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PROPAGATION OF WAVES AT AN INTERFACE BETWEEN TWO THERMOELASTIC HALF-SPACES WITH DIFFUSION*

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Abstract. Theory of generalized thermo-elastic diffusion is applied to study the reflection and the transmission of P and SV waves at an interface of two dissimilar thermo-elastic solids with diffusion. The boundary conditions at the interface are satisfied by the suitable potentials to obtain a system of eight non-homogeneous equations in amplitude ratios of various reflected and transmitted waves. These amplitude ratios depend on the angle of incidence of P and SV waves, thermo-diffusion parameters and other material constants. The numerical values of these reflection and transmission coefficients are shown graphically against the angle of incidence for a particular model. The graphical results are compared with those without diffusion.
Key words: generalized thermo-elasticity, reflection and transmission, diffusion, amplitude ratios.

1. Introduction

Non-classical theories known as generalized thermo-elasticity were introduced into the literature in an attempt to eliminate the shortcomings of the classical dynamical thermo-elasticity [1]. Lord and Shulman [2], incorporated a flux rate term into Fourier's law of heat conduction and formulated a

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generalized theory involving a hyperbolic heat transport equation, admitting finite speed for thermal signals. Green and Lindsay [3], developed a temperature rate dependent thermoelasticity by including temperature rate among the constitutive variables, that does not violate the classical Fourier's law of heat conduction, when the body under consideration has a centre of symmetry and this theory also predicts a finite speed of heat propagation. Chandrasekharaiah [4], referred to this wave like thermal disturbance as 'second sound'. A survey article of representative theories in the range of generalized thermoelasticity is due to Hetnarski and Ignaczak [5]. Sinha and Sinha [6] and Sinha and Elsibai [7, 8], were the first to study the reflection and transmission phenomenon in generalized thermo-elasticity.

Diffusion is defined as the random walk of an ensemble of particles, from regions of high concentration to regions of lower concentration. Thermo-diffusion in an elastic solid is due to the field of temperature, mass diffusion and that of strain. Nowacki [9–10] studied dynamical problem on diffusion in solids using the coupled thermo-elastic model. Dudziak and Kowalski [11] discussed the theory of thermo-diffusion for solid. Olesiak and Pyryev [12] studied a coupled quasi-stationary problem of thermo-diffusion for elastic cylinder. They discussed the influences of cross effects arising of the coupling of the fields of temperature, mass diffusion and strain. Due to these cross effects the thermal excitation results in an additional mass concentration, which generates the additional temperature field. Sherief et al. [13] developed a theory of generalized thermo-elastic diffusion following Lord and Shulman [2] theory, which also allows the finite speeds of propagation of waves. Singh [14, 15] studied the wave propagation in thermo-elastic solid with diffusion in context of both Lord-Shulman and Green-Lindsay theories. Abo-Dahab and Singh [16] studied the effects of magnetic field on wave propagation in a thermo-elastic solid with diffusion. Various other problems on the theory of generalized thermo-elastic diffusion were also studied during last few years [17–22].

The diffusion phenomenon is of great concern due to its many geophysical and industrial applications. The concept of thermo-diffusion is helpful to oil companies for more efficient extraction of oil from oil deposits. The problems on reflection and refraction of elastic waves at a plane separating two media have wide applications in seismic-reflection surveys. In the present paper, the problem of reflection and transmission at an interface between two dissimilar thermoelastic half-spaces with diffusion is considered. The effects of diffusion are shown graphically on various reflected and transmitted waves for the incidence of both P and SV waves.

2. Governing equations

The governing equations for an isotropic, homogenous elastic solid with generalized thermo-diffusion at constant reference temperature T_0 in the absence of body forces are [13]:

(i) Equation of motion

$$\mu \boldsymbol{u}_{i,jj} + (\lambda + \mu)\boldsymbol{u}_{j,ij} - \beta_1 \Theta_{,i} - \beta_2 C_{,i} = \rho \ddot{\boldsymbol{u}}_i. \tag{1}$$

(ii) Heat conduction equation

$$\rho C_E(\dot{\Theta} + \tau_0 \ddot{\Theta} + \beta_1 T_0(\dot{e} + \tau_0 \ddot{e}) + a T_0(\dot{C} + \tau_0 \ddot{C}) = K \Theta_{,ii}. \tag{2}$$

(iii) Mass diffusion equation

$$D\beta_2 e_{,ii} + D a \Theta_{,ii} + \dot{C} + \tau \ddot{C} - D b C_{,ii} = 0. \tag{3}$$

(iv) Constitutive equations

$$\begin{aligned} \sigma_{ij} &= 2\mu e_{ij} + \delta_{ij}(\lambda e_{kk} - \beta_1 \Theta - \beta_2 C), \\ \rho T_0 S &= \rho C_E \Theta + \beta_1 T_0 e_{kk} + a T_0 C, \\ P &= -\beta_2 e_{kk} + bC - a\Theta, \end{aligned} \tag{4}$$

where, $\beta_1 = (3\lambda + 2\mu)\alpha_t$, $\beta_2 = (3\lambda + 2\mu)\alpha_c$, α_t is the coefficient of linear thermal expansion, α_c is the coefficient of linear diffusion expansion, λ, μ are Lame's coefficients, ρ is the density of the medium, T is absolute temperature, T_0 is temperature of the medium in its natural state, $\Theta = T - T_0$ is the change in temperature such that $|\Theta/\mathrm{T}_0| \ll 1$, σ_{ij} are components of the stress tensor, e_{ij} are components of the strain tensor and $e = e_{ii}$, u_i are components of displacement vector, ρ is density of the medium, S is entropy per unit mass, P is chemical potential per unit mass, C is mass concentration, C_E is specific heat at constant strain, K is coefficient of thermal conductivity, D is thermo-diffusion constant, τ_0 is thermal relaxation time, τ is diffusion relaxation time, a is a constant to measure the thermo-diffusion effects, b is a constant to measure the diffusive effects, δ_{ij} is Kronecker delta. The superposed dots denote the time derivatives with respect to time. The comma notation is used to denote the partial derivatives in the above equations.

