Journal of Theoretical and Applied Mechanics, Sofia, 2011, vol. 41, No. 1, pp. 19–32

AN ENGINEERING APPROACH FOR WAVE NUMBER DETERMINATION

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[Received 06 October 2009. Accepted 04 October 2010]

Abstract. An engineering approach to radiation damping evaluation for both concrete and embankment dams has been proposed in some recent papers of the authors. The approach employs some elements of the Wave Mechanics to solve the problem of the dynamic dam-foundation interaction, e.g. the velocity of seismic shear wave, the frequency dependent wave number values, etc. The present paper demonstrates the possibilities of obtaining and estimating these parameters. The wave number values of Santa Felicia earthfill dam are obtained for five different types of excitation. The variations in the wave number value are determined on the recorded field data and recently developed Method of Identification and Prognostication of dynamic characteristic of dams. A generalization of the obtained results is presented. Possible practical applications of the proposed approach include an estimation of the total and the radiation damping for existing dams, as well as prediction of the dynamic characteristics of dams under construction, and analyzing the dam-water-foundation interaction effects. The proposed approach can be employed in the solution of similar problems for other civil engineering structures as tall buildings, towers, etc.

Key words: dam, damping, dynamic, earthquake, frequency, wave number.

1. Introduction

The solution of a wave equation following the opinion of some authors is still in its infancy and requires the detailed knowledge of the participated parameters. Two of them are frequency dependent: wave numbers and damping factors for all of the mode shapes of interest. The unavailability of sufficient mode shapes and frequencies is a serious limitation. In many works by this reason the dynamic behaviour is treated by frequency independent parameters.

A small number of frequencies and mode shapes was used as a common tendency until recently. Lately, the interest for estimation of higher mode shapes and corresponding frequencies of vibration marks significant progresses, but the recorded vibration frequencies are not complete enough. The types of mode shapes are often unknown or badly determined. The recently proposed Method of Identification and Prognostication of dynamic characteristics of dams (MIP) can be applied [1] in order to overcome these difficulties. It is possible to achieve conformity between recorded and calculated mode shapes and frequencies of vibration by the implementation of the MIP.

The possibility of comparing damping ratio values especially for dams is of a great interest. Problems with their valuation and comparison arise because they are usually determined at given conditions at different dynamic excitation, and at the separate structure. The recent values obtained for the same structure at other dynamic excitations cannot be compared with those previously obtained.

The results of our investigations show that the damping coefficients of a given dam subjected to different excitations at different time history can be compared, as well as to be analyzed and estimated more profoundly. The frequency dependent wave numbers for Santa Felicia earth fill dam under five different excitations are obtained. Their changes are determined according to available experimental data of three full-scale tests and two earthquakes. This work presents the results for the period of vibrations and the damping coefficients obtained by means of the MIP, versus averaged wave numbers regarded as global characteristics of dynamic/seismic behaviour of the investigated dam.

The approach to determining these basic wave parameters presented in this paper can be successfully applied for calculation of wave or dynamic characteristics of dams and parameters of dam-water-foundation interaction. Future research could be targeted at an estimation of the real seismic shear wave velocities, correlated with the dam vibration frequencies.

2. Method of identification and prediction

In the present paper the Method of Identification and Prediction of dynamic characteristics of concrete dam (MIP) was used [1]. Four sets of calculations by an own research computer programmes named TRIDIDAD are carried out. The results obtained for undamped natural vibration frequencies are displayed against the elasticity modulus in the log-log co-ordinate system as inclined straight lines corresponding to the type of mode shapes. A family of parallel lines is obtained. The resonant periods of the investigated structure obtained by a dynamic full-scale test or by the recorded earthquake are plotted on these lines according to the type of their mode shapes. The points are connected and the result is a curve consisting of two parts. The intersection point obtained between them divides the mode shapes into low and high ones. This curve describes the change of dynamic characteristics of the system and is called Resonant Travel Time Curve (RTTC). Such characteristic type of curves is obtained for concrete gravity dams [1, 2], arch dams [1], earth fill dams [3], and high-rise buildings [3].

3. Information of dam and its seismic/dynamic excitations

Santa Felicia earth fill dam is located on Piru Creek, about 7 km northeast of the town Piru, and about 65 km northwest of the city of Los Angeles, California. The dam is 83 m high, 389 m long at the crest, 364 m long across the valley at the base and 9 m wide at the crest (Fig. 1) [4, 5].

