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BASIC THESIS FOR FORMAL FORCE ANALYSIS OF REAL MECHANICAL PARTS

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Force analysis of mechanical assembly (MA) is intended to solve the problem of discovering all the forces and moments in their quantitive and qualitative aspects, which load up all the parts of the assemblies, when the incoming forces are known.

The purpose of the work is to create a data base which will permit to do the formal force analysis. Certain statements and resulting consequences are proved to solve the problem.

Statement 1. *An active force F_i is transmitted from one part to another adjacent one: a) along the normal n to the contact elements of the two parts (without taking into account the friction force and at an angle between F_i and n <> 90°) or b) along the line t, that makes a angle σ with n equal to the friction angle between the contact elements of the two parts (accounting for the friction force and at an angle between F_i and n <> 90°).*

In other words, this statement is a fundamental principle for bearing reactions accepted and proved in mechanics [1-3].

From statement 1 it follows:

Consequence 1. *An active force F_i can't be transmitted through a connection of two adjacent parts: a) at an angle between F_i and n (t) equal to 90° (without /with/ taking into account the friction force).*

Statement 2. *An active moment is transmitted from one part to another adjacent one: a) along the line n (t) if the two parts are movable or b) along one*

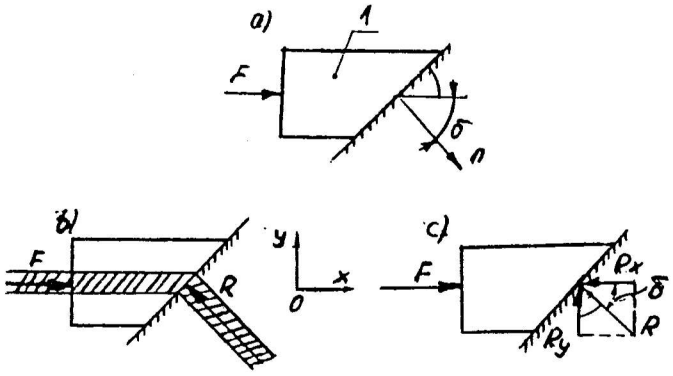

Fig. 1

or more lines n_i (t_i), $i = 1, 2, \ldots, p$, to the contact elements of the parts or c) by transforming to an active force in case of transmission with one force.

Proof. A. Each moment M_i that aims to rotate a material body round an axis perpendicular to the plane of the moment can be substituted for force F on the plane α, so that $M = F.h$, where h is distance between the axis and the force. The passing of a force from one part to another one proceeds according to statement 1, the first part of statement 2 beeing proved.

B. The moment M_i can be substituted for a set of forces $F_1, F_2, \ldots, F_p$, so that $\sum F_i.h_i = M_i$ $(i = 1, 2, \ldots, p)$, where h_i is a distance between the axis and the force F_i. According to the superposition principle the action of the forces F_i is equal to the action of M_i. The proof will be done if statement 1 is applied to each force F_i in the set.

C. The body is loaded by moment M and force F $(M = F.h)$. Applying Dalamber principle (principle of dynamic force analysis) [4] to the uniformly moving body shows that force F is unbalanced. Or the force F is taken from bearings of the body and therefore F is an active force.

Consequence 2. *Moment M_i passes from one part to another one by force or by moment.*

Consequence 3. *An active moment M_i can be transmitted through the connection of two parts at line perpendicular to the normal towards the junction at following conditions: a) A normal force N exists in the junction and b) $M \leq \mu o.N.h$ (this condition has to be taken into account if there is not a movement between the two parts).*

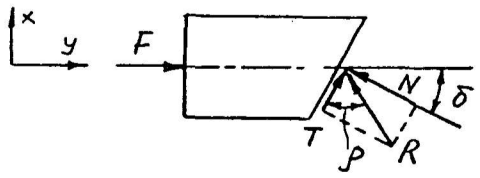

Fig. 2

Statement 3. *A new force appears in the connection if an incoming force and the principal normal of the connection lie in one coordinate plane and the angle between them is different from* 90°.

Proof. Part 1 following the conditions of statement 3 is drawn in Fig. 1. The reaction R of the adjacent part has to be normal to the contact surface (Fig. 1b) according to statement 1. The body 1 is in balance under the influence of the forces and the reactions. Analytic conditions of that balance [2,3] are:

(1) $$\sum X_i = 0$$

(2) $$\sum Y_i = 0$$

(3) $$\sum M_{io} = 0,$$

where X_i and Y_i are the projections of all active forces and reactions on an arbitrary coordinate system (Fig. 1c), while M_{io} are the moments of these forces towards the arbitrary point. The concretization of (1) is the equation.

$$\sum X_i = F - Rx = 0 \quad (i = 1, 2) \quad \text{or} \quad F = Rx$$

Therefore there is new force with a size of (Fig. 1c)

(5) $$Ry = F.\mathrm{tg}\,\delta$$

The treatment with friction is similar (Fig. 2).

