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SPATIAL NUMERICAL MODEL OF THE THERMAL STRESS STATE OF ELASTIC – CREEPING CONCRETE BLOCKS*

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1. Introduction

During their construction and service life the blocks of mass concrete structures undergo significant temperature changes, causing considerable stresses. The thermal stress state (TSS) of such blocks is transient and always spatial, even when the field of the temperature changes is $1-D$ or uniform.

Up to the present time the TSS of concrete blocks is calculated prevailently by $2-D$ numerical models employing an averaged through the third dimension temperature change field (TCHF). The plane model is much more economical and feasible, regarding the necessary machine resources and the output data processing, but leads to a simplification of the fenomenon. For an assesment of the plane model approximation and a study of the $3-D$ TSS full picture, a spatial numerical model for TSS of discretely growing by lifts blocks on an elastic foundation was developed and a correspondent programme was carried out.

2. Basic assumptions

The basic assumptions of the model are as follows: a) The block is an elastic-creeping body with a linear stress – strain relationship and age functions, elasticity and creep; b) The coefficient of the temperature expansion α is constant; c) The coefficient of lateral strains μ is the same with the elastic and viscous deformations; d) The creep

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function is the same in tension and compression; e) Young's modulus and the creep doesn't depend upon the temperature.

To a great extent these assumptions [1] are valid for the concrete when only the tensile stresses reach the strength, while the compression is far below that limit.

3. Basic equation of the mathematical model

To the implementation of the $3-D$ model of TSS the FEM with 8-points isoparametric elements was applied. An essential problem was the type of the creep consideration. Zenkievich [4] proposed an iterational procedure for stress calculation in every time interval but no convergence criterion, limiting the interval length, was mentioned.

The analysis of two Chinese papers [2, 5] brings to the conclusion that in the case they use in an implicit way the strain-stress relation of an elastic-creeping body:

$$\varepsilon(t) = \int_{\tau_0}^{t} \delta(t,\tau)\frac{\partial\sigma(\tau)}{\partial\tau}d\tau + \alpha\Delta T \tag{1}$$

where: $\delta(t,\tau)$ is the unit long time deformation, ΔT is the temperature change in the whole period. In the numerical solution developed by Van Gobin [2] the period is divided into shorter intervals with $\partial\sigma/\partial\tau$ =const which makes possible the exact solution of the integral. Here again convergence criterion is not discussed but the criterion derived in [1] could be used in the case.

The method, made use of in this report, is based on [3]. The basic equation is the stress-strain relation:

$$\{\sigma(t)\} = \int_{\tau_0}^{t} [D(t,\tau)].\left(\left\{\frac{\partial\epsilon(\tau)}{\partial\tau}\right\} - \left\{\frac{\partial\varepsilon_0(\tau)}{\partial\tau}\right\}\right).d\tau, \tag{2}$$

where $D[t,\tau] = p(t,\tau).[M] = K(t,\tau).E(\tau).[M]$.

In the above formula $p(t,\tau)$ is the relaxation function; $[M]$ is the unit elasticity matrix; $\{\sigma\},\{\varepsilon\}$ and $\{\varepsilon_0\}$ are the vectors respectively of the stresses, strains and imposed volumetric changes; $E(\tau)$ is Young's modulus and $K_r(t,\tau) \leq 1$ is the relaxation coeficient at the moment t for the stress, induced at the concrete age τ.

The relaxation function $p(t,\tau)$ can be determinated either by an analitical solution of (1) and determination of the resolvent of $\delta(t,\tau)$, which is rather difficult, or by selection of an empirical formula for $p(t,\tau)$ based on experimental data [5] or numerically obtained data by means of a known $\delta(t,\tau)$ as in [3].

