Journal of Theoretical and Applied Mechanics, Sofia, 2011, vol. 41, No. 1, pp. 51–60

FLUID MECHANICS

NUMERICAL STUDY OF THE ELECTRO-THERMO-CONVECTIVE FLOW PATTERNS IN DIELECTRIC LIQUID LAYER SUBJECTED TO UNIPOLAR INJECTION

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[Recieved 05 October 2009. Accepted 28 February 2011]

Abstract. In this article we study the electro thermal convection in a dielectric liquid layer placed between two electrodes subjected to the simultaneous action of an electric field and a thermal gradient. The full set of equations describing the electro-thermo-convective phenomena is directly solved using a finite volume method. The electro convective motion development is analysed in detail in four configurations: 1) strong injection from the upper or lower electrode; 2) when the electric potential and heating are applied simultaneously in the same time and in different time. We also study the heat transfer enhancement due to electro-convection. The evolution of the Nusselt number in terms of various non dimensional electrical parameters is presented and analyzed.
Key words: electro-thermo-convection, heat transfer, dielectric liquid, charge injection.

1. Introduction

The combined effects of an electric field and a thermal gradient simultaneously applied to a horizontal dielectric liquid layer leads to very complex physical flow interactions. It has been shown experimentally [1, 2] that the heat transfer across an insulating liquid can be increased by an order of magnitude. This augmentation of heat transfer is due to the development of secondary motions that result from two principal body forces: Coulomb forces acting on any free charge present in the liquid and dielectric forces. We consider here the dielectric liquid of very low conductivity when the free space charge comes from ion injection created at one electrode by electrochemical reactions. The charge induced by the injection process is dominant often by more than one order of magnitude compared to that induced by the volume electrical conduction. The convection induced by charge injection in a dielectric liquid in EHD is a problem as fundamental as the one of Rayleigh-Benard in non isothermal fluid mechanics. The action of the electric field on the space charge density arising from a unipolar injection has the same destabilizing role than the thermal field when the fluid is heated from below in Rayleigh-Benard problem [3]. However, both convections are not identical from a physical point of view. The mechanisms at the origin of the motion of the fluid are quite different in both cases: in Rayleigh-Benard convection the heat transfer is governed by thermal diffusion whereas the ion migration is the relevant mechanism in the electric charge transfer in electro-convection. The coupling between the conservation equations of momentum, electric charge and energy is ensured via the Coulomb and the buoyancy forces, when the space charge only results from ion injection. This coupling results from the direct interaction between the velocity, temperature and charge perturbations and from the non direct interaction between the velocity and the charge. Most of the authors who have been working so far on electro or electro-thermo convective problems in horizontal planar layers of dielectric liquids mainly undertake the stability analysis [4–7]. A few numerical simulations only have been attempted on pure EHD convection problem [8–10].

We solve for the first time in the present work the electro-hydro-dynamic problem coupled with energy and Navier-Stokes equations in a 2D cavity. We developed a direct numerical simulation based on a finite volume method. We focus here on the convective mechanisms responsible for fluid motion when an ion injection from the lower or upper electrode into an insulating liquid is applied. The spatio-temporel evolution of the electro-thermo convective flow in the dielectric liquid layer is analyzed in detail. The influence of the induce electro-convection on the heat transfer is studied by the time evolution of the Nusselt number.

2. Statement of the problem

2.1. Basic governing equations

We consider a dielectric liquid layer of thickness H enclosed between two electrodes of length L_x. The layer is subjected to a potential difference $\Delta V = V_0 - V_1$, thermal gradient $\Delta\theta = \theta_1 - \theta_0$ and the charge injection from lower electrode, presented in Fig. 1.

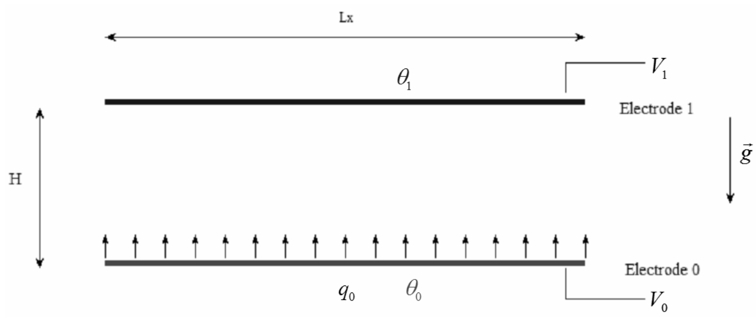

Fig. 1

The general set of equations for an incompressible fluid expressing the conservation of mass and momentum (the Navier-Stokes equations) including electrical and buoyancy effects, energy balance equation under the Boussinesq assumption, charge density conservation, the Gauss theorem and the definition of the electric field towards electric potential V, takes the form:

