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SOLID MECHANICS

STRUCTURAL SYSTEMS SECURITY ZONES*

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Abstract. This presentation is based on some reviews' of published papers and some new results. We propose the determination of security zones for structural system or the zones of good quality for structural materials. In the particular case, we verify if our system (or material) under consideration lies in these zones and what is the corresponding stock of security (quality). We introduce the nondimensional parameters characterized by the loading to realize this verification, the dimensions of the system and the mechanical properties of the structural material. These parameters come form the space of events. We apply the different criteria for the carrying capacity loss of the system and on this base we build up the limit surfaces. We introduce the sub zone of admissible parameter values. The zone between the limit surfaces and the corresponding admissible sub zone forms the security zone (or the zone of a good material quality). On this base, we can verify, what the stock of the system security is (or for the quality of the material). We give some examples. The special attention is paid to the seismic security zone and to the calculation of the seismic risk for the multi-storey building under seismic action.
Key words: structural system, security zones.

1. Introduction

We take into account a design or estimation procedures for different structural systems – some structural elements with different dimensions or structural materials [1, 2, 3]. We examine this system according to the loss of carrying capacity applying different criteria.

In the particular case we verify if our system under investigation lies in the security zone and what is the stock of security if there is any.

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We apply the following method to realize this verification. We introduce the non-dimensional parameters characterizing the loading, the dimensions of the system and the mechanical properties of the chosen structural material. All this parameters are taken form the space of events for the system. We apply different criteria for the loss of the carrying capacity of the system under investigation according to the strength of material, do not permit a large deformation of the system, the loss of stability of some system elements etc. We build on this basis the corresponding limit surfaces in the space of the chosen parameters. The security zone is determined. We verify the security of the existing system in every particular case for design or for estimation if the representing point (for particular values of the parameters) lies or does not in the security zone. This information is very important to ensure the security of the entire system.

We propose some examples of the method application.

2. Test example

We will present a simple test example: structural system of cantilever beams with a rectangular cross section (see Fig. 1). The beams are subjected to the uniform distributed loading $q(t)$ and the axially applied pressure force P_0. The time t is in a given time interval$[t_I, t_f]$. The structural material process is a linear elastic one with a linear plastic hardening (see Fig. 2).

We assume the following simple criteria for the loss the carrying capacity of the cantilever beam under the presented loading and the given material mechanical properties:

$$P_0 = \text{const}, \quad \frac{h}{l} < \frac{1}{2}, \quad x \in [0, l]\,; \quad b_0 = \text{const}, \quad t \in [t_I, t_f]$$

$$\varepsilon = \varepsilon^e + \varepsilon^p$$

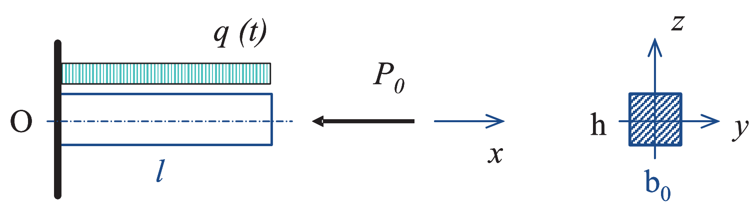

Fig. 1. Cantilever beams with rectangular cross section and uniform distributed loading $q(t)$ with fixed axially applied pressure force

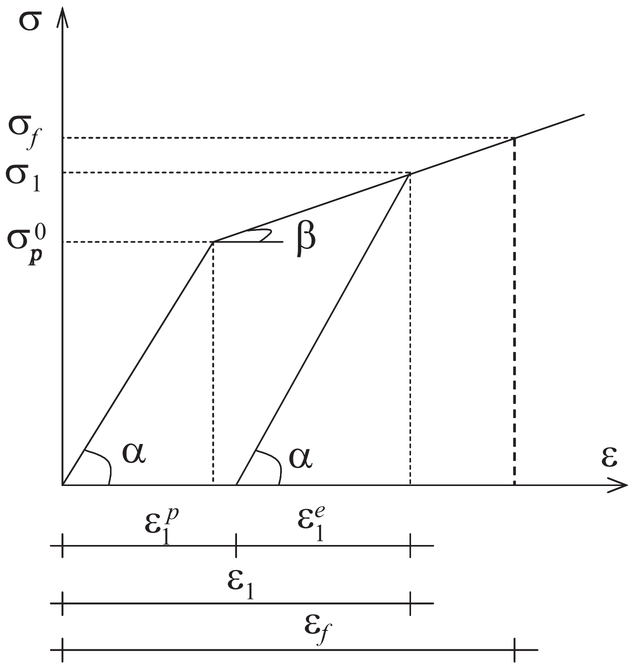

Fig. 2. Diagram $\sigma - \varepsilon$ for extension: σ – extension stress; ε – linear strain