With the help of following Helmholtz representation of displacement components u_1 and u_3 in terms of potentials ϕ and ψ

$$u_1 = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z}, \quad u_3 = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x}, \tag{5}$$

the equations (1)–(3) in $x-z$ plane are written as

$$\mu\nabla^2\psi = \rho\ddot{\psi}, \tag{6}$$

$$(\lambda + 2\mu)\nabla^2\phi - \beta_1\Theta - \beta_2 C = \rho\ddot{\phi}, \tag{7}$$

$$K(\Theta_{,11} + K\Theta_{,33}) = \rho C_E\tau_m\dot{\Theta} + \beta_1 T_0\tau_m\frac{\partial}{\partial t}\nabla^2\phi + aT_0\tau_m\dot{C}, \tag{8}$$

$$D\beta_2\nabla^4\phi + Da(\Theta_{,11} + \Theta_{,33}) - Db(C_{,11} + C_{,33}) + \tau_n\dot{C} = 0, \tag{9}$$

where, $\tau_m = 1 + \tau_0\frac{\partial}{\partial t}$ and $\tau_n = 1 + \tau\frac{\partial}{\partial t}$. The equation (6) is uncoupled and the equations (7)–(9) are coupled in ϕ, Θ and C. The solution of equation (6) corresponds to the propagation of SV wave with phase speed $\sqrt{\mu/\rho}$. The solutions of the equations (7)–(9) are now sought in the form of the time-harmonic plane waves

$$\{\phi, \Theta, C\} = \{\phi_0, \Theta_0, C_0\}\exp[ik(x\sin\theta + z\cos\theta - vt)], \tag{10}$$

where ϕ_0, Θ_0 and C_0 are constants, k is the wave number, v is the phase speed and θ is the angle which the propagation vector makes with the normal to the interface. The equations (7)–(9) lead to a system of three homogenous equations in ϕ_0, Θ_0 and C_0, which has a non-trivial solution if the following cubic equation is satisfied,

$$\xi^3 + L\xi^2 + M\xi + N = 0, \tag{11}$$

here, $L = -(\varepsilon + \varepsilon\varepsilon_2\varepsilon_1^2 + d_1 + d_2 + \lambda + 2\mu)$, $M = (\lambda + 2\mu)(d_1 + d_2 + \varepsilon\varepsilon_2\varepsilon_1^2) + d_1d_2 + d_2\varepsilon - 2\varepsilon\varepsilon_1\varepsilon_2 - \varepsilon_2$, $N = -d_1d_2(\lambda + 2\mu) + \varepsilon_2 d_1$, $\xi = \rho c^2$, $d_1 = \frac{K}{C_E\tau_m^1}d_2 = \frac{\rho Db}{\tau_n^1}$, $\varepsilon = \frac{\beta_1^2 T_0}{\rho C_E}$, $\varepsilon_1 = -\frac{a}{\beta_1\beta_2}$, $\varepsilon_2 = \frac{\rho D\beta_2^2}{\tau_n^1}$, $\tau_m^1 = \tau_0 + \frac{\iota}{\omega}$, $\tau_n^1 = \tau + \frac{\iota}{\omega}$, $\omega = kv$, where ω is the frequency of the incident plane wave. The cubic equation (11) has three complex phase speeds c_1, c_2 and c_3. Let $c_j^{-1} = v_j^{-1} - i\omega^{-1}q_j$ $(j = 1, 2, 3)$, where v_j and q_j are phase speeds and attenuation coefficients. The phase speeds v_1, v_2 and v_3 $(v_1 > v_3 > v_2)$ correspond to P wave, mass diffusion (MD) wave and thermal (T) wave, respectively [14].

3. Reflection and transmission

We consider an interface between two thermo-elastic solid half-spaces with diffusion as shown in Fig. 1. The boundary conditions for incidence of P and SV waves at an interface are satisfied by the particular solutions in two half-spaces, if the incident P or SV wave gives rise to a reflected SV and three reflected coupled longitudinal waves (i.e., P, MD and T waves) in medium M_1 and four similar waves transmitted in the medium M_2. The quantities with primes correspond to medium M_2. The required boundary conditions at an interface between two half spaces for motion in $x-z$ plane are, the continuity of the normal force stress, tangential force stress, tangential displacement component, normal displacement component, heat flux, temperature, mass flux, chemical potential, i.e.:

$$\begin{aligned} &\sigma_{zz} = \sigma'_{zz}, \quad \sigma_{zx} = \sigma'_{zx}, \quad u_x = u'_x, \quad u_z = u'_z, \\ &q_z = q'_z, \quad \Theta = \Theta', \quad \eta_z = \eta'_z, \quad P = P', \end{aligned} \tag{12}$$

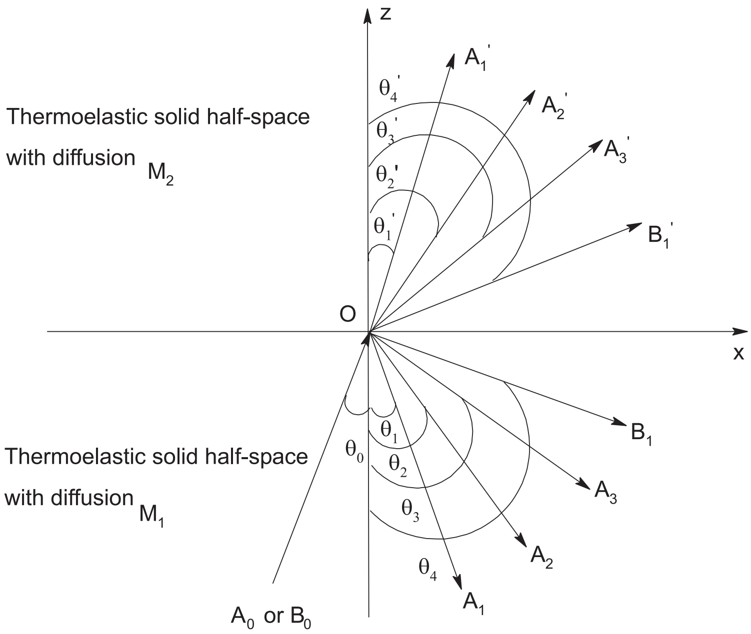

Fig. 1. Geometry of the model

where,

$$\sigma_{zz} = 2\mu\left[\frac{\partial^2\phi}{\partial z^2} + \frac{\partial^2\psi}{\partial x\partial z}\right] + \lambda\nabla^2\phi - \beta_1\Theta - \beta_2 C,$$

$$\sigma_{zx} = \mu\left[2\frac{\partial^2\phi}{\partial x\partial z} + \frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial z^2}\right].$$

$$q_z = -\frac{K}{1+\tau_0\frac{\partial}{\partial t}}\frac{\partial\Theta}{\partial z},$$

$$\eta_z = -\frac{D}{1+\tau\frac{\partial}{\partial t}}\frac{\partial P}{\partial z}.$$

Similar expressions with primes correspond to medium M_2.