$$\nabla.\vec{u} = 0, \tag{2.1}$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} = -\nabla\tilde{p} + \Delta\vec{u} + RTCq\vec{E} + \frac{Ra}{Pr}\theta\,\vec{e}_z, \tag{2.2}$$

$$\frac{\partial \theta}{\partial t} + \vec{u}.\nabla\theta = \frac{1}{Pr}\Delta\theta, \tag{2.3}$$

$$\frac{\partial q}{\partial t} + \nabla.\left(q(\vec{u} + R\vec{E})\right) = 0, \tag{2.4}$$

$$\Delta V = -Cq, \quad \vec{E} = -\nabla V, \tag{2.5}$$

where $\vec{u}$ is the velocity, p the static pressure, q the electric charge density in the liquid, $\vec{E}$ the electric field, ε the permittivity of the fluid, θ the absolute temperature and V the electric potential, $q\vec{E}$ is the Coulomb force. We introduce the following non-dimensional scales: H for length, the applied voltage

$\Delta V = V_0 - V_1$ for electric potential, $\Delta V/H$ for electric field, $\varepsilon_0 V/H^2$ for charge density, $\varepsilon_0 K \Delta V^2/H^3$ for current density, υ/H for velocity (υ is the kinematic viscosity of the liquid), $\rho_0 \upsilon^2/H^2$ for pressure, θ for temperature and H^2/υ for time. The following dimensionless quantities appear: $Ra = g\beta\Delta\theta\, H^3/\nu\,\kappa$, $T = \varepsilon\Delta V/\rho\nu K$, $C = q_0 H^2/\varepsilon\Delta V$, $M^2 = \varepsilon/\rho K^2$, $\mathrm{Pr} = \nu/\kappa$, $R = T/M^2$, where Ra is the Rayleigh number with g the gravity and κ the thermal diffusivity, T is the instability parameter, C is a measure of the injection level with q_0 the injected charge,M is the mobility parameter, Pr is the Prandtl number, R is electrical Reynolds number. We consider here the case of strong injection so that the dielectric force $-1/2E^2\nabla\varepsilon$ is much lower than the Coulomb force and could be neglected [10, 11].

2.2. Initial and boundary conditions

The boundary conditions in the case of heating and injection from lower electrode are presented in Fig. 2. At the lateral walls: here we consider the rigid walls with no-slip boundary conditions.

We consider the initial conditions for the case 2 and case 3, respectively:

$$(2.6)\quad u(x,t=0)=0,\quad (x,t=0)=0,\quad p(x,t=0)=0,$$
$$V(x,t=0)=1,\quad q(x,t=0)=1,\quad \theta(x,t=0)=1,$$

$$(2.7)\quad u(x,t=0)=0,\quad (x,t=0)=0,\quad p(x,t=0)=0,$$
$$V(x,t=0)=0,\quad q(x,t=0)=0,\quad \theta(x,t=0)=1.$$

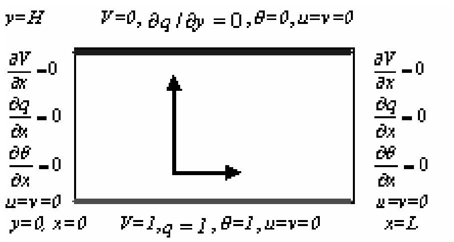

Fig. 2. Boundary conditions

3. Numerical method

The equations are integrated using the finite volume method [12]. The numerical procedure is based on the Augmented Lagrangian method [13], which transforms the Navier-Stokes equations into a problem of minimization. Here, the primitive variables $\vec{U}$ and P are determined by the algorithm of Uzawa [14]. These equations are integrated over finite volume using a staggered grid and a semi-implicit second order in time and space accurate schemes. The linear systems are solved using the Bi-CGSTAB [15] method with a preconditioning based on a modified and incomplete Gauss factorization MILU [16]. The calculation is fully transient and to skip from instant t_n to $t_{n+1} = t_n + \Delta t$ we define according the Augmented Lagrangian method and the Uzawa algorithm, an iterative procedure. Special care must be provided while solving (2.4) because of its hyperbolic nature. The SMART algorithm [17] has been employed in that way.

4. Results and discussion

4.1. Flow structure

We consider tree general cases. Two possible situations of charge injection (from lower and from upper electrode) are also considered for case 2 and case 3.

Case 1: It corresponds to pure thermo-convection (without electric field and any injection of electric charges). It is our reference case.

Case 2: The thermal and electrical fields act simultaneously in the beginning of the simulation ($t = 0$). In the both cases (injection from above and below), for $T < 320$ the flow patterns in steady state regime consist of 2 cells while for $T > 320$ the flow patterns changes into a 4 cell.

