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FLUID MECHANICS

CONVECTION HEAT AND MASS TRANSFER IN A POWER LAW FLUID WITH NON CONSTANT RELAXATION TIME PAST A VERTICAL POROUS PLATE IN THE PRESENCE OF THERMO AND THERMAL DIFFUSION*

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Abstract. The paper investigates convection heat and mass transfer in power law fluid flow with non relaxation time past a vertical porous plate in presence of a chemical reaction, heat generation, thermo diffusion and thermal diffusion. The non – linear partial differential equations governing the flow are transformed into ordinary differential equations using the usual similarity method. The resulting similarity equations are solved numerically using Runge–Kutta shooting method. The results are presented as velocity, temperature and concentration profiles for pseudo plastic fluids and for different values of parameters governing the problem. The skin friction, heat transfer and mass transfer rates are presented numerically in tabular form. The results show that these parameters have significant effects on the flow, heat transfer and mass transfer.
Key words: Power law fluid, pseudo plastic fluid, chemical reaction, convection heat, thermo diffusion, thermal diffusion, non-constant relaxation time.

1. Introduction

The study of non-Newtonian fluids has gained much attention recently in view of its promising applications in engineering and industry. Such fluids exhibit a non linear relationship between the stresses and the rate of strain. Most slurries, suspensions and dispersions, polymer solutions, melts and solutions of naturally occurring high-molecular-weight, synthetic polymers, pharmaceutical formulations, cosmetics and toiletries, paints, pagebreak biological fluids, synthetic lubricants and foodstuffs, exhibit complex rheological behaviour which

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is not experienced when handling ordinary low-molecular-weight Newtonian fluids such as air, water, silicon oils, etc.

Abdul-Rahim and Ali [1] discussed the problem of coupled heat and mass transfer by natural convection from a vertical impermeable semi-finite flat plate embedded in a non-uniform non-metallic porous medium in the presence of thermal dispersion effects. It is found that the variable porosity of the porous medium and the effect of thermal dispersion result in an increase in the local Nusselt number.

Afify [2] investigated the effect of chemical reaction on free convective flow and mass transfer of a viscous incompressible fluid and electrically conducting fluid over a stretching in the presence of a constant transverse magnetic field.

Ahmet and Muharrem [3] investigated the pressure gradient flow rate relationship for steady state non-isothermal pressure driven flow of a non-Newtonian fluid in a channel including the effect of viscous heating. The viscosity of the fluid depends on both temperature and shear rate. The effects of the activation energy parameter and the Brinkman number as well as the power law index and material time constant on the flow are studied. Ali [4] presented the numerical solution of the problem of steady, laminar, buoyancy-induced flow by natural convection along a vertical permeable surface immersed in a thermally-stratified environment in the presence of magnetic field and heat generation or absorption effects. Ali [5] studied the asymptotic solution for small and large values of the distance from the leading edge of the plate for the analysis of the problem the convection-radiation interaction heat transfer in boundary-layer flows over a semi-infinite flat plate with temperature dependent effective viscosity embedded in a fluid-saturated porous medium in the presence of a magnetic field. Ali [6] and [7] investigated hydromagnetic flow and heat transfer over a non-isothermal permeable surface stretching with a power-law velocity with heat generation and suction/injection effects and in the presence of a non-uniform transverse magnetic field and boundary-layer flow of a viscous, incompressible, electrically conducting and heat-absorbing fluid along a semi-infinite vertical permeable moving plate in the presence of a uniform transverse magnetic field and thermal and concentration buoyancy effects, respectively.

Ali and Abdul-Rahim [8] examined the problem of coupled heat and mass transfer by mixed convection in a linearly stratified stagnation flow in the presence of an externally applied magnetic field and internal heat generation or absorption effect. Also, Ali and Ali [9] investigated the problem of coupled heat and mass transfer by natural convection from a permeable sphere in the presence of an external magnetic field and thermal radiation effect. Bird [10, 11] examined unsteady pseudo plastic flow near a moving wall. The power–law index ranges from $n = 1/3$ to $n = 5/6$. He estimated for each n the similarity value r_1 for which the fluid velocity has fallen off to 1% of the velocity of the moving wall. The result shows that r_1 decreases as n increases.

Chung and Wuladana [12] examined the nonlinear stability of steady flow and temperature distribution of a Newtonian fluid in a channel heated from below where the viscosity is a function of temperature. Hassanien et al. [13] presented a boundary layer analysis for the problem of flow and heat transfer from a power law fluid to a continuous stretching sheet with variable wall temperature. Their result showed that the friction factor and heat transfer rate depend strongly on the flow parameters. Howell et al. [14] examined the momentum and heat transfer occurring in the laminar boundary layer on a continuously moving and stretching two dimensional surface in non Newtonian fluid. Their results include situation when the velocity is nonlinear and when the surface is stretched linearly. Ibrahim et al. [15] investigated the method of similarity reduction for problems of radiative and magnetic field effects on free convection and mass transfer flow past a semi-infinite flat plate. Kelly et al. [16] studied a non-conventional fluid dynamic problem by means of the boundary layer approximation. They presented analysis of the asymptotic behaviour of the solutions for small and high values of the non-dimensional numbers that govern the energy and the diffusion equation.