The appropriate plane wave solutions incident and reflected waves in half-space M_1 $(z < 0)$

$$\begin{aligned}\phi = {} & A_0\exp[\iota k_1(x\sin\theta_0 + z\cos\theta_0 - v_1 t)] \\ & + A_1\exp[\iota k_1(x\sin\theta_1 - z\cos\theta_1 - v_1 t)] \\ & + A_2\exp[\iota k_2(x\sin\theta_2 - z\cos\theta_2 - v_2 t)] \\ & + A_3\exp[\iota k_3(x\sin\theta_3 - z\cos\theta_3 - v_3 t)],\end{aligned} \tag{13}$$

$$\begin{aligned}\Theta = {} & \varsigma_1 A_0\exp[\iota k_1(x\sin\theta_0 + z\cos\theta_0 - v_1 t)] \\ & + \varsigma_1 A_1\exp[\iota k_1(x\sin\theta_1 - z\cos\theta_1 - v_1 t)] \\ & + \varsigma_2 A_2\exp[\iota k_2(x\sin\theta_2 - z\cos\theta_2 - v_2 t)] \\ & + \varsigma_3 A_3\exp[\iota k_3(x\sin\theta_3 - z\cos\theta_3 - v_3 t)],\end{aligned} \tag{14}$$

$$\begin{aligned}C = {} & \eta_1 A_0\exp[\iota k_1(x\sin\theta_0 + z\cos\theta_0 - v_1 t)] \\ & + \eta_1 A_1\exp[\iota k_1(x\sin\theta_1 - z\cos\theta_1 - v_1 t)] \\ & + \eta_2 A_2\exp[\iota k_2(x\sin\theta_2 - z\cos\theta_2 - v_2 t)] \\ & + \eta_3 A_3\exp[\iota k_3(x\sin\theta_3 - z\cos\theta_3 - v_3 t)],\end{aligned} \tag{15}$$

$$\begin{aligned}\psi = {} & B_0\exp[\iota k_4(x\sin\theta_0 + z\cos\theta_0) - v_4 t)] \\ & + B_1\exp[\iota k_4(x\sin\theta_4 - z\cos\theta_4 - v_4 t)].\end{aligned} \tag{16}$$

The appropriate plane wave solutions for refracted waves in half-space M_2 $(z > 0)$

$$\begin{aligned}\phi' &= A_1' \exp[\iota' k_1(x\sin\theta_1' + z\cos\theta_1' - v_1' t)]\\ &\quad + A_2' \exp[\iota k_2'(x\sin\theta_2' + z\cos\theta_2' - v_2' t)]\\ &\quad + A_3' \exp[\iota k_3'(x\sin\theta_3' + z\cos\theta_3' - v_3' t)],\end{aligned}\tag{17}$$

$$\begin{aligned}\Theta' &= \varsigma_1' A_1' \exp[\iota k_1'(x\sin\theta_1' + z\cos\theta_1' - v_1' t)]\\ &\quad + \varsigma_2' A_2' \exp[\iota k_2'(x\sin\theta_2' + z\cos\theta_2' - v_2' t)]\\ &\quad + \varsigma_3' A_3' \exp[\iota k_3'(x\sin\theta_3' + z\cos\theta_3' - v_3' t)],\end{aligned}\tag{18}$$

$$\begin{aligned}C' &= \eta_1' A_1' \exp[\iota k_1'(x\sin\theta_1' + z\cos\theta_1' - v_1' t)]\\ &\quad + \eta_2' A_2' \exp[\iota k_2'(x\sin\theta_2' + z\cos\theta_2' - v_2' t)]\\ &\quad + \eta_3' A_3' \exp[\iota k_3'(x\sin\theta_3' + z\cos\theta_3' - v_3' t)],\end{aligned}\tag{19}$$

$$\psi' = B_1' \exp[\iota k_4'(x\sin\theta_4' + z\cos\theta_4' - v_4' t)],\tag{20}$$

here, the wave normal of the incident P or SV wave makes angle θ_0 with the negative direction of z-axis and reflected P, MD, T, and SV waves makes angles θ_1', θ_2', θ_3' and θ_4' with z-axis, and:

$$\varsigma_i = k_i^2 G_i(\rho v_i^2 - \lambda - 2\mu), \quad \eta_i = k_i^2 H_i(\rho v_i^2 - \lambda - 2\mu), \quad (i = 1, 2, 3)\tag{21}$$

where

$$G_i = \frac{\varepsilon\rho v_i^2(\varepsilon_1\varepsilon_2 - d_2 + \rho v_i^2)}{d_1\varepsilon_2 + \rho v_i^2[\varepsilon(d_2 - \rho v_i^2) - \varepsilon_2 - 2\varepsilon\varepsilon_1\varepsilon_2]},$$

$$H_i = \frac{\varepsilon_2[\rho v_i^2(\varepsilon\varepsilon_1 + 1) - d_1]}{d_1\varepsilon_2 + \rho v_i^2[\varepsilon(d_2 - \rho v_i^2) - \varepsilon_2 - 2\varepsilon\varepsilon_1\varepsilon_2]}.$$

The expressions with primes similar to as given in equation (21) correspond to the medium M_2. The ratios of amplitudes of the reflected and refracted wave to the amplitude of the incident P-wave namely $\frac{A_1}{A_0}$, $\frac{A_2}{A_0}$, $\frac{A_3}{A_0}$, $\frac{B_1}{A_0}$, $\frac{A_1'}{A_0}$, $\frac{A_2'}{A_0}$, $\frac{A_3'}{A_0}$, and $\frac{B_1'}{A_0}$ gives the amplitude ratios for reflected P, reflected MD, reflected T, reflected SV, refracted P, refracted MD, refracted T and refracted SV, respectively.

The potentials given by the equations (13)–(20) will satisfy the boundary conditions (12) at the interface $z = 0$ if the wave numbers k_1, k_2, k_3, k_4, k'_1, k'_2, k'_3, k'_4 and the angles are connected by the following relations:

$$k_1 \sin\theta_1 = k_2 \sin\theta_2 = k_3 \sin\theta_3 = k_4 \sin\theta_4 = k'_1 \sin\theta'_1 = k'_2 \sin\theta'_2 = k'_3 \sin\theta'_3 = k'_4 \sin\theta'_4, \tag{22}$$

$$k_1 v_1 = k_2 v_2 = k_3 v_3 = k_4 v_4 = k'_1 v'_1 = k'_2 v'_2 = k'_3 v'_3 = k'_4 v'_4. \tag{23}$$

We can obtain the modified Snell's law from relations (22) and (23) as

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} = \frac{\sin\theta_3}{v_3} = \frac{\sin\theta_4}{v_4} = \frac{\sin\theta'_1}{v'_1} = \frac{\sin\theta'_2}{v'_2} = \frac{\sin\theta'_3}{v'_3} = \frac{\sin\theta'_4}{v'_4}. \tag{24}$$