Because of a lack of experimental data we assumed the method given in [3] representing the relaxation function as a product of age function $\varphi(\tau)$ and strain duration function $f(t)$:

$$p(t,\tau) = \sum_{j=1}^{s} f_j(t).\varphi_j(\tau). \tag{3}$$

The numerical solution of (2) is accomplished by division of the calculation period into smaller intervals. At the end t_k of the current interval k the integral equation (2) transforms into:

$$\{\sigma(t_k)\} = [D(t_k, \tau_k)]\{\Delta\varepsilon_k\} + \{A_k\}, \quad \text{where} \tag{4}$$

$$\{A_k\} = \sum_{i=1}^{k-1}[D(t_k, \tau_i)]\{\Delta\varepsilon_i\} - \sum_{i=1}^{k}[D(t_k, \tau_i)]\{\Delta\varepsilon_{0,i}\}, \quad \tau_i = \frac{t_{i-1} + t_i}{2}. \tag{5}$$

This approximation presumes that ε and $p(t_k, \tau)$ in the interval $t_i - t_{i-1}$ are linear functions.

Employing (4) and FEM, finally a sistem of linear algebraic equations for nodal displacements $\{u_k\}$ determination with given volumetric changes $\{\varepsilon_0\}$ is arrived at.

The above exposed numerical solution of (2) should be carried out successively step by step for every interval. In each interval k only the increment of the displacements $\{\Delta u_k\}$ is unknown and is to be determined [3]. The equation's sistem for interval $t_k - t_{k-1}$ looks as follows:

$$\left(\sum_{e=1}^{N_{\text{el}}} \int_{V_e} [B_e]^t [D(t_k, \tau_k)] [B_e] .dV\right) \{\Delta u_k\} + \sum_{e=1}^{N_{\text{el}}} \int_{V_e} [B_e]^t \{A_k\} = 0.$$

The calculation of $\{A_k\}$ by formula (5) requires the storage of the increments $\{\Delta\varepsilon_i\}$ of all previous intervals. However if the formula of $p(t, \tau)$ has the structure of (3), a recurrent expression for $\{A_k\}$ could be derived. If $p(t, \tau) = f(t).\varphi(\tau)$ for $\{A_k\}$ the following expression is valid:

$$\{A_k\} = \{S_k\} - [D(t_k, \tau_k)]\{\Delta\varepsilon_{0,k}\} \quad \text{where}$$

$$\{S_k\} = f(t_k) \sum_{i=1}^{k-1} \varphi(\tau_i)[M] \left(\{\Delta\varepsilon_i\} - \{\Delta\varepsilon_{0,i}\}\right).$$

After some simple transformations we obtain:

$$\{S_{k+1}\} = \frac{f(t_{k+1})}{f(t_k)}\{S_k\} + f(t_{k+1}).\varphi(\tau_k)[M] \left(\{\Delta\varepsilon_k\} - \{\Delta\varepsilon_{0,k}\}\right). \tag{6}$$

As it can be seen from (6) for the calculation of $\{A_{k+1}\}$ it is necessary to keep only the values of $\{S_k\}$, $\{\Delta\varepsilon_k\}$ and $\{\Delta\varepsilon_{0,k}\}$.

With this model the final stress at the moment t_k is obtained as a sum of the relaxed to t_k stress increments $\{\Delta\sigma_i\}$ of all previous intervals. The exposed approximation of (2) is absolutely convergent because the relaxation coefficient $K_r(t, \tau)$ is a monotonously decreasing function and always $K_r(t,\tau) > 0$. This solution compared with the proposed by Zienkewich method is direct and doesn't need iterations. Compared with Van Gobin's method [2] it is absolutely convergent.

On the basis of the proposed method a programme for spatial modelling of TSS of discretely growing blocks is under developement. The first study it served for, was the comparison of the TSS, obtained by spatial and plane models. Hereby, as an illustration, are given some results of a simple problem of TSS of a rectangular block with uniform temperature changes ΔT =const. In this case TSS is caused only by the foundation restraint and the spatial effect is influenced by the relation between the dimensions of the block and the foundation in the horizontal plane. For making clear the influence of that factor, the TSS was determined by different calculation schemes given in Fig. 1.a. Fig. 1.b shows the distribution of σ_x along the central vertical axis of the block. Very briefly some of the conclusions are as follows.