ε^e – elastic strain; ε^p – plastic strain; σ_p^0 – initial plastic limit; σ_f – stress fracture limit; $E = \operatorname{tg}\alpha$ – elastic modulus; $E_p = \operatorname{tg}\beta$ – plastic modulus; ε_f – strength limit deformation, which depends on the chosen material.

2.1. Strength criterion

(1)
$$\max_{x,t} = \{M_z(x,t)\} \leq M_{ad}, \quad M_{ad} = k_M M_F$$

where M_{ad} is the admissible momentum; k_M is the coefficient of security, M_F is the fracture momentum (see Fig. 3).

(2)
$$M_F = \left\{ \frac{1}{2}\left(\sigma_F + \sigma_P^0\right) h_p z_p + \frac{1}{3}\sigma_p^0 h_p z_p \right\} b_0$$

where $\dfrac{2h_e}{h} = \dfrac{\varepsilon_p^0}{\varepsilon_F}$, $h_e = \dfrac{h}{2} - h_p$

(3)
$$h_p = \frac{h}{2} - \frac{2}{h}\frac{\varepsilon_p^0}{\varepsilon_F}$$

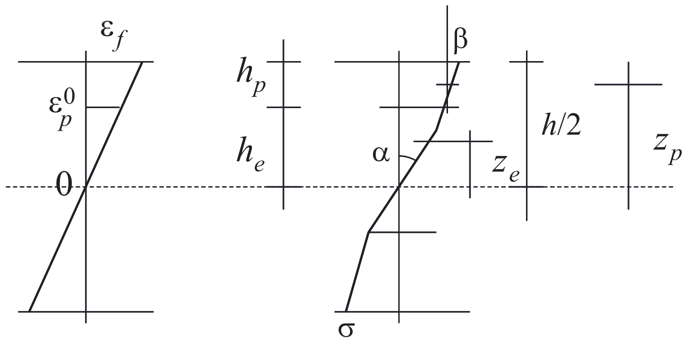

Fig. 3. Stress and strain in a cross section at fracture limit

$$z_p = \frac{h}{2} - \frac{2}{3}\,h_p, \quad z_e = \frac{2}{3}\,h_e.$$

2.2. Deformational criterion

(4) $$\max_{x,t} = \{W\,(x,t)\} \leq W_{ad},$$

where W_{ad} is the admissible deflection, $x \in [0,l]$, $t \in [t_I, t_f]$.

2.3. Stability criterion

(5) $$N\,(x,t) = -P_0, \quad N_{ad} = k_N\,N_{cr},$$

$$N_{cr} = \frac{\nu^2 K_p}{a^2}, \quad a = \frac{l}{3}, \quad K_p = E_p J, \quad J = \frac{b_0 h^3}{12},$$

where

(6) $$N \leq N_{ad},$$

N_{ad} – admissible pressure force; N_{cr} – critical force; k_N – the coefficient of security; ν – correction coefficient for elastic-plastic properties of the structural materials; a – effective length.

We assume the following non-dimensional parameters.

2.3.1 For loading

(7) $$Q = \frac{q_M}{q_0}, \quad q_0 = \text{const},$$

$$q_M = \frac{1}{(t_f - t_I)} \int_{t_I}^{t_f} q\,(t)\; dt.$$

2.3.2 For beam geometry

(8) $$\xi = \frac{h}{l}.$$

2.3.3 For material characteristics

(9) $$\eta = \frac{N_c\, W_c}{M_F}.$$

The admissible sub zone (ξ, η) in the space of events is formed on the basis of an expert estimation by $\xi \in [\xi_1, \xi_2]$ and $\eta \in [\eta_1,\, \eta_2]$, where $\xi_1, \xi_2, \eta_1, \eta_2$ are assumed limit values.

We determine the following limit surfaces in the space of events (Q, ξ, η).