The use of potentials (13)–(20) in boundary conditions (12) with the help of above Snell's law, results into a system of eight non-homogenous equations in amplitude ratios of reflected and refracted waves as:

$$\sum a_{ij} Z_j = b_i \qquad (i, j = 1, 2, \ldots, 8), \tag{25}$$

here,

$$a_{11} = -\left[2\mu\left\{1 - \sin^2\theta_0\left(\frac{v_1}{v}\right)^2\right\} + \lambda + \beta_1 G_1(\rho v_1^2 - \lambda - 2\mu) + \beta_2 H_1(\rho v_1^2 - \lambda - 2\mu)\right]\left(\frac{v}{v_1}\right)^2,$$

$$a_{12} = -\left[2\mu\left\{1 - \sin^2\theta_0\left(\frac{v_2}{v}\right)^2\right\} + \lambda + \beta_1 G_2(\rho v_2^2 - \lambda - 2\mu) + \beta_2 H_2(\rho v_2^2 - \lambda - 2\mu)\right]\left(\frac{v}{v_2}\right)^2,$$

$$a_{13} = -\left[2\mu\left\{1 - \sin^2\theta_0\left(\frac{v_3}{v}\right)^2\right\} + \lambda + \beta_1 G_3(\rho v_3^2 - \lambda - 2\mu) + \beta_2 H_3(\rho v_3^2 - \lambda - 2\mu)\right]\left(\frac{v}{v_3}\right)^2,$$

$$a_{14} = 2\mu \sin\theta_0 . \left[\left\{ 1 - \sin^2\theta_0 \left(\frac{v_4}{v}\right)^2 \right\}^{1/2} \right] \left(\frac{v}{v_4}\right),$$

$$a_{15} = \left[2\mu' \left\{ 1 - \sin^2\theta_0 \left(\frac{v_1'}{v}\right)^2 \right\} + \lambda' + \beta_1' G_1' (\rho' v_1'^2 - \lambda' - 2\mu') + \beta_2' H_1' (\rho' v_1'^2 - \lambda' - 2\mu') \right] \left(\frac{v}{v_1'}\right)^2,$$

$$a_{16} = \left[2\mu' \left\{ 1 - \sin^2\theta_0 \left(\frac{v_2'}{v}\right)^2 \right\} + \lambda' + \beta_1' G_2' (\rho' v_2'^2 - \lambda' - 2\mu') + \beta_2' H_2' (\rho' v_2'^2 - \lambda' - 2\mu') \right] \left(\frac{v}{v_2'}\right)^2,$$

$$a_{17} = \left[2\mu' \left\{ 1 - \sin^2\theta_0 \left(\frac{v_3'}{v}\right)^2 \right\} + \lambda' + \beta_1' G_3' (\rho' v_3'^2 - \lambda' - 2\mu') + \beta_2' H_3' (\rho' v_3'^2 - \lambda' - 2\mu') \right] \left(\frac{v}{v_3'}\right)^2,$$

$$a_{18} = 2\mu' \sin\theta_0 . \left[\left\{ 1 - \sin^2\theta_0 \left(\frac{v_4'}{v}\right)^2 \right\}^{1/2} \right] \left(\frac{v}{v_4'}\right),$$

$$a_{21} = 2 \sin\theta_0 \left[\left\{ 1 - \sin^2\theta_0 . \left(\frac{v_1}{v}\right)^2 \right\}^{1/2} \right] \left(\frac{v}{v_1}\right),$$

$$a_{22} = 2 \sin\theta_0 \left[\left\{ 1 - \sin^2\theta_0 . \left(\frac{v_2}{v}\right)^2 \right\}^{1/2} \right] \left(\frac{v}{v_2}\right),$$

$$a_{23} = 2 \sin\theta_0 \left[\left\{ 1 - \sin^2\theta_0 . \left(\frac{v_3}{v}\right)^2 \right\}^{1/2} \right] \left(\frac{v}{v_3}\right),$$

$$a_{24} = \left[1 - 2\sin^2\theta_0.\left(\frac{v_4}{v}\right)^2\right]\left(\frac{v}{v_4}\right),$$

$$a_{25} = 2\left(\frac{\mu'}{\mu}\right)\sin\theta_0\left\{1 - \sin^2\theta_0\left(\frac{v_1'}{v}\right)^2\right\}^{1/2}\left(\frac{v}{v_1'}\right),$$

$$a_{26} = 2\left(\frac{\mu'}{\mu}\right)\sin\theta_0\left\{1 - \sin^2\theta_0\left(\frac{v_2'}{v}\right)^2\right\}^{1/2}\left(\frac{v}{v_2'}\right),$$

$$a_{27} = 2\left(\frac{\mu'}{\mu}\right)\sin\theta_0\left\{1 - \sin^2\theta_0\left(\frac{v_3'}{v}\right)^2\right\}^{1/2}\left(\frac{v}{v_3'}\right),$$

$$a_{28} = -\left(\frac{\mu'}{\mu}\right)\left\{1 - 2\sin^2\theta_0\left(\frac{v_4'}{v}\right)^2\right\}\left(\frac{v}{v_4'}\right)^2,$$

$$a_{31} = \sin\theta_0, \quad a_{32} = \sin\theta_0, \quad a_{33} = \sin\theta_0,$$

$$a_{34} = \left[\left\{1 - \sin^2\theta_0\left(\frac{v_4}{v}\right)^2\right\}^{1/2}\right]\left(\frac{v}{v_4}\right),$$

$$a_{35} = -\sin\theta_0, \quad a_{36} = -\sin\theta_0, \quad a_{37} = -\sin\theta_0,$$

$$a_{38} = \left[\left\{1 - \sin^2\theta_0\left(\frac{v_4'}{v}\right)^2\right\}^{1/2}\right]\left(\frac{v}{v_4'}\right),$$

$$a_{41} = \left[\left\{1 - \sin^2\theta_0\left(\frac{v_1}{v}\right)^2\right\}^{1/2}\right]\left(\frac{v}{v_1}\right),$$

$$a_{42} = \left[\left\{1 - \sin^2\theta_0\left(\frac{v_2}{v}\right)^2\right\}^{1/2}\right]\left(\frac{v}{v_2}\right),$$

$$a_{43} = \left[\left\{1 - \sin^2\theta_0\left(\frac{v_3}{v}\right)^2\right\}^{1/2}\right]\left(\frac{v}{v_3}\right), \qquad a_{44} = -\sin\theta_0$$

$$a_{45} = \left[\left\{1 - \sin^2\theta_0 \left(\frac{v_1'}{v}\right)^2\right\}^{1/2}\right]\left(\frac{v}{v_1'}\right),$$