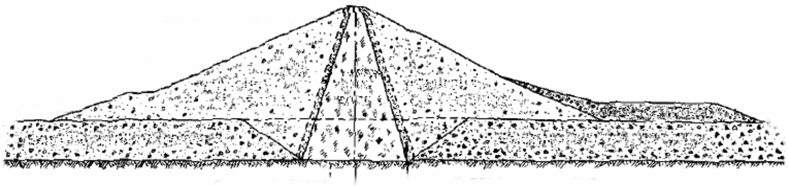

Fig. 1. Dam profile (after [4])

This study concerns with results from observations of the effect of two earthquakes and three full-scale tests. The first earthquake is well-known 1971 San Fernando earthquake with $M_L = 6.3$ and an epicentre distance of 33 km, and the second is 1976 Southern California earthquake with $M_L = 4.7$ and an epicentre distance of 14 km. More than 80 vibration frequencies ware recorded in the three directions – transversal (T), longitudinal (L) and vertical (V). In the full-scale tests, carried out during 1978, three types of dynamic excitations were used including forced vibration test (FVT), ambient vibration test (AVT), and hydro dynamically generated forces (HDT). Three-dimensional measurements of the motions of 25 stations along the crest and 7 stations on the downstream slope were recorded and analyzed by the authors cited above during the dynamic tests. The recorded vibration frequencies are 150 in number.

4. Finite element model of the Santa Felicia earth fill dam

The dam is analyzed in the assumption of linear behaviour for the material as a homogeneous and symmetric 3-D finite element system. Material characteristics are Poisson ratio $\nu = 0.35$ and specific weight $\gamma = 0.0207$ MN/m^3. The boundary conditions along the dam-foundation interface are of the rigid constraint type. The boundary conditions allow obtaining symmetric and anti-symmetric mode shapes for the nodal points of the central cross section. The model of the reservoir is of constant depth, extending to the upstream direction at about three times of the dam height (Fig. 2). The modelled water level is about 61 m and corresponds to the one reported in [5]. The dam-reservoir interaction is taken into consideration by solving the equations of motion and the hydrodynamic equations simultaneously.

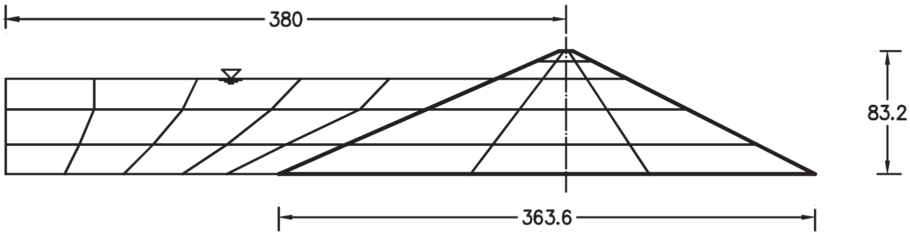

Fig. 2. Finite element mesh in the plane of central cross section of the dam and in the reservoir (all dimensions in meters)

5. Earlier results obtained by MIP

The huge volume of results obtained by the dynamic tests of the Santa Felicia earth fill dam, are represented as separate RTTC, as reported earlier [6, 7]. In this manner, it is already possible to interpret so complicated problem as the dynamic behaviour of earth fill dams with the application of the MIP.

The registered earthquake data have been also interpreted with the MIP and are shown in Fig. 3, together with the obtained curve for the forced vibration test.

The resonant periods obtained by a FVT and these recorded during the 1971 San Fernando earthquake (SF E'71) and during the 1976 South California earthquake (So C'76) are plotted on the inclined lines according to the type of their mode shapes (Fig. 3). The additional curves, presented in Fig. 3, namely the translation of a part of the RTTC of the FV curve to the probable intersection point of the San Fernando earthquake curve (the curve **4-a-b**), is presented in order to facilitate its drawing. The curve **4-a-c** corresponds to the section from the RTTC for the 1976 South California earthquake. The dam cracking occurring during the San Fernando earthquake has caused a lengthening of the vibration period of about 0.025 s, which is shown in Fig. 3 by the projection of the line **b-5** on the vertical axis for the mode shape marked as T5. More information can be obtained from Ref. [8].