Case 3: The thermal field acts first (at $t = 0$), the electric one later (at $t = 5$). We represent the stream function at different time in Fig. 3 till the steady state regime is reached for the cases: case 1 (without injection) and case 3 for injection from bellow and from above. The system reaches a stationary state very fast when the injection is from below by time $t = 5.6$, for not so high values of T ($T = 200$) and the injection does not modify the initial flow structure of 2 cells. We obtain 4 cells for $T \geq 320$. The flow patterns fundamentally differ for injection from above for the value of T ($T > 170$). There are two thermo-convective cells at the beginning of the simulation. The secondary structures appear and grow in the area between instants $t = 5.2$ and $t = 7.0$ when injection begins. They lead finally to another stationary state in time $t = 7.2$ with four cells as it is in the pure electro-convection (without heating). The effect of the electrical field on flow is very strong and we obtain the establishment of 4 cells patterns in the stationary state (see Fig. 3). In that case the electro-convection dominates over the thermo-convection.

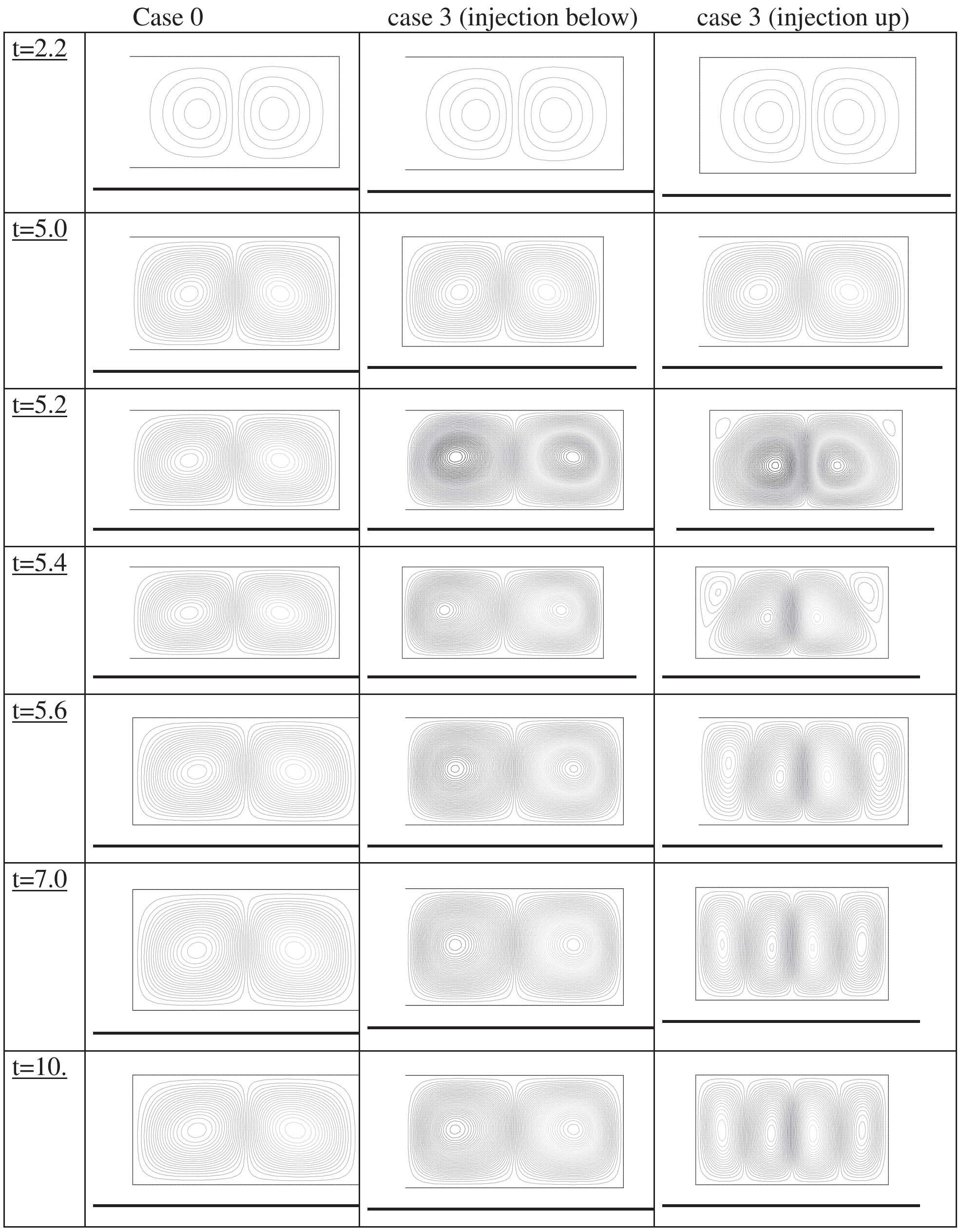

Fig. 3. The stream function evolution in case 1 (pure thermo-convection) and case 3 (injection from bellow and from above) for $Ra = 10000$, $T = 200$, $M = 10$, $Pr = 10$

