P
P
P
β
β
$\Delta\alpha_c$
i_x
O_M
$R_M = R \cos \theta_{f1}$
sin i_x sin β
sin $\Delta\alpha_c$ = sin i_x cos β / cos δ_{M1}
R
θ_{f1}
sin δ_{M1} = cos i_x sin δ_{f1} − sin i_x sin β cos δ_{f1}
O_M
P
δ_{M1}
δ_{f1}
δ_{f1}
δ_{M1}
cos i_x
cos δ_{M1} = cos i_x cos δ_{f1} +
+ sin i_x sin β sin δ_{f1}

Fig. 2

The helical motion will produce a difference between the inside and outside blade angles $\Delta\alpha_\varepsilon$, which is a result of the hypoid offset in the pinion finishing system. The difference is calculated according to the following formula (see Fig. 2):

$$\sin\Delta\alpha_\varepsilon = \frac{\sin i_x \cdot \cos\beta}{\cos\delta_{M1}}. \tag{11}$$

The helical motion index in pinion finishing is defined as follows:

$$p = \frac{H}{2\pi} = \cos\beta \cdot \mathrm{tg}\, i_x \cdot R_M, \tag{12}$$

where the spiral lead is equal to:

$$H = 2\cdot\pi\cdot R_M \cdot \mathrm{tg}\, i_X \cdot \cos\beta = 2\cdot\pi\cdot R_M \cdot \mathrm{tg}\,\gamma \tag{13}$$

and

$$\mathrm{tg}\, i_x \cdot \cos\beta = \mathrm{tg}\,\gamma \text{ – the lead angle.}$$

Just as in gear finishing, the main direction of tilt of the cutter spindle axis is determined on the basis of the distribution of the unit vector associated with the axis.

The difference between the pressure angle and the blade angle, which is compensated for by tilting the cutter spindle during pinion finishing, is equal to:

$$\Delta\alpha_1 = \Delta\alpha_{0\Sigma} - \sin\beta \cdot \mathrm{tg}\,\Theta_{f1} \cdot 3438.$$

The angle of the main tilt plane of the cutter spindle is defined by the following formula:

$$\mathrm{tg}\,\lambda_1 = \frac{C}{F} \tag{14}$$

according to Fig.3

$$C = -B \cdot \cos\Delta\alpha_\varepsilon + A \cdot \sin\Delta\alpha_\varepsilon \tag{15}$$

and

$$B = \sin\Delta\alpha_1 \cdot \cos\beta \tag{16}$$

$$A = \cos\Delta\alpha_1 \cdot \cos\delta_{f1} + \sin\Delta\alpha_1 \cdot \sin\beta \cdot \sin\delta_{f1} \tag{17}$$

