Journal of Theoretical
and Applied Mechanics
Sofia 1993, Year XXIV No 3

MECHANICAL–MATHEMATICAL MODEL OF BALANCING SPRING SUSPENSION WITH ASYMMETRIC SPRINGS

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1. Formulation

Heavy-freight semi-trailers are widely used at long distance cargo transportation. Their operational qualities depend strongly on the semi-trailer car suspension.

In the recent several years a group from the Technical University of Rousse and the Technological Motor Transport Institute of Shumen has been working on improving the semi-trailer car balancing spring suspension. The method used in the investigations was the mechanical-mathematical modelling of vertical vibrations at random microprofile. To improve the operational characteristics of trailer–semi-trailer mechanical system the following general trends were formulated: increasing vertical accelerations in longitudinal direction of semi-trailer deck; improving the contact between the tyres and the road microprofile; decreasing the weight of the semi-trailer car structure; decreasing the load in suspension elements and improving their reliability.

Juxtapositional investigations on different balancing suspension structures have been done [1, 2, 3]. In contrast to the mechanical-mathematical models in reference sources [4, 5] characteristic for the developed until now models is the representing of the spring by two elastic elements which describe its bending at both ends. This approach is being further developed in the present work by introducing asymmetry in the spring which is an element of the balancing suspension. This means that the spring has different stiffness and length of its both parts with regard to the axle which supports it. Thus, the car suspension parameters and characteristics can be more exactly optimized.

The objective of this work was to develop a mechanical-mathematical model of asymmetric balancing spring suspension for semi-trailer car in the trailer–semi-trailer system.

The purpose of the work was:

- To form a mechanical-mathematical model of the trailer–semi-trailer mechanical system;
- To develop a computer software system for vertical vibrations modelling, where from the dynamic characteristics for the mechanical system behaviour would be acquired;
- To do a preliminary investigation for estimating the influence of the spring asymmetry upon the operational characteristics of the investigated system.

2. Mechanical–mathematical model

When creating the model some assumptions were made, which have been already used in such mechanical systems [4]. For the present we assume that the model represents a constrained holonomic system with perfect bonds. System's vibrations around the equilibrium position are small. The trailer and semi-trailer are considered perfectly rigid bodies, the longitudinal plane of which is a symmetry plane, and which have uniform motion. Only vertical forces act on the mechanical system. The coordinate axes are drawn through the centers of gravity and are principal central axes. The tyres are in a point contact with the road profile, their leveling ability being considered through filtering of the random microprofile high-frequency components.

The trailer–semi-trailer mechanical system is shown in Fig. 1. The following designations are introduced: M_1, M_2, J_1, J_2 – masses and mass inertia moments of the trailer and semi-trailer; m_1, m_2, m_3, m_4 – axles masses; c_{s1}, c_{s2} – trailer spring coefficient of elasticity; c_1, c_2, c_3, c_4 – coefficients of elasticity of the separate parts in the car balance suspension springs; c_{t1}, c_{t2}, c_{st} – coefficient of elasticity of the tyres; k_{s1}, k_{s2}, k_1, k_2, k_3, k_4, k_{t1}, k_{t2}, k_{st} – coefficients of reduced hydraulic friction in the suspension and tyres; a, b, l_1, l_2, l_3, d_1, d_2 – distance between mechanical system's characteristic points; f_1, f_2 – length of each of the two balance beam parts; e_1, e_2, e_3, e_4 –length of each of the separate semi-trailer spring parts; q_1, q_2, q_3, q_4 – current random values of microprofile ordinates under corresponding tyres; z_1, z_2, φ_1, ζ_1, ζ_2, ζ_3, ζ_4 – generalized coordinates.

The differential equations of the mechanical system motion are as follows:

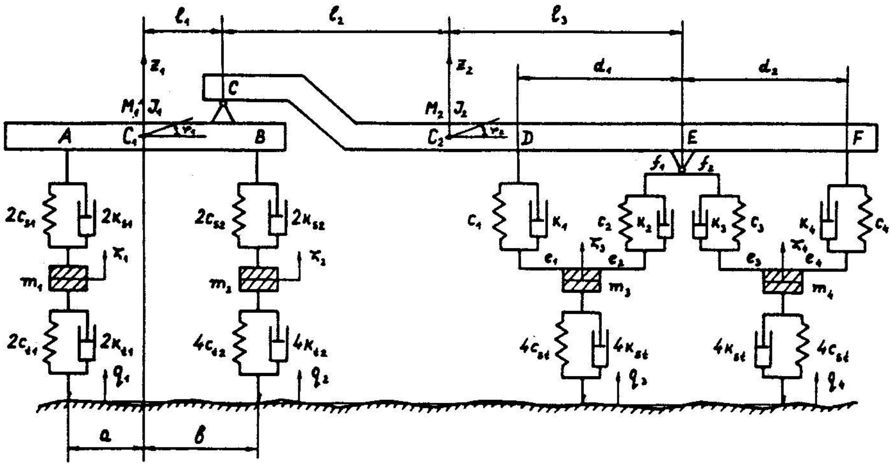

Fig. 1

$$
\begin{aligned}
\ddot{z}_1 &= k_{1,1}z_1 + k_{1,2}z_2 + k_{1,3}\varphi_1 + k_{1,4}\zeta_1 + k_{1,5}\zeta_2 + k_{1,6}\zeta_3 + k_{1,7}\zeta_4 + k_{1,8}\dot{z}_1 + \\
&\quad + k_{1,9}\dot{z}_2 + k_{1,10}\dot{\varphi}_1 + k_{1,11}\dot{\zeta}_1 + k_{1,12}\dot{\zeta}_2 + k_{1,13}\dot{\zeta}_3 + k_{1,14}\dot{\zeta}_4, \\
\ddot{z}_2 &= k_{2,1}z_1 + k_{2,2}z_2 + k_{2,3}\varphi_1 + k_{2,4}\zeta_1 + k_{2,5}\zeta_2 + k_{2,6}\zeta_3 + k_{2,7}\zeta_4 + k_{2,8}\dot{z}_1 + \\
&\quad + k_{2,9}\dot{z}_2 + k_{2,10}\dot{\varphi}_1 + k_{2,11}\dot{\zeta}_1 + k_{2,12}\dot{\zeta}_2 + k_{2,13}\dot{\zeta}_3 + k_{2,14}\dot{\zeta}_4, \\
\ddot{\varphi}_1 &= k_{3,1}z_1 + k_{3,2}z_2 + k_{3,3}\varphi_1 + k_{3,4}\zeta_1 + k_{3,5}\zeta_2 + k_{3,6}\zeta_3 + k_{3,7}\zeta_4 + k_{3,8}\dot{z}_1 + \\
&\quad + k_{3,9}\dot{z}_2 + k_{3,10}\dot{\varphi}_1 + k_{3,11}\dot{\zeta}_1 + k_{3,12}\dot{\zeta}_2 + k_{3,13}\dot{\zeta}_3 + k_{3,14}\dot{\zeta}_4, \\
\ddot{\zeta}_1 &= k_{4,1}\dot{z}_1 + k_{4,3}\varphi_1 + k_{4,4}\zeta_1 + k_{4,8}\dot{z}_1 + k_{4,10}\dot{\varphi}_1 + k_{4,11}\dot{\zeta}_1 + k_{4,15}q_1 + k_{4,16}\dot{q}_1, \\
\ddot{\zeta}_2 &= k_{5,1}z_1 + k_{5,3}\varphi_1 + k_{5,5}\zeta_2 + k_{5,8}\dot{z}_1 + k_{5,10}\dot{\varphi}_1 + k_{5,12}\dot{\zeta}_2 + k_{5,17}q_2 + k_{5,18}\dot{q}_2, \\
\ddot{\zeta}_3 &= k_{6,1}z_1 + k_{6,2}z_2 + k_{6,3}\varphi_1 + k_{6,6}\zeta_3 + k_{6,7}\zeta_4 + k_{6,8}\dot{z}_1 + k_{6,9}\dot{z}_2 + k_{6,10}\dot{\varphi}_1 + \\
&\quad + k_{6,13}\dot{\zeta}_3 + k_{6,14}\dot{\zeta}_4 + k_{6,19}q_3 + k_{6,20}\dot{q}_3, \\
\ddot{\zeta}_4 &= k_{7,1}z_1 + k_{7,2}z_2 + k_{7,3}\varphi_1 + k_{7,6}\zeta_3 + k_{7,7}\zeta_4 + k_{7,8}\dot{z}_1 + k_{7,9}\dot{z}_2 + k_{7,10}\dot{\varphi}_1 + \\
&\quad + k_{7,13}\dot{\zeta}_3 + k_{7,14}\dot{\zeta}_4 + k_{7,21}q_4 + k_{7,22}\dot{q}_4,
\end{aligned}
$$

where $k_{i,j}$ ($i = 1,\ldots,7$ and $j = 1,\ldots,22$) are coefficients depending on system's parameters.

3. Computer software system

A software system was designed for modelling the vertical vibrations at the following two motion conditions:

- Stationary-motion upon random microprofile which consists of centered Gauss random process with a definite fraction-rational spectral density;
- Non-stationary-moving over a single sinusoid unevenness of type $q = q_0(1 - \cos\gamma)/2$, where q_o is the maximum height of unevenness, and $\gamma = 2\pi.\Delta s.i/l$, $i = 0, 1, \ldots, l/\Delta s$. Here Δs is the road step, and l is the unevenness length.

As a evaluation criterion for the mechanical system operational characteristics the following root-mean-square values were chosen: of accelerations of points along the length of the trailer and the semi-trailer; deformations of spring elastic elements and tyres; forces in springs, tyres, supports, balance beam, rods and stock.

4. Results

A partial investigation was done on the influence of asymmetry of springs upon operational characteristics of heavy-freight semi-trailer PR32 coupled to MADARA LIAZ trailer. It was determined that:

- Comparing the load in the four ends of semi-trailer car springs showed that the front end of the front spring was most heavily bent, as in certain conditions it changed up to 2.5 times;
- Increasing the stiffness c_1 of the front end of the front semi-trailer car spring makes its bending increase to 147% the load being distributed on the other spring ends.

5. Conclusions

The partial investigations showed that the asymmetry in springs of a balancing spring suspension influences significantly the semi-trailer operational characteristics, and therefore it should undergo purposeful investigations.

The developed mechanical-mathematical model and computer software system give possibilities for revealing entirely new sources for improvement of the mechanical system operational characteristics at balancing spring suspension design.

The developed mechanical-mathematical model and computer software system can be used in design of combined pneumatic-spring balancing suspension and in organizing its active control.

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Presented at the 7th Congress on Theoretical and Applied Mechanics, Sofia, August 30 – September 1, 1993
section: "Applied Mechanics"