4.2. Heat transfer enhancement

The authors demonstrated in the experimental works [1, 2] that the electro-convection induced by unipolar injection increases the heat transfer. We compute the mean Nusselt number $\overline{Nu} = \int_0^L \left(\frac{\partial T}{\partial y}\right)_{y=0} dx$ in a cavity of length $L_x = 2$, at the lower plate. Here, we consider also the cases of pure thermo-convection (without electric field), injection from bellow and injection from above. The time evolution of the Nusselt number for $C = 10$, $Ra = 10000$ and $T = 200$ is presented in Fig. 4. The values of Ra and T are above the respective critical ones ($Ra_c = 1720$, $T_c = 161$). We see that in the both cases with injection the value of the Nusselt number increases in comparison with case 1, so the heat transfer increases. This amount is at about 30% for injection from below and 37.5% for injection from above. This increase of Nusselt is linked to the dynamic mixture due to turbulent motion in flow. The mixing is increased as it is seen in Fig. 3 where the stream function lines are thicker and it is stronger for injection from above when the electric and gravity forces act in the same direction.

Figure 5 shows the mean value of the Nusselt number for different Ra

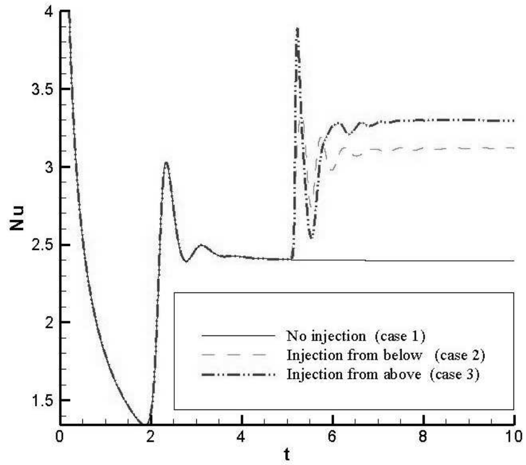

Fig. 4. Time evolution of Nusselt number in cases of pure thermo-convection, injection from below and from above for $C = 10$, $Ra = 10000$, $Pr = 10$, $T = 200$, $M = 10$

and T for extended length of the cavity ($L_x = 10$). One can clearly see that above $T = 250$ the electrical forces dominate and for $T = 500$ Nu does not depend on Ra. In the experi-mental works [1, 2] Nu is plotted as a function of Ra, for different values of the applied voltage. Above a voltage of 6 kV the Nusselt number does not depend on the Rayleigh number. Both approaches give the same tendency: the electrical forces increase the heat transfer and from a certain value of the electrical parameter T they completely dominate the heat transfer, which does not depend on Ra any longer.

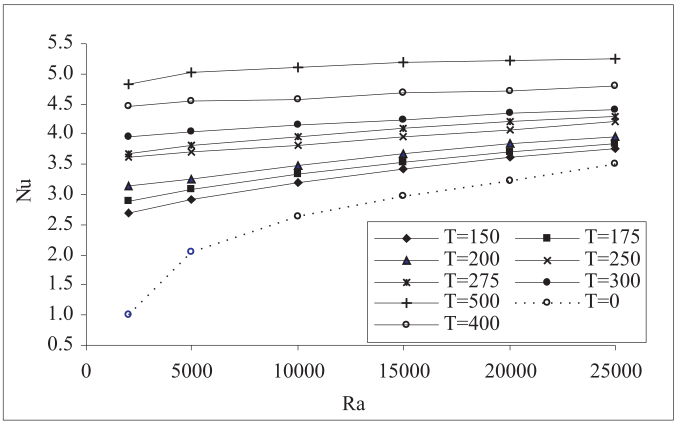

Fig. 5. Nusselt number with Ra for different values of T

5. Conclusions

The effect of the charge injection on the heat transfer in a dielectric liquid layer is presented by a direct numerical simulation. We obtained that the unipolar injection change radically the thermo-convective flow patterns. Their number depends on the ratio between the two instability criterions: the Rayleigh number Ra and the electrical parameter T as well as on the characteristic parameters Pr and M number. The injection from below and injection from above gives a four roll structure when the electric effects are dominant (pure electro-convection), whereas when the buoyancy effect is dominant (pure thermo-convection) it gives rise to a two roll patterns. The electro-convection significantly increases the turbulence and therefore the heat transfer. The Nusselt number becomes almost independent on the Rayleigh number for high sufficient values of T. Our results are in a good qualitative agreement with the available experimental data.

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