If Q_{st} is the value of the Q when the strength criterion is fulfilled the corresponding strength limit surface is as follows:

(10) $$Q\,(\xi, \eta) = Q_{st}.$$

If Q_{def} is the value of the Q when the deformational criterion is fulfilled the corresponding deformational limit surface is as follows:

(11) $$Q\,(\xi, \eta) = Q_{def}.$$

If Q_{cr} is the value of Q when the stability criterion is fulfilled the corresponding limit surface is as follows:

(12) $$Q\,(\xi, \eta) = Q_{cr}.$$

We have a point (ξ', η') in the admissible sub zone in some particular case under estimation. We calculate the corresponding value $Q' = (\xi', \eta')$, which gives the point (Q', ξ', η') in the space (Q, ξ, η). The security of the beam under consideration is estimated on the basis of the difference $\Delta Q' = Q'_l\,(\xi', \eta') - Q'\,(\xi', \eta')$ where $Q'_l\,(\xi', \eta') = \min\{Q'_{st}, Q'_{\text{det}}, Q'_{cr}\}$. If $\Delta Q' > 0$ the value:

(13) $$\chi = \frac{\Delta Q'}{Q'_l} 100 \text{ in } \%,$$

gives the stock of security for the beam under consideration (see Fig. 4)

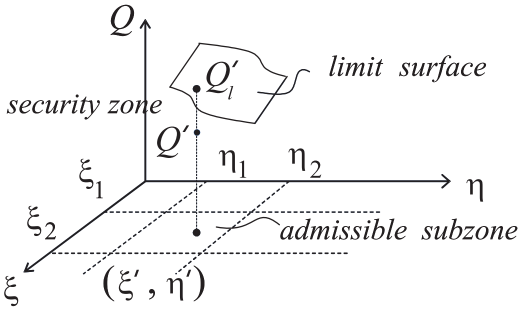

Fig. 4. Stock security estimation $\Delta Q'$ in the security zone belonging to the space of events (Q, ξ, η)

3. Applications

3.1. Safety zones for inelastic structures under impulsive loading

We take into account a high building system with three storeys under horizontal impulsive storey loading $F' = m' A_0 \sin(\theta t)$ [1]. The principal system parameters are as follows (see Fig. 5): m' – is the concentrated storey mass; A_0– is the constant amplitude; P' – is the storey pressure force; l – is the building height; l_s – is the storey height; l_p – is the plastic zone height; l_e – is the elastic zone height; θ – is the frequency of the impulsive loading F'; Process time $t \in [0, T]$, where T is the period of the dynamic action; k_{pl} is the plastic rigidity; k_{el} is the elastic rigidity.

We assume the same criteria for the carrying capacity of the system like in the test example (item 2). Nevertheless, the important problem remains how to calculate the strength momentum M_F concerning the storey level. This

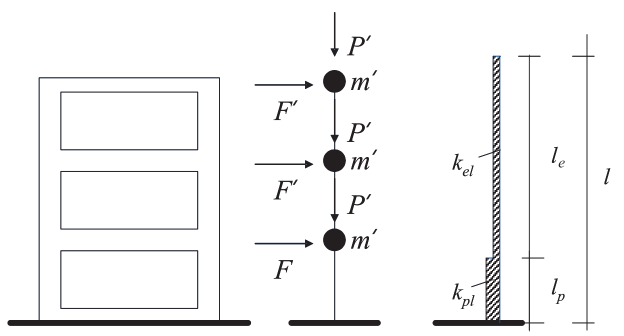

Fig. 5. A high storey building system under impulsive loading

requires special procedure taking into account the storey structural system, including the existing against-earthquake walls [9]. The same additional precision requires also the stability criterion. The deformational criterion concerns the model of cantilever beam.