$$a_{46} = \left[\left\{1 - \sin^2\theta_0 \left(\frac{v_2'}{v}\right)^2\right\}^{1/2}\right]\left(\frac{v}{v_2'}\right),$$

$$a_{47} = \left[\left\{1 - \sin^2\theta_0 \left(\frac{v_3'}{v}\right)^2\right\}^{1/2}\right]\left(\frac{v}{v_3'}\right), \qquad a_{48} = \sin\theta_0$$

$$a_{51} = \left\{1 - \sin^2\theta_0 \left(\frac{v_1}{v}\right)^2\right\}^{1/2}\left(\frac{v}{v_1}\right)^3,$$

$$a_{52} = \left[\left\{1 - \sin^2\theta_0 \left(\frac{v_2}{v}\right)^2\right\}^{1/2}\right]\left\{\frac{G_2(\rho v_2^2 - \lambda - 2\mu)}{G_1(\rho v_1^2 - \lambda - 2\mu)}\right\}\cdot\left(\frac{v}{v_2}\right)^3,$$

$$a_{53} = \left[\left\{1 - \sin^2\theta_0 \left(\frac{v_3}{v}\right)^2\right\}^{1/2}\right]\left\{\frac{G_3(\rho v_3^2 - \lambda - 2\mu)}{G_1(\rho v_1^2 - \lambda - 2\mu)}\right\}\cdot\left(\frac{v}{v_3}\right)^3, \quad a_{54} = 0,$$

$$a_{55} = \chi_1\left[\left\{1 - \sin^2\theta_0 \left(\frac{v_1'}{v}\right)^2\right\}^{1/2}\right]\cdot\left\{\frac{G_1'(\rho' v_1'^2 - \lambda' - 2\mu')}{G_1(\rho v_1^2 - \lambda - 2\mu)}\right\}\cdot\left(\frac{v}{v_1'}\right)^3,$$

$$a_{56} = \chi_1\left[\left\{1 - \sin^2\theta_0 \left(\frac{v_2'}{v}\right)^2\right\}^{1/2}\right]\cdot\left\{\frac{G_2'(\rho' v_2'^2 - \lambda' - 2\mu')}{G_1(\rho v_1^2 - \lambda - 2\mu)}\right\}\cdot\left(\frac{v}{v_2'}\right)^3,$$

$$a_{57} = \chi_1\left[\left\{1 - \sin^2\theta_0 \left(\frac{v_3'}{v}\right)^2\right\}^{1/2}\right]\cdot\left\{\frac{G_3'(\rho' v_3'^2 - \lambda' - 2\mu')}{G_1(\rho v_1^2 - \lambda - 2\mu)}\right\}\cdot\left(\frac{v}{v_3'}\right)^3,$$

$$a_{58} = 0, \quad \chi_1 = \frac{K'}{K}\left(\frac{1 - i\omega\tau_0}{1 - i\omega\tau_0'}\right), \quad a_{61} = 1,$$

$$a_{62} = \left\{\frac{G_2(\rho v_2^2 - \lambda - 2\mu)}{G_1(\rho v_1^2 - \lambda - 2\mu)}\right\} \cdot \left(\frac{v_1}{v_2}\right)^2, \; a_{63} = \left\{\frac{G_3(\rho v_3^2 - \lambda - 2\mu)}{G_1(\rho v_1^2 - \lambda - 2\mu)}\right\} \cdot \left(\frac{v_1}{v_3}\right)^2,$$

$$a_{64} = 0, \quad a_{65} = -\left\{\frac{G_1'(\rho' v_1'^2 - \lambda' - 2\mu')}{G_1(\rho v_1^2 - \lambda - 2\mu)}\right\} \cdot \left(\frac{v_1}{v_1'}\right)^2,$$

$$a_{66} = -\left\{\frac{G_2'(\rho' v_2'^2 - \lambda' - 2\mu')}{G_1(\rho v_1^2 - \lambda - 2\mu)}\right\} \cdot \left(\frac{v_1}{v_2'}\right)^2,$$

$$a_{67} = -\left\{\frac{G_3'(\rho' v_3'^2 - \lambda' - 2\mu')}{G_1(\rho v_1^2 - \lambda - 2\mu)}\right\} \cdot \left(\frac{v_1}{v_3'}\right)^2, \quad a_{68} = 0,$$

$$a_{71} = \left[\left\{1 - \sin^2\theta_0 \left(\frac{v_1}{v}\right)^2\right\}^{1/2}\right]$$
$$\{-\beta_2 - bH_1(\rho v_1^2 - \lambda - 2\mu) + aG_1(\rho v_1^2 - \lambda - 2\mu)\} \left(\frac{v}{v_1}\right)^3,$$

$$a_{72} = \left[\left\{1 - \sin^2\theta_0 \left(\frac{v_2}{v}\right)^2\right\}^{1/2}\right]$$
$$\{-\beta_2 - bH_2(\rho v_2^2 - \lambda - 2\mu) + aG_2(\rho v_2^2 - \lambda - 2\mu)\} \left(\frac{v}{v_2}\right)^3,$$

$$a_{73} = \left[\left\{1 - \sin^2\theta_0 \left(\frac{v_3}{v}\right)^2\right\}^{1/2}\right]$$
$$\{-\beta_2 - bH_3(\rho v_3^2 - \lambda - 2\mu) + aG_3(\rho v_3^2 - \lambda - 2\mu)\} \left(\frac{v}{v_3}\right)^3,$$

$$a_{74} = 0,$$

$$a_{75} = -\chi_2 \left[\left\{1 - \sin^2\theta_0 \left(\frac{v_1'}{v}\right)^2\right\}^{1/2}\right]$$
$$\{-\beta_2' - b'H_1'(\rho v_1'^2 - \lambda' - 2\mu') + a'G_1'(\rho v_1'^2 - \lambda' - 2\mu')\} \left(\frac{v}{v_1'}\right)^3,$$

$$a_{76} = -\chi_2 \left[\left\{ 1 - \sin^2\theta_0 \left(\frac{v_2'}{v} \right)^2 \right\}^{1/2} \right]$$
$$\{-\beta_2' - b'H_2'(\rho v_2'^2 - \lambda' - 2\mu') + a'G_2'(\rho v_2'^2 - \lambda' - 2\mu')\} \left(\frac{v}{v_2'} \right)^3,$$

$$a_{77} = -\chi_2 \left[\left\{ 1 - \sin^2\theta_0 \left(\frac{v_3'}{v} \right)^2 \right\}^{1/2} \right]$$
$$\{-\beta_2' - b'H_3'(\rho v_3'^2 - \lambda' - 2\mu') + a'G_3'(\rho v_3'^2 - \lambda' - 2\mu')\} \left(\frac{v}{v_3'} \right)^3,$$