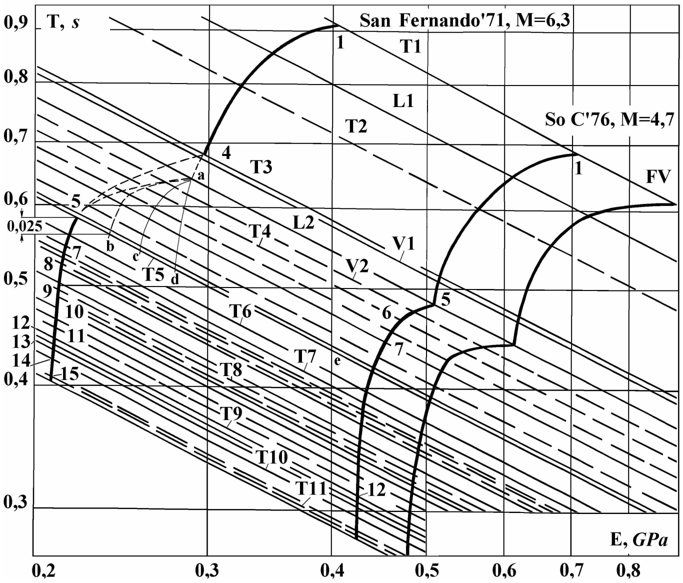

Fig. 3. Resonant travel time curves for seismic response of the Santa Felicia dam and their comparison with curve of forced vibration test

The obtained results provide the unique possibility to compare the response of the Santa Felicia Dam during five different types of excitation. The damping ζ_1 and the period of the first transversal symmetric mode shape $T_{1,D}$ are known from the tests, undamped period for the same mode $T_{1,U}$ is obtained by the next well known classical expression:

$$T_{1,U} = T_{1,D}\sqrt{1-\zeta_1^2}. \tag{5.1}$$

Later on, the frequency dependent damping factors ζ_i are determined, using the numerically defined periods of the undamped vibrations $T_{i,U}$ and the experimentally identified and predicted by the MIP periods of damped vibrations $T_{i,D}$, $i = 2, \dots, n$, where n is the number of the mode shapes under consideration. The calculated damping factors and the corresponding periods (according to the selected San Fernando earthquake) are given in columns two and three in Table 1. The fourth column contains the values of the equivalent elasticity modulus, used for numerical determination of damped vibration periods. Column five in Table 1 represents the shear modulus values, calculated as:

$$G = \frac{E}{2(1+\nu)}. \tag{5.2}$$

The seismic shear wave velocities V_S are calculated as:

$$V_S = \sqrt{G/\rho} \tag{5.3}$$

where ρ is the material density.

The values of the calculated wave numbers (Ω, rad/m) are written in the last column of Table 1.

Table 1. Frequency-dependent parameters of Santa Felicia earth fill dam during the 1971 San Fernando earthquake obtained by MIP

No	T_D, s	T_U, s	ξ_{tot}	E_D, GPa	G, GPa	V_S, m/s	Ω, rad/m
1	0.9091	0.8996	0.14400	0.4025	0.1491	265.501	0.0260
2	0.8520	0.7905	0.37315	0.3480	0.1289	246.873	0.0299
3	0.7948	0.7079	0.45478	0.3230	0.1196	237.840	0.0332
4	0.6849	0.5782	0.53608	0.2950	0.1093	227.297	0.0404
5	0.6760	0.5687	0.54071	0.2930	0.1085	226.526	0.0410
6	0.6400	0.5327	0.55419	0.2870	0.1063	224.194	0.0438
7	0.6350	0.5151	0.58481	0.2680	0.0993	216.646	0.0457
8	0.6300	0.4966	0.61525	0.2520	0.0933	210.080	0.0475
9	0.6250	0.4788	0.64274	0.2420	0.0896	205.869	0.0488
10	0.6170	0.4644	0.65832	0.2350	0.0870	202.870	0.0502
11	0.6030	0.4438	0.67696	0.2270	0.0841	199.387	0.0523
12	0.5930	0.4310	0.68691	0.2230	0.0826	197.622	0.0536
13	0.5880	0.4265	0.68840	0.2220	0.0822	197.179	0.0542
14	0.5624	0.4058	0.69229	0.2160	0.0800	194.496	0.0574
15	0.5605	0.4043	0.69261	0.2158	0.0799	194.406	0.0577
16	0.5478	0.3942	0.69442	0.2127	0.0788	193.004	0.0594
17	0.5384	0.3866	0.69592	0.2120	0.0785	192.687	0.0606
18	0.5348	0.3839	0.69625	0.2110	0.0781	192.232	0.0611
19	0.5260	0.3773	0.69680	0.2100	0.0778	191.775	0.0623
20	0.5210	0.3737	0.69684	0.2095	0.0776	191.547	0.0630