Schemes I and III yield greater σ_x compared to plane deformation $2-d$ model (PD). The difference is smaller with $B/L = 1$ and grows with decrease of B/L reaching 1,3 times with $B/L = 0,1$. Schemes II and IV always yield values of σ_x between those, obtained by PD (with $B/L = 1$) and plane stress $2 - D$ model (PS) (with $B/L \approx 0$).

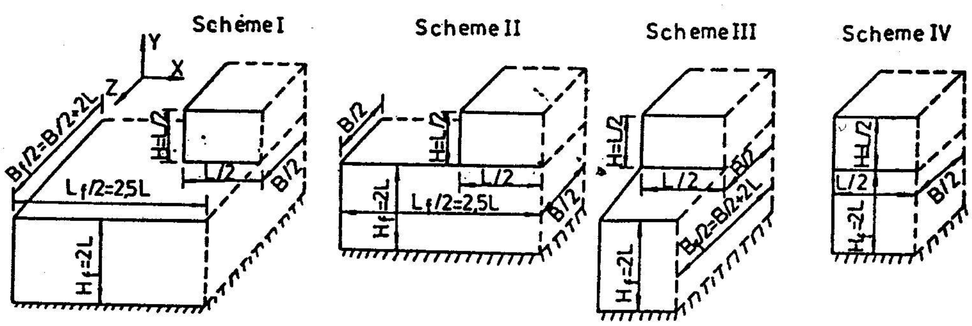

Fig. 1.a
Distribution of σ_x and σ_z along the central vertical axis of the blok

4. Conclusions

On the basis of the theory of the elastic - creeping body and FEM, a threedimensional mathematical model for TSS of discretely growing blocks has been developed and a programme on the model basis has been carried out. The studies accomplished by $3-D$ and $2-D$ models of TSS of a concrete block on an elastic foundation with diffferent dimensions show that the difference between the results of the models varies with the change of the ratio B/L of the block and the ratio B_{bl}/B_f. Approximative methods should be sought for consideration of the spatial effect in $2-D$ modelling of TSS of concrete blocks.

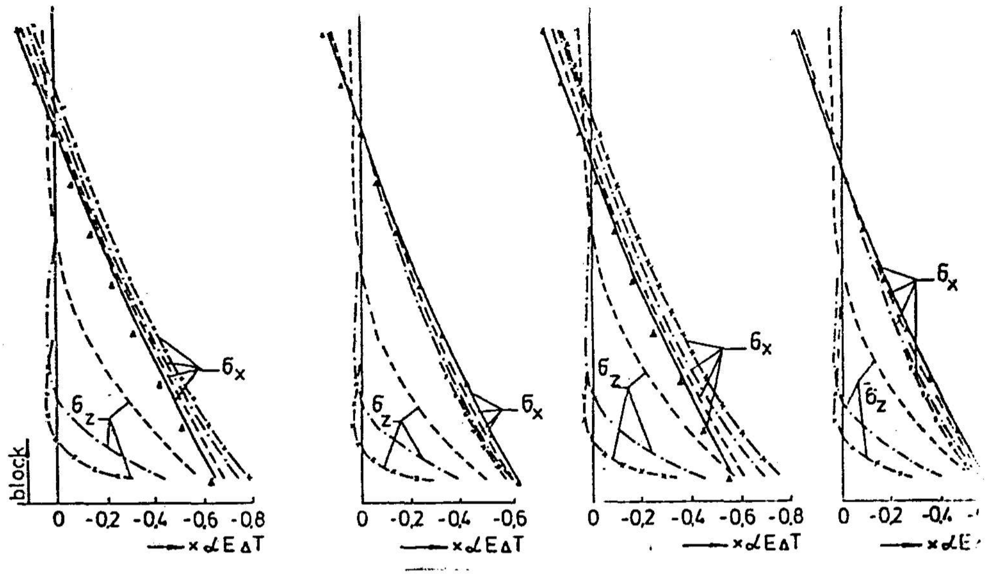

Fig. 1.b
Distribution of σ_x and σ_z along the central vertical axis of the blok
—$B = L$ – – –$B = L/2$ – · – $B = L/5$ — — $B = L/10$ ▲▲▲P_D

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