Statement 4. *If M_i is a rotation moment that causes or can cause rotation of a material body round an axis s and if the angle Γ between the normal n and line l*

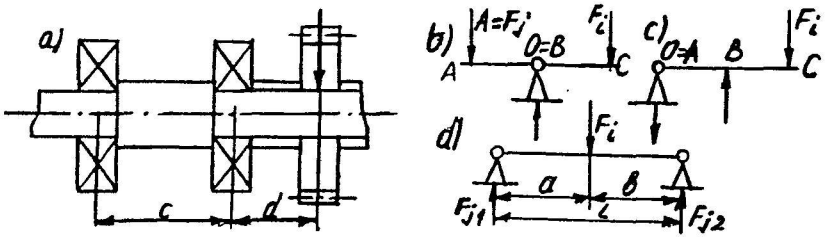

Fig. 3

(the line l is on plane $\alpha \perp s$ and connects $p.A = \alpha \cap s$ and $p.B$ – the origin of n) is on plane and $\Gamma <> 90°$ then new force F' appears in the junction.

Proof. The force F has to be perpendicular to line l and in size will be $F = M/r$, where r is the length of l, if the body revolves round an axis s without appearing of additional radial forces. If $F, n <> 90°$, i.e. $l, n <> 90°$ according to statement 3 an additional force F' appears.

Statement 5. *If the force F_i is parallel to one of the coordinate axes and intersects the normal n and n is not parallel to any coordinate axis, then two forces F' and F'', that are parallel to two other coordinate axes, appear in the junction.*

Statement 6. *If M_i is a rotation moment, that causes or can cause rotation of a material body round an axis s and if the angle Γ between n and l (the line l lies on the plane $\alpha \perp s$ and connects $p.A = \alpha \cap s$ and $p.B$ – the origin of n) is not on the plane and $\Gamma <> 90°$, then additional forces F' and F'' appear in the junction.*

The proof of statement 5 (statement 6) is as (to) the proof of statement 3 (statement 4).

Parts like "lever of the first kind" and "lever of the second kind" are often popular in MP (Fig. 3). The first of them can be named "force lever" and the second ones — "dividing lever". The analysis of the two levers is a basis for the following statement.

Statement 7. *If the force F_i acts on a part-force lever (a part, containing three connections where the incoming force acting on the end connection), then the force F_i can be transmitted across the nearer connection ($p.A$) and a new force F_i' appears in the farer connection ($p.B$). The forces F_i and F_i' are opposite and $|F_i'| = F_i((l+d)/c)$ – Fig. 3 if the normals of the three connections are parallel.*

Consequence 4. *A part-lever (force lever or dividing lever) is loaded with three forces and the intermediate force in opposite to the end forces.*

Statement 8 [4,5]. *Inertia forces appear in every moving body, which acceleration is* $a <> 0$.

Consequence 5. *An active force* F_i *(an active moment* M_i*) does not change its direction and size when it passes from one part to another.*

The last consequence follows from statements 2-6. The next consequence results from consequence 5.

Consequence 6. *An active force* F_i *(an active moment* M_i*) can pass through a part only once.*

Statement 9. *If an active force* F_i *(an active moment* M_i*) in the* MA *set on the ground (directly or indirectly with some other* MA*) does not equalize another force (moment) of* MA*, that force (moment) goes to the ground or to another* MA.

Proof. A) **Force** F_i. A force F_i which has passed through a part can not return back but it can not disappear because every force incoming to the part causes reactions in the connections of the part with the adjacent parts of the assembly [2].

Therefore the force F_i leaves MA if it doesn't balance with an opposite one but with the same direction force in any part of MA. The connections of MA with the technical environment are the ground or another MA with which the first MA has a connection. So that these are all the possibilities for the exit of the force F_i.

B) **Moment** M_i. As the moment M_i can be substituted by a force or some forces and statement 9 had been proved for a force, then statement 9 is valid for a moment.

It should be mentioned that the forces and the moments going to the ground are internal for the ground. From statement 9 it results

Consequence 7. *A force can not disappear.*

From the statement 9 and the law of Newton it follows

Consequence 8. *A material body can not be loaded only with one force or only with one moment or only with one moment and one force.*

From consequence 1 it follows

Consequence 9. *If a material body is loaded only with two forces their directions can not be perpendicular to each other.*

From statements 3 and 5 and the fact of the appearance of a new force it results

Consequence 10. *The incoming force and the leaving force or forces are parallel.*

Statements an results include active, friction, weight and inertia forces (moments) and two kind of parts simple and part rods. They are deduced on the basic of the work hypothesis for absolutely solid and represent a precondition for doing formal force analysis of real MA.

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