6. Wave number versus dimensionless frequency

Many authors represent the model properties by the dimensionless frequency:

$$\alpha_0 = r\omega/V_S, \tag{6.1}$$

where ω is circular frequency, V_S – shear wave velocity, and r – parameter (radius or other characteristic linear dimension of the structures). Reference [9] has shown a dimensionless frequency factor. This is a coefficient that at first sight seems very complex but in practice it is an interesting representation of expression (6.1). Some authors have disclosed that various problems that have arisen during the usage of expression (6.1). For example, the authors of paper [10] have concluded that the coefficient α_0 does not correlate well with the stiffness degradation. In article [11] a significant modification of α_0 was proposed. This correction can be considered as an indirect confirmation of the problems relevant to the determination of the dimensionless frequency α_0 and its correlation with other factors.

One of the reasons during the practical application of (6.1) is caused by the inappropriate choice of the parameter r. This parameter is frequency dependent, as well as all remaining parameters including stiffness, mass, etc. Another reason is the absence of some frequencies and mode shapes in the test results. Therefore, the frequency of the fundamental mode shape ω is used more frequently. The third reason is related to the inertness. There is an assumption that the dynamic interaction, manifested as wave radiation from the dam body towards the dam foundation, originated with the first mode shape of vibration. This assumption is still applied doubtlessly, despite some observed contradictory results [12], which have shown that this interesting phenomenon originates with the excitation of high mode shapes.

The variables in this equation must be determined by an adequate manner as frequency dependent parameters because of the above-quoted reasons, related to problematic application and calculation of α_0 (6.1). More information on these problems is given in References [3, 13].

The main purpose for usage of the dimensionless frequency is to achieve an approximation of relationship between the earth foundation characteristics – shear modulus and density. The same problem is relevant to the dam body material characteristics. There is certain information for earth foundation characteristics, but data about the dam body are very scarce. Very small number of studies is dedicated to shear wave velocities measurement, among others Miho Dam [14] and Fukada Dam [15]. Unfortunately, the results obtained are not bound by the frequencies of vibration, as well as by the wave numbers, respectively, so they are unusable.

The wave number Ω participates in equation (6.1) as:

$$\Omega = \omega / V_S. \tag{6.2}$$

The wave numbers for all mode shapes of interest can be easily determined by equation (6.2) using the shear wave velocity V_S obtained by expression (5.3) for corresponding circular frequency ($\omega = 2\pi/T$). The wave number values for Santa Felicia earth fill dam during the 1971 San Fernando earthquake, calculated as per the above mentioned manner, are shown in the last column of Table 1.

Excellent works dedicated to wave number determination have been published recently [16]. They concern the theoretical and experimental determination of shear wave velocities and wave numbers in a medium-rise building. At present, according their authors, no two- or three- dimensional theories exist that can interpret and quantify changes in wave numbers in a structure. They have used the elementary low frequency approximation for computing wave numbers within the building. The results obtained showed that for moderate and small ground motion amplitudes the waves propagate through the structure in essentially a linear manner, in spite of some little differences in lower bounds of wave numbers during ambient vibration tests and a good accordance during forced vibration test. The relationship between frequencies and wave numbers was obtained and it showed a linear trend. However, these researchers have noticed nonlinear effects of foundation concerning higher frequencies of vibration during recorded earthquake excitations. The authors of these advanced investigations conclude that the relative changes in the observed wave numbers may become useful tool for locating and interpreting the effects of damage, as well as for the structural health monitoring. The method proposed in [16] could be used for the measurement of the dam wave parameters.

7. Application of the wave number results

The results for the calculated damping factors for the five excitation cases are given in Table 2. The calculated results for the wave numbers and their corresponding average values (Ω_{av}) are presented in Table 3.

Figure 4 shows frequencies of dam vibration versus wave-numbers. Figure 5 shows the total damping versus wave numbers. In Figs 4 and 5, the graphs “SF E’71” give results recorded during 1971 San Fernando earthquake ($M_L = 6.3$), and the graphs “S-C E’76” represent results recorded during 1976