$$F = D \cdot \cos\delta_{M1} - E \cdot \sin\delta_{M1} \tag{18}$$

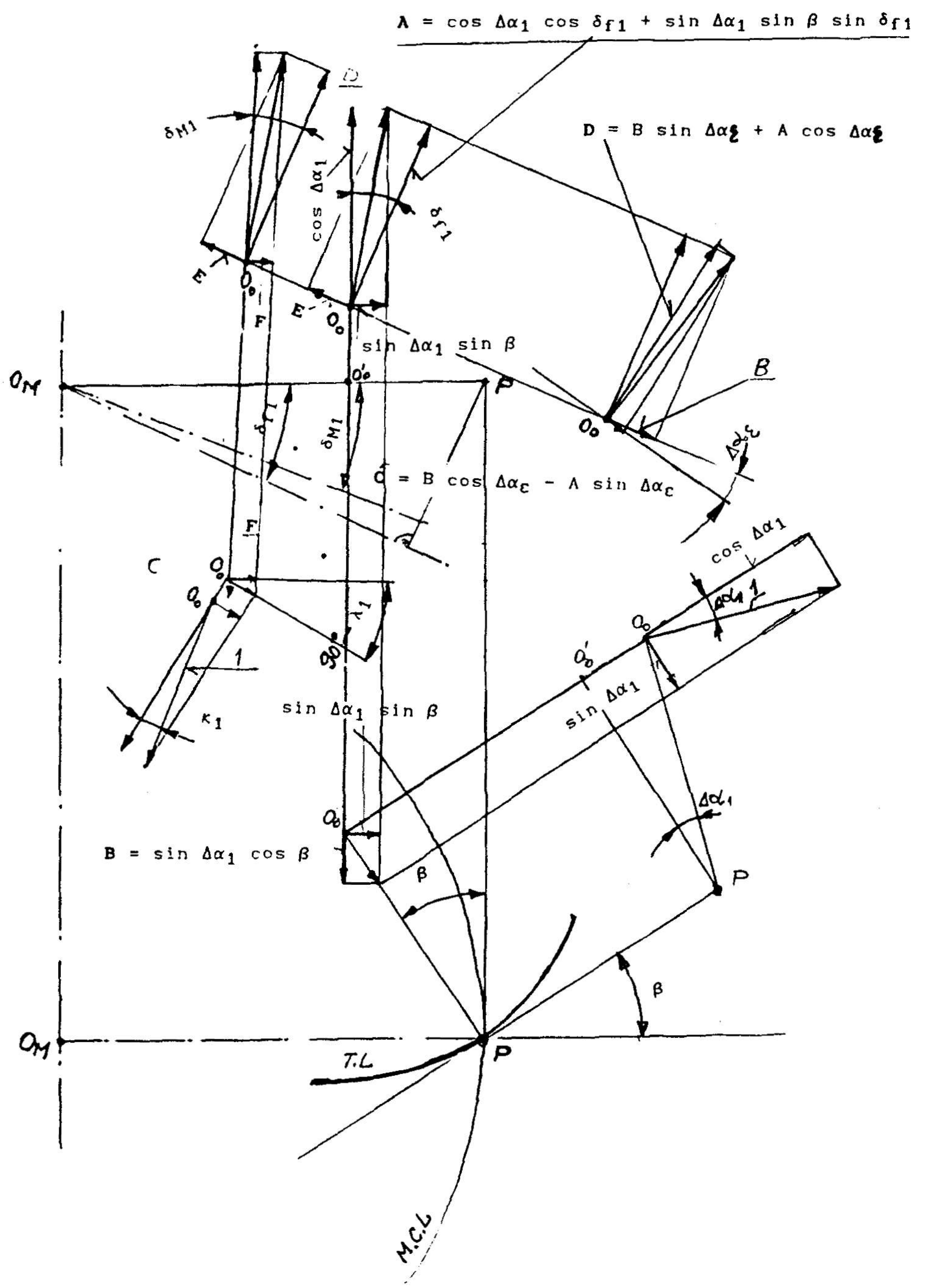
A = cos Δα1 cos δf1 + sin Δα1 sin β sin δf1
D = B sin Δαξ + A cos Δαξ
C = B cos Δαc − A sin Δαc
B = sin Δα1 cos β
sin Δα1 sin β
cos Δα1
sin Δα1
T.L
M.C.L

Fig. 3

(19) $$D = B \cdot \sin \Delta\alpha_\varepsilon + A \cdot \cos \Delta\alpha_\varepsilon$$

(20) $$E = \cos \Delta\alpha_1 \cdot \sin \delta_{f1} - \sin \Delta\alpha_1 \cdot \sin \beta \cdot \cos \delta_{f1}$$

The tilt angle of the cutter spindle is determined in the following way:

(21) $$\sin \kappa_1 = \frac{C}{\cos \lambda_1}$$

The geometrical quantities (chiefly angles) of the technological system given above make it possible to calculate the other setting parameters for the machines.

The formulas used contain the uncorrected angle of the line measured in the middle of the face width, as recommended by Gleason.

6. Final remarks

The above geometrical analysis concerning the main technological quantities of the systems generated according to the SGDH method is a deciphered set of formulas used by the Gleason Works Ltd.

Russian (former Soviet) sources do not provide such detailed calculations concerning the SGDH method.

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