$$a_{78} = 0, \qquad \chi_2 = \frac{D'}{D} \left(\frac{1 - i\omega\tau}{1 - i\omega\tau'} \right),$$

$$a_{81} = \{\beta_2 + bH_1(\rho v_1^2 - \lambda - 2\mu) - aG_1(\rho v_1^2 - \lambda - 2\mu)\} \left(\frac{v}{v_1} \right)^2,$$

$$a_{82} = \{\beta_2 + bH_2(\rho v_2^2 - \lambda - 2\mu) - aG_2(\rho v_2^2 - \lambda - 2\mu)\} \left(\frac{v}{v_2} \right)^2,$$

$$a_{83} = \{\beta_2 + bH_3(\rho v_3^2 - \lambda - 2\mu) - aG_3(\rho v_3^2 - \lambda - 2\mu)\} \left(\frac{v}{v_3} \right)^2, \quad a_{84} = 0,$$

$$a_{85} = -\{\beta_2' + b'H_1'(\rho v_1'^2 - \lambda' - 2\mu') - a'G_1'(\rho v_1'^2 - \lambda' - 2\mu')\} \left(\frac{v}{v_1'} \right)^2,$$

$$a_{86} = -\{\beta_2' + b'H_2'(\rho v_2'^2 - \lambda' - 2\mu') - a'G_2'(\rho v_2'^2 - \lambda' - 2\mu')\} \left(\frac{v}{v_2'} \right)^2,$$

$$a_{87} = -\{\beta_2' + bH_3'(\rho v_3'^2 - \lambda' - 2\mu') - a'G_3'(\rho v_3'^2 - \lambda' - 2\mu')\} \left(\frac{v}{v_3'} \right)^2, \quad a_{88} = 0,$$

and, Z_j and b_j, $(j = 1, 2, \ldots 8)$ are as follows:

(a) For incident P wave ($v = v_1$):

$$Z_1 = \frac{A_1}{A_0}, \quad Z_2 = \frac{A_2}{A_0}, \quad Z_3 = \frac{A_3}{A_0}, \quad Z_4 = \frac{B_1}{A_0},$$
$$Z_5 = \frac{A_1'}{A_0}, \quad Z_6 = \frac{A_2'}{A_0}, \quad Z_7 = \frac{A_3'}{A_0}, \quad Z_8 = \frac{B_1'}{A_0},$$
$$b_1 = -a_{11}, \quad b_2 = a_{21}, \quad b_3 = -a_{31}, \quad b_4 = a_{41},$$
$$b_5 = a_{51}, \quad b_6 = -a_{61}, \quad b_7 = a_{71}, \quad b_8 = -a_{81}.$$

(b) For incident SV wave ($v = v_4$):

$$Z_1 = \frac{A_1}{B_0}, \quad Z_2 = \frac{A_2}{B_0}, \quad Z_3 = \frac{A_3}{B_0}, \quad Z_4 = \frac{B_1}{B_0},$$
$$Z_5 = \frac{A_1'}{B_0}, \quad Z_6 = \frac{A_2'}{B_0}, \quad Z_7 = \frac{A_3'}{B_0}, \quad Z_8 = \frac{B_1'}{B_0},$$
$$b_1 = a_{14}, \quad b_2 = -a_{24}, \quad b_3 = a_{34}, \quad b_4 = -a_{44},$$
$$b_5 = 0, \quad b_6 = 0, \quad b_7 = 0, \quad b_8 = 0.$$

4. Application to a particular model

A particular model is chosen to study the amplitude ratios numerically, where the following material constants are considered at $T_0 = 27\,^\circ C$:

For medium M_1:

$\lambda = 5.775 \times 10^{11}$ dyne/cm^2, $\mu = 2.646 \times 10^{11}$ dyne/cm^2, $\rho = 2.7$ gm/cm^3,
$C_e = 0.216$ cal/gm°C, $K = 0.00492$ cal/cm s°C, $\tau_0 = 0.05$ s,
$\tau = 0.04$ s, $\alpha_t = 0.05$ °C, $\alpha_c = 0.06$ cm^3/gm,
$a = 0.005$ cm^2/s^2 °C, $b = 0.05$ cm^5/gm s^2, $D = 0.5$ gm s/cm^3.

For medium M_2:

$\lambda' = 5.334 \times 10^{11}$ dyne/cm^2, $\mu' = 2.312 \times 10^{11}$ dyne/cm^2, $\rho' = 2.6$ gm/cm^3,
$C_e = 0.203$ cal/gm°C, $K' = 0.00483$ cal/cm s°C, $\tau_0' = 0.05$ s,
$\tau' = 0.035$ s, $\alpha_t' = 0.04$ °C, $\alpha_c' = 0.05$ cm^3/gm,
$a' = 0.004$ cm^2/s^2 °C, $b' = 0.04$ cm^5/gm s^2, $D' = 0.4$ gm s/cm^3.

The system of eight non-homogeneous equations (25) is solved for the above material constants for amplitude ratios by using a Fortran Program of Gauss elimination method.

4.1. Discussion for incidence P wave

The amplitude ratios of various reflected and transmitted waves are obtained for the range of $1^\circ \leq \theta_0 \leq 89^\circ$ of angle of incidence, when P wave