Table 2. Damping factors for Santa Felicia earthfill dam

No	SF E'71	S-C E'76	AVT'78	FVT'78	HDT'78
1	0.14400	0.03106	0.07001	0.09200	0.05601
2	0.37315	0.37907	0.45345	0.43664	0.33996
3	0.45478	0.44778	0.51427	0.49889	0.43912
4	0.53608	0.51967	0.56643	0.55680	0.50155
5	0.54071	0.52378	0.56886	0.55894	0.50497
6	0.55419	0.53754	0.57947	0.56735	0.51528
7	0.58481	0.56432	0.58598	0.60360	0.51707
8	0.61525	0.58292	0.59114	0.63052	0.52200
9	0.64274	0.59799	0.61006	0.64339	0.54379
10	0.65832	0.60630	0.62070	0.64937	0.55797
11	0.67696	0.61429	0.62895	0.65692	0.56808
12	0.68691	0.61809	0.63241	0.66043	0.57268
13	0.68840	0.61959	0.63293	0.66272	0.57424
14	0.69229	0.62596	0.63844	0.66893	0.58047
15	0.69261	0.62650	0.63945	0.66987	0.58080
16	0.69442	0.62886	0.64035	0.67156	0.58251
17	0.69592	0.62976	0.64210	0.67253	0.58326
18	0.69625	0.62993	0.64290	0.67289	0.58366
19	0.69680	0.63023	0.64291	0.67390	0.58434
20	0.69684	0.63035	0.64329	0.67465	0.58441

South California earthquake ($M_L = 4.7$). The results obtained by full scale tests are marked as follows: "AVT'78" for ambient vibration test, "FVT'78" for forced vibration test, and "HDT'78" for hydrodynamic test.

Figure 4 explicitly visualizes that for the lower mode shapes, i.e. for wave number values from 0.0261 to 0.0438 (for the two earthquakes and FVT), and to 0.0472 (for AVT and HDT), the relationships are almost linear, excluding the first mode shapes of full scale tests. The linear trend could be seen also for high mode shapes, i.e. for wave number values from 0.0472 (from 0.05 for SF E'71) to 0.0626, excluding the San Fernando earthquake when the cracks at the left dam abutment were formed. In the medium area for all curves it can be seen the moment of dam-foundation interaction originating and its influence on the period of dam vibrations. The results agree well with those ones obtained in [12].

The practical benefit from the results expressed in Fig. 4 is that it visualizes the dependency between the periods of first mode shape and/or the earthquake magnitude correlated to the wave number. The curves clearly show that for periods of 0.7–0.9 s, relevant to earthquakes with magnitude between 4.7 – 6.3 Richter scale, similar curves could be built successfully. Conclusively, one could say that occurrence of similar seismic behaviour in this range is expected.

Table 3. Wave-number values and average wave number

No	SF E'71	S-C E'76	AVT'78	FVT'78	HDT'78	Ω_{av}, rad/m
1	0.0260	0.0258	0.0265	0.0264	0.0260	0.0261
2	0.0299	0.0296	0.0296	0.0295	0.0291	0.0295
3	0.0332	0.0333	0.0332	0.0330	0.0333	0.0332
4	0.0404	0.0404	0.0406	0.0402	0.0405	0.0404
5	0.0410	0.0410	0.0413	0.0409	0.0412	0.0411
6	0.0438	0.0437	0.0440	0.0435	0.0439	0.0438
7	0.0457	0.0453	0.0454	0.0456	0.0454	0.0455
8	0.0475	0.0471	0.0471	0.0473	0.0472	0.0472
9	0.0488	0.0486	0.0486	0.0489	0.0488	0.0488
10	0.0502	0.0501	0.0501	0.0504	0.0502	0.0502
11	0.0523	0.0525	0.0525	0.0526	0.0526	0.0525
12	0.0536	0.0541	0.0540	0.0542	0.0542	0.0540
13	0.0542	0.0547	0.0546	0.0547	0.0548	0.0546
14	0.0574	0.0575	0.0573	0.0574	0.0576	0.0574
15	0.0577	0.0577	0.0575	0.0575	0.0578	0.0577
16	0.0594	0.0591	0.0590	0.0590	0.0593	0.0592
17	0.0606	0.0603	0.0602	0.0603	0.0605	0.0604
18	0.0611	0.0608	0.0606	0.0607	0.0610	0.0608
19	0.0623	0.0619	0.0617	0.0618	0.0621	0.0619
20	0.0630	0.0625	0.0623	0.0624	0.0627	0.0626

Figure 5 shows the relationships between coefficients of total damping and wave numbers for all 20 mode shapes, included in this investigation. Figure 5 gives reasons for the following conclusions to be made. The damping during HDT is lowest while for the low mode shapes the damping during AVT is highest. It is interesting to note that damping as well as dynamic behaviour of dam in these two cases is almost similar. The damping during FVT is higher for high mode shapes, but the damping during 1971 San Fernando earthquake after cracking (for $\Omega > 0.05$) is the highest one.