The non-dimensional characteristic parameters are chosen as follows:

3.1.1 For loading

$$\bar{P}(\tau) = \frac{1 - \cos\left(\bar{\theta}\,\tau\right)}{\theta}, \quad \bar{\theta} = \theta T, \quad \tau = \frac{t}{T}, \quad \tau \in [0,1], \tag{14}$$

and also:

$$\bar{F}' = \sin\left(\bar{\theta}\,\tau\right), \qquad \bar{F}' = \frac{F'}{m' A_0}. \tag{15}$$

3.1.2 For model dimensions with material characteristics and loading parameters

$$\mu = \frac{W_c}{T^2 A_0}, \quad \lambda = \frac{W_c M_F}{m' A_0 l^2}. \tag{16}$$

Now the space of events is $\left(\bar{P}', \mu, \lambda\right)$ and the admissible sub zone is $\{(\mu_1, \mu_2)\,; (\lambda_1, \lambda_2)\}$. The limit surfaces corresponding to the assumed criteria are as follows [1]:

$$\begin{aligned} \bar{P}'(\mu,\lambda) &= \bar{P}_{st} \\ \bar{P}'(\mu,\lambda) &= \bar{P}_{def} \ , \\ \bar{P}'(\mu,\lambda) &= \bar{P}_{cr} \end{aligned} \tag{17}$$

where $\bar{P}_{st}$ is the value of $\bar{P}'$ when the strength criterion is fulfilled; $\bar{P}_{def}$ – when the deformation criterion is fulfilled and $\bar{P}_{cr}$ – when the stability criterion is fulfilled.

We build the security zone in the space $\left(\bar{P}', \mu, \lambda\right)$. We obtain the difference $\Delta\bar{P}' = \left\{\bar{P}_l - P^*\right\}$ in the particular case (μ^*, λ^*), where $\bar{P}_l = \min\{\bar{P}^*_{st}, \bar{P}^*_{def}, \bar{P}^*_{cr}\}$ and $P^* = \bar{P}'(\mu^*, \lambda^*)$.

The corresponding security stock parameter is:

$$\chi = \frac{\Delta\bar{P}'}{\bar{P}_l} 100 \text{ in } \%. \tag{18}$$

The usually calculated model for high building with many storeys is a cantilever beam with concentration mass on every storey.

3.2. Seismic security zones and seismic risk

Now a day, the problem to estimate the seismic risk for a given building is very important. We may decide on the basis of corresponding analysis if this building requires some additional reinforcement. There are many publications concerning this estimation [6, 7, 8]. Usually, they calculate the seismic risk applying some probabilistic approaches, which are completely different from the deterministic manner to obtain the seismic forces, according to the existing norms. We apply the normative acceleration for the given building place on the basis of the national seismic map and the normative requirements. We calculate and apply to the building deterministic normative forces. The gap between the two approaches possesses many practical difficulties. We propose some semi-probabilistic method to avoid this which combines both points of view using our determination of the security zone.

We will give an example to demonstrate our procedure using the same three storey building and the corresponding structural model with storey concentration masses on the cantilever beam like in item 3.1. Now, the horizontal storey force F' appears from the seismic action and has static character according to the Euro code 8.

The seismic force on the storey level is given as follows according to the Bulgarian seismic norms [8]:

$$F'_{norm_i} = CRk_e\beta_s\eta'_iQ', \tag{19}$$

where (i) is the natural mode of the cantilever beam model $(i = 1, 2, \dots, n)$; C is the coefficient estimates the building importance; R is the coefficient taking into account the eventual non-elastic response of the building under normative seismic action; k_e is the coefficient defining the relation between the normative acceleration a_{norm} and earth acceleration g; β_s is the spectral coefficient depending on the period T_{norm} of the normative seismic action according to the assumed relation; η'_i is the coefficient of i-th-natural mode representing the deformation of the model beam under normative seismic acceleration (It depends on the displacement X_{ki}, where (k) is the number of the storey level); $Q' = m'g$, where $m' = m_k$ is the concentration storey mass, $(k = 1, 2, \dots, n$ is the number of the storey).

Now, we calculate the corresponding semi-probabilistic horizontal storey force $\tilde{F}'$, which depends on the probability $0 \le P_a(j) \le 1$ that the acceleration $a\ (j)$ will take place $(a\ (j) \ge a_{norm},\ j = 1, 2, \dots, m,\ a_{norm}$ – normative acceleration), according to the long period of seismic observations on the given place (taking into account the local seismic map). It depends also on the probability $0 \leq P_{\omega}(j) \leq 1$ that the i-th-natural mode will realize under the seismic action with the acceleration $a\ (j)$.