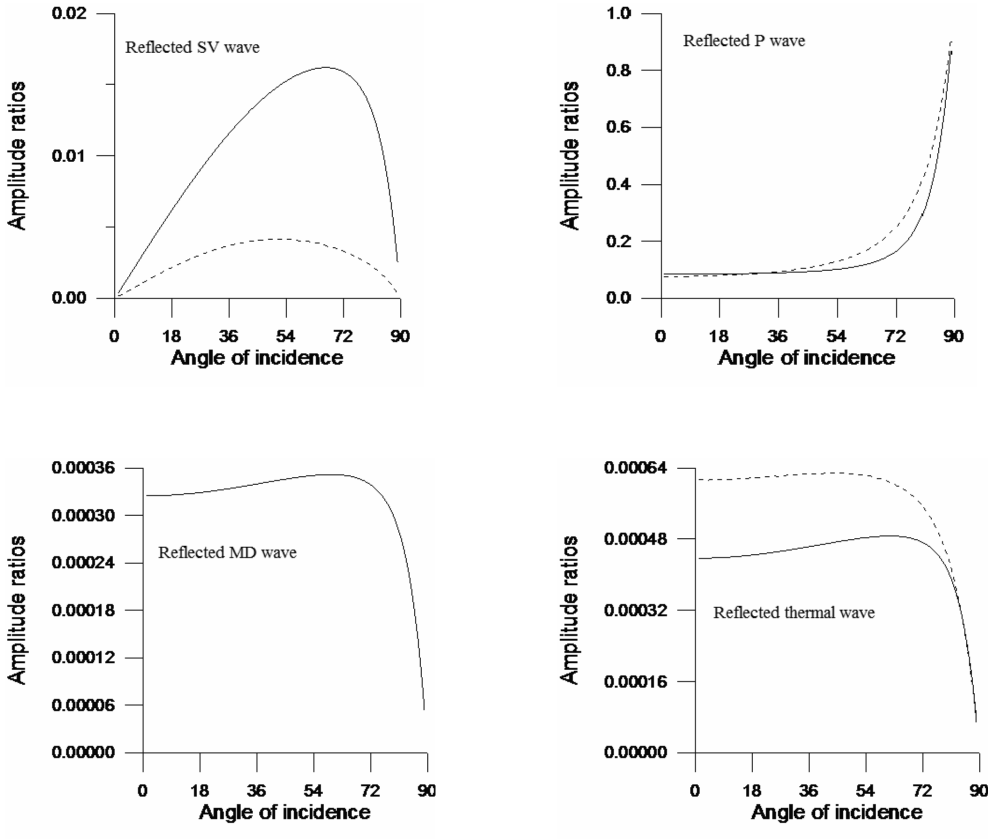

Fig. 2. Diffusion effects on the reflected waves for the incidence of P wave

is incident. These amplitude ratios of reflected and refracted waves are shown graphically against the angle of incidence in Figs. 2 and 3, respectively. The dotted curves in these figures show the variation of amplitude ratios in absence of diffusion.

The amplitude ratio of the reflected SV wave increases slowly to its maxima and then decreases sharply at angles near grazing incidence. In absence of diffusion, the solid curve reduces to the dotted curve, where the amplitude decreases at each angle of incidence. The amplitude ratio of the reflected P wave first increases very slowly and it increases very sharply at angles near grazing incidence. In absence of diffusion, the solid curve reduces to the dotted curve as shown in Fig. 2. The presence of diffusion changes the amplitude at each angle of incidence. The amplitude ratio of the reflected MD wave is a bit smaller than amplitude ratios of other reflected waves at each angle of

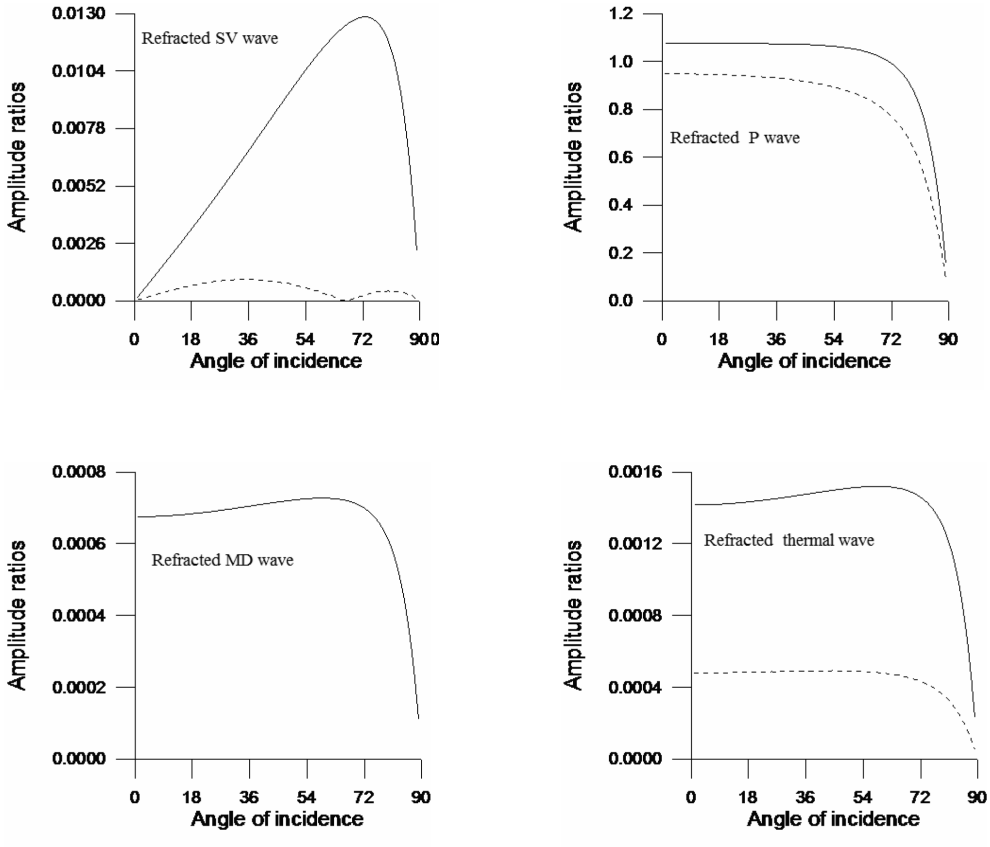

Fig. 3. Diffusion effects on the refracted waves for the incidence of P wave

incidence. It increases slowly with the angle of incidence and decreases sharply at angles near grazing incidence. This reflected wave disappears in absence of diffusion. The amplitude ratio of reflected thermal wave is smaller than the reflection coefficients of the reflected P and SV waves, but it is slightly more than that of the reflected MD waves at each angle of incidence. It also increases slowly with the angle of incidence and decreases sharply at the angle near grazing incidence. The amplitude increases at each angle of incidence in absence of diffusion as shown in Fig. 2.

The amplitude ratio of refracted SV wave increases slowly to its maxima and it decrease very sharply at angles near grazing incidence. The variation of amplitude ratio of refracted SV wave is similar to that of reflected SV wave. The value of amplitude ratio of refracted SV decreases significantly at each angle of incidence as shown by solid and dotted curves in Fig. 3. The amplitude of refracted P decreases very slowly with the angle of incidence starting from normal incidence and it decrease very sharply at the angles near grazing incidence. The presence of diffusion increases the amplitude ratio of refracted P wave at each angle of incidence as shown by dotted and solid curves in Fig. 3. The variations of amplitude ratios of refracted MD and thermal waves are similar to those for reflected MD and thermal waves. The refracted MD wave disappears in absence of diffusion. The amplitude of refracted thermal wave decreases at each angle of incidence in absence of diffusion, whereas the amplitude of reflected thermal wave increases at each angle of incidence.