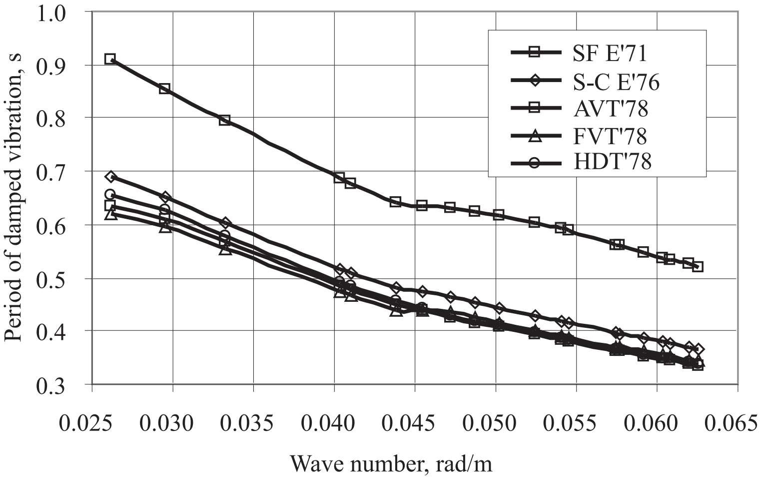

Fig. 4. Periods of damped vibration versus wave numbers

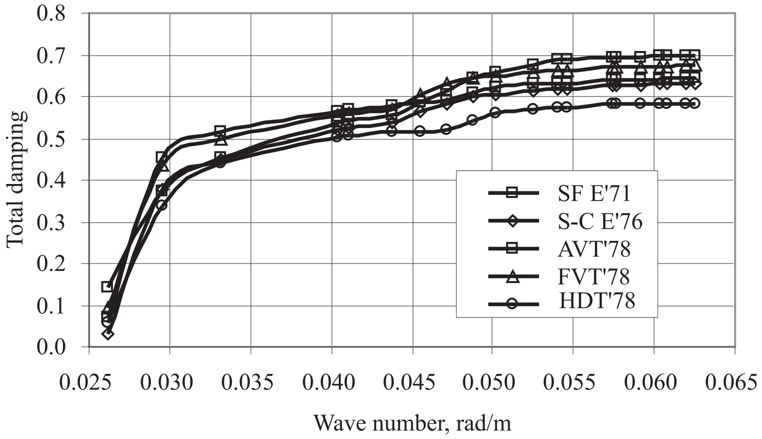

Fig. 5. Total damping versus wave number

Figure 5 shows for the first time a comparison of the dam total damping values obtained at different time history and type of excitations. As seen from Fig. 5, the change in the damping ratio shows that the dissipative capacity of the system under investigation does not change significantly even in case of a strong earthquake, while the increasing of the vibration periods during the strong earthquake is significant, as shown in Fig. 4.

The average wave numbers, regarded as global characteristics of dynamic/seismic behaviour of the investigated dam, facilitate the better comparison between the damping factors determined for different tests (Fig. 5), as well as the corresponding vibration periods (Fig. 4).

Figures 4 and 5 show dam essential parameters of dynamic/seismic behaviour. The increase in load intensity or local Richter magnitude (M_L) causes increase in vibration periods and the dissipative capacity of the system.

8. Conclusions

- The simplified engineering approach, presented in this paper, could be considered as a successful step towards deeper understanding of the wave equation parameters, especially their values and range of variation.
- The Method of Identification and Prognostication of dynamic characteristics of dams gives possibilities to obtain two important wave parameters, relevant to dynamic dam behaviour: wave numbers and damping factors, as well as the manner of their comparison.
- For the first time, the results obtained by the MIP for the period of vibrations and the damping coefficients, are presented as a function of averaged wave numbers, regarded as global characteristics of dynamic/seismic behaviour of the investigated dam.
- The results obtained are, in general, in accordance with the ones, obtained by other authors but they are visualized and represented in much more comprehensive and helpful manner.
- Major practical conclusion, regarding the future full scale tests of dams, is the requirement from simultaneous measurement of the shear wave velocities and the resonant frequencies. These measurements could be facilitated and focused thanks to the proposed method for determination of the range and values of the wave numbers variations. The approach can be applied for deepening of the theoretical knowledge and gaining of practical usefulness regarding estimation of real dynamic/seismic dam behaviour.

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