We propose the following semi-probabilistic expression:

$$\tilde{F}_i' = CR\beta_s\tilde{k}_e\tilde{\eta}_j'Q', \tag{20}$$

where C, R, β_s, Q' are the normative deterministic coefficients; $\tilde{k}_e = \tilde{P}_a k_e$, $\tilde{P}_a = 1 + \sum_{j=1}^{m} P_a(j)$, k_e is the normative deterministic coefficient; $\tilde{\eta}_i'$ depends on the probabilistic displacements, $\tilde{X}_{ki} = P_i\, X_{ki}$, where $\tilde{P}_i = 1 + \sum_{j=1}^{m} P_{\omega}(j)$, $\tilde{X}_{ki}$ are the normative displacements for the i-th-natural mode and the $k-$ storey.

We apply the same criteria like in item 3.1 for the loss of the carrying capacity for the system with required additional precision concerning the limit values. We assume also the same material characteristics.

Now we introduce the following non-dimensional parameters

3.2.1 For loading

$$f_i' = \frac{\tilde{F}_i'}{F_0}, \tag{21}$$

$F_0 = \text{const}$ (for non-dimensionalization), for the i-th-natural building vibration obtained like natural mode of the model cantilever beam.

Practically, if we take into account the s-first natural modes we can propose the following representative loading parameter:

$$\tilde{q} = \max\left(f_i'\right), \quad i = 1, 2, \ldots, s. \tag{22}$$

3.2.2 For the mixed effects

$$\xi = \frac{m'\ A_{norm}W_c}{M_p^0}n, \tag{23}$$

where n is the number of storey, M_p^0 is the beginning of the plastic deformation in the cross sectionfor the model elastic-plastic material of the cantilever beam.

3.2.3 For the limit values

$$\eta = \frac{N_{cr}l'}{M_F}, \tag{24}$$

where l' is the length between the storeys; N_{cr} and M_F have the same meaning like in item 3.1.

The space of events is formed by $\{\tilde{q}, \xi, \eta\}$. We build the admissible sub zone $\{\xi_1, \eta_1\}$ and $\{\xi_2, \eta_2\}$ on the basis of an expert estimation of the limit values $\{\xi_1, \xi_2, \eta_1, \eta_2\}$.

We obtain using previously proposed procedure the limit surface for the strength, deformation and the stability criterion:

$$\begin{aligned} \tilde{q} &= \tilde{q}_s(\xi, \eta) \\ \tilde{q} &= \tilde{q}_{def}(\xi, \eta) \ , \\ \tilde{q} &= \tilde{q}_{cr}(\xi, \eta) \end{aligned} \tag{25}$$

where $q_s^* = \tilde{q}_s(\xi^*, \eta^*)$, $q_{def}^* = \tilde{q}_{def}(\xi^*, \eta^*)$ and $q_{cr}^* = \tilde{q}_{cr}(\xi^*, \eta^*)$.

We have the value (ξ^*, η^*) in the admissible zone in some particular cases for estimation. The corresponding loading parameter value $\tilde{q}^*$ permits us to calculate the difference $\Delta\tilde{q}^* = \tilde{q}_l^* - \tilde{q}_{cr}^*$, where $\tilde{q}_l^* = \min\left(\tilde{q}_s^*, \tilde{q}_{def}^*, q_{cr}^*\right)$. We obtain the seismic risk $\chi^* = \dfrac{\Delta\,\tilde{q}^*}{\tilde{q}_l^*}100$ /in %/ for the system under consideration if $\Delta\tilde{q}^* > 0$.

This semi-probabilistic risk estimation is similar to the normative seismic calculation.

3.3 Zone for structural material quality

The proposed method for estimation the carrying capacity of structural system is to be applied for obtaining the carrying capacity stock of structural material. We will give an example with reinforced concrete [4, 5]. The essential problem for this composite is the common work of its two components – the concrete matrix and the steel bar reinforcement. We will take into account the following more important case for the loss of material carrying capacity:

(I) the connection between the two materials is damaged. Practically, this means that the steel bar is pulled out of the concrete matrix;

(II) In the neighbourhood of the bar some extensional micro-cracks appear in the concrete. The concrete is failed.

The system is presented in the Fig. 6:

We propose the following non-dimensional parameters (see Fig. 6)

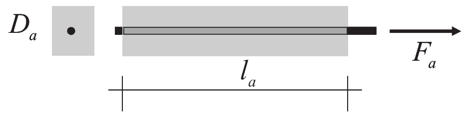

Fig. 6. Reinforced concrete structural system

3.3.1 For loading

$$q = \frac{F_a}{F_0}, \tag{26}$$

where F_0 is a fixed initial force value.