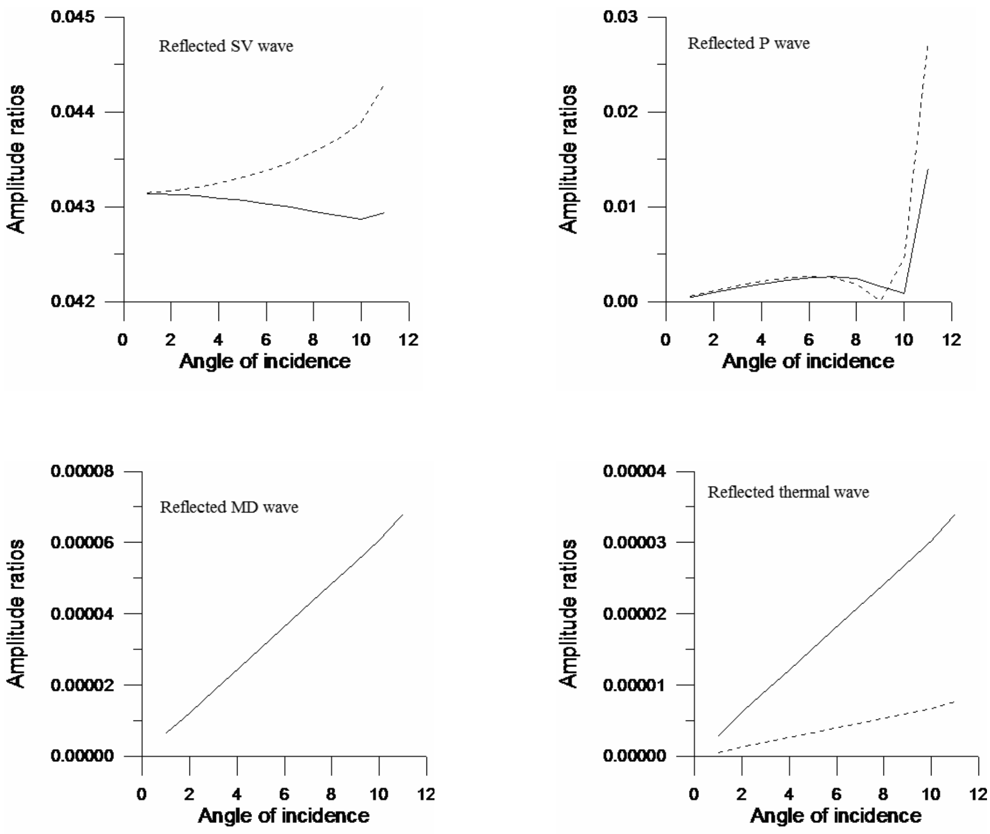

Fig. 4. Diffusion effects on the reflected waves for the incidence of SV wave

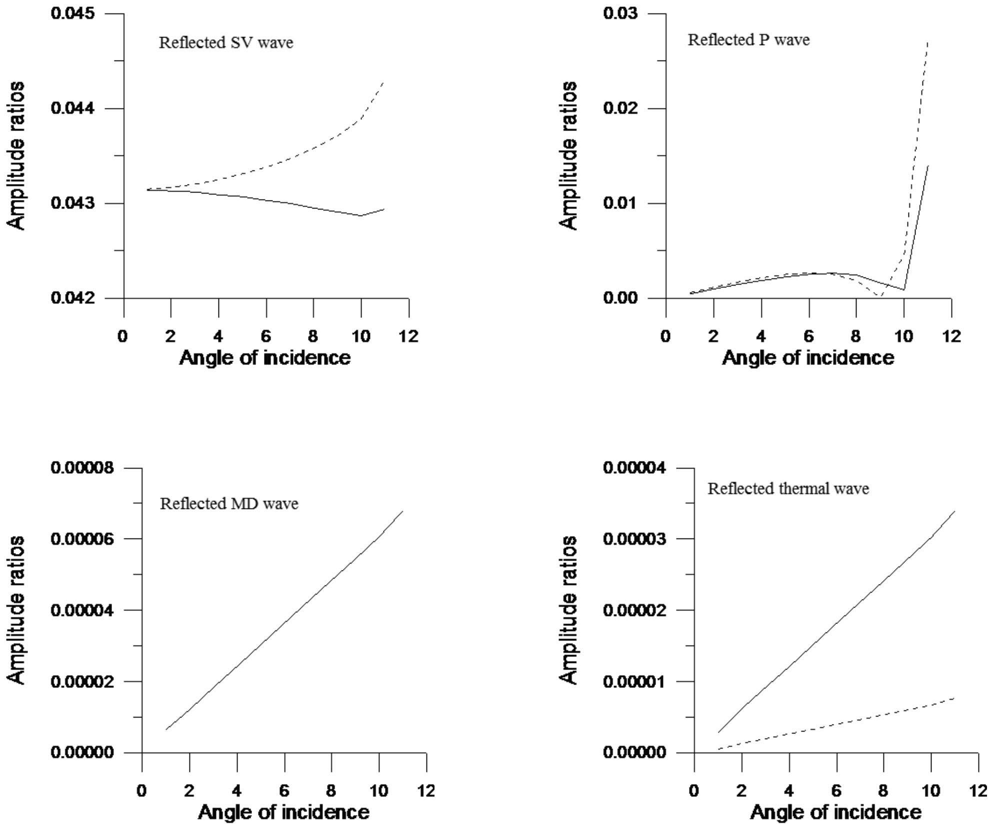

Fig. 5. Diffusion effects on the refracted waves for the incidence of SV wave

4.2. Discussion for incidence SV wave

The reflection and transmission coefficients of various reflected and transmitted waves are obtained for the range of angle of incidence $1^\circ \le \theta_0 \le 11^\circ$, when SV wave is incident at the interface. Total internal reflection occurs beyond this critical angle of incidence for reflected and transmitted P, MD and thermal waves. The amplitude ratios of reflected and transmitted waves are shown graphically with the angle of incidence in Figs. 4 and 5, respectively. The dotted curves in these figures show the variation of amplitude ratios in absence of diffusion. Reflected and refracted MD waves disappear in absence of diffusion.

5. Conclusions

A problem of reflection and transmission of plane waves at an interface between two thermo-elastic half-spaces with diffusion is considered. Boundary conditions at the interface between two thermo-elastic half-spaces with diffusion are formulated. The appropriate potentials or solutions are chosen to satisfy the boundary conditions at the interface. A system of eight non-homogenous equations in reflection and transmission coefficients (amplitude ratios) is obtained with the modified Snell's law. These amplitude ratios are computed for incident P and SV waves. The variations of these amplitude ratios are shown graphically in presence, as well as in absence of diffusion. Effects of diffusion on reflected and refracted waves are observed.

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