3.3.2 For geometry

$$\xi = \frac{D_a}{l_a}. \tag{27}$$

This parameter presents the contact zone between the steel bar and the concrete matrix $S_{cont} = \pi D_a l_a$, when D_a is the bar diameter.

3.3.3 For material properties

$$\eta = \frac{E_c}{E_a} \frac{\sigma_s}{\sqrt{3}\tau_f}, \tag{28}$$

where E_c is the elastic modulus of the concrete; E_a is the elastic modulus of the steel; σ_s is the concrete extension strength; τ_f is the shear strength of the connection between steel and concrete.

We assume linear elasto-plastic damage model for the extensional properties of the concrete, presented in the Fig. 7 with its stress-strain diagram.

σ_d^0 (point 2) is the stress in the beginning of the damage; σ_F (point 4) is fractural stress; σ_s (point 3) is the extensional strength with corresponding strain ε_s after which some intensive damage appears; σ_p^0 (point 1) is the stress in the beginning of the non-elastic deformation.

Now, the space of events is built up by (q, ξ, η). The admissible zone is $\{(\xi_1, \xi_2), (\eta_1, \eta_2)\}$, where $\xi_1, \xi_2, \eta_1, \eta_2$ are the estimated values from the authorized experts.

We propose the following carrying capacity criteria for the composite:

(I) first criterion:

$$q = q_I, \quad q_I = \frac{F_{pullout}}{F_0}, \tag{29}$$

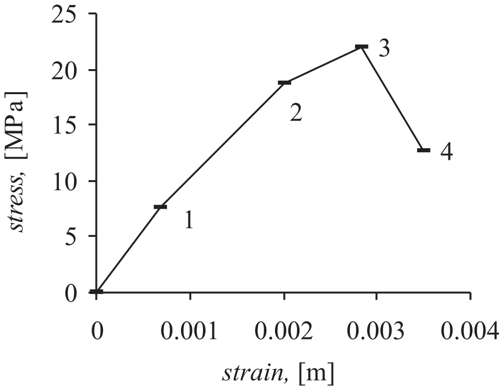

Fig. 7. Stress-strain diagram for C20/25 concrete

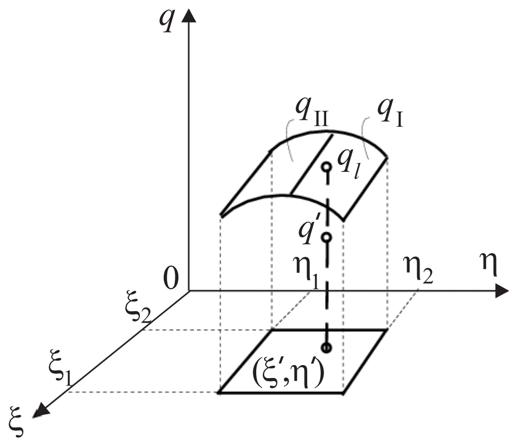

Fig. 8. The space of events and the stock of good material quality

where $F_{pullout}$ is the force, when the steel bar becomes to be pulled out of the concrete matrix.

(II) second criterion:

(30) $$\max\left(\sigma_{ext}\right) \leq \sigma_{addm},$$

where σ_{addm} is the admissible stress.

We obtain the limit surfaces on the basis of these two criteria:

(31) $$q\left(\xi, \eta\right) = q_I,$$

$$q\,(\xi, \eta) = q_{II}. \tag{32}$$

The zone in space of event (q, ξ, η), between sub-zone and the limit surfaces, gives information about the good quality of the reinforced concrete (see Fig. 8).

We calculate the corresponding value $q' = q\,(\xi', \eta')$ for the given particular case (ξ', η'). The difference $\Delta q' = q_l - q'$ estimates the state of the composite material, where $q_l = \min\,(q_I, q_{II})$. The material is failed if $\Delta q' \leq 0$. We obtain some stock of the good quality if $\Delta q' > 0$.

$$\chi' = \frac{q_l - q'}{q_l} 100 \text{ in } \%. \tag{33}$$

This verification is very important to have the information about the applied reinforced concrete.

4. Conclusion

The presented procedure gives a good possibility to realize a design or to make any estimation of the structural system which ensures carrying capacity. It can be applied in different cases, arising from the practice.

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