Makinde [17] examined the free convection flow with thermal radiation and mass transfer past a moving vertical porous plate. The plate is maintained at a uniform temperature with uniform species concentration and the fluid is considered to be gray, absorbing – emitting. Marusic – Paloka [18] examined the steady flow of a dilatant non-Newtonian fluid obeying the power law in unbounded channels and pipes. A proof of existence and uniqueness of the solution for Leray's problem is given as well as the delay estimate for the solution. For the existence result, he applied Galerkins procedure using monotonicity of the principal of the operator and the continuity of the inertial term. The effect of the thermal radiation, suction, thermal diffusion and heat generation on convection heat and mass transfer in a power law fluid were also investigated in [19, 20, 21, 22, 23].

Pascal [24] investigated the spread of a gravity current consisting of a fluid of Non-Newtonian power law rheology along a rigid horizontal plane under a shallow layer with a free theory approximation to establish a two layer model which couples the dynamics of the two layers. Sivasankaran et al. [25] investigated the natural convection heat and mass transfer fluid past an inclined semi–infinite porous surface with heat generation using Lie group analysis. Their result revealed that the velocity and temperature of the fluid increases with the heat generation parameter. Also, the velocity of the fluid increases with the porosity parameter and temperature and concentration decreases with the increase in the porosity parameter. Uzun [26] presented the finite difference solution for laminar heat transfer of a non-Newtonian power law fluid in the thermal entrance region of arbitrary cross sectional ducts with constant wall temperature. In his study, the effects of axial heat conduction, viscous dissipation and thermal energy sources with the fluid were neglected.

Yu-shu and Karsten [27] review their previous work relying on the development of a three dimensional, fully implicit, integral finite difference simulation for simple and multi-phase flow of non-Newtonian fluids in porous fractured media. Yurusoy and Pakdemirli [28] examined the exact solution of boundary layer equations of a non-Newtonian fluid over a stretching sheet by the method of lie group analysis and they found that the boundary layer thickness increases when the non-Newtonian behaviour increases.

Due to the non-linear dependence, the analysis of the behaviour of the non-Newtonian power law fluids tends to be more complicated and subtle in comparison with that of the Newtonian fluids. In general, the equations of motion for non-Newtonian fluids are of higher order than the Navier–Stokes equations and thus, one needs some conditions in addition to the usual adherence boundary condition. Hence, there is a need for a method which provides means of obtaining other conditions necessary for the solution. One of such methods is the Runge–Kutta shooting method. In addition, to the best of our knowledge the combined effects of the suction, chemical reaction, thermo diffusion and thermal diffusion on the convection heat and mass transfer flow in a power law fluid with a non constant relaxation time have not been studied.

The relations between the fluxes and the driven potential are important when heat and mass transfer occur simultaneously in a moving fluid. It has been found that an energy flux can be generated not only by temperature gradients but by composition gradient as well. The energy caused by a composition gradient is called the Dufour or the diffusion-thermo effect, also the mass fluxes can also be created by the temperature gradients and this is called the Soret or thermal diffusion effect.

This paper examines the combined effects of the suction, chemical reaction, thermo diffusion and thermal diffusion on the convection heat and mass transfer flow in a power law fluid with a non constant relaxation time past a vertical porous plate. The results are presented as velocity, temperature and concentration profiles for pseudo plastic fluid and for different values of parameters entering into the problem. The skin friction, rate of heat transfer and mass transfer are presented numerically in tabular form.

2. Mathematical formulation

Consider an unsteady natural convection flow of a non-Newtonian power law fluid past a moving porous plate with non constant relaxation time. Let the x-axis be taken along the plate in the vertically upward direction and the y-axis be taken normal to it. Let, u and v be the velocity component along the x and y directions, respectively. If x-axis is chosen along the plate and y-axis perpendicular to it, then the investigated flow does not depends on x. Hence, the continuity equation becomes:

$$\frac{\partial v}{\partial y} = 0, \tag{1}$$

i.e the velocity component in y-direction, v is a function of time only equalled by the suction velocity,

$$v = -\frac{c}{At^{\frac{n}{n+1}}}, (2)$$

(c, is the suction parameter). The surface is maintained at a constant temperature T_w which is higher than the constant temperature T_∞ of the surrounding and concentration C_w is greater than the constant concentration C_∞ The fluid properties are assumed to be constant. The governing equations of continuity since the plate is vertically upward, momentum (see [11]), energy and concentration for the unsteady flow can be written as:

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} + \lambda t^b \frac{\partial^2 u}{\partial t^2} = -\mu \frac{\partial}{\partial y}\left(-\frac{\partial u}{\partial y}\right)^n + g\beta(T - T_\infty) + g\lambda^*(C - C_\infty), \tag{3}$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{Q(T - T_\infty)}{\rho c} + \frac{D_m K_T}{\rho c}\frac{\partial^2 C}{\partial y^2}, \tag{4}$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y} + \frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial y^2} - R(C - C_\infty). \tag{5}$$

Where, the non-constant relaxation time is given by $\lambda t^b \frac{\partial^2 u}{\partial t^2}$, When λ is zero, the fluid is pseudo plastic and when the power law exponent is unity ($n = 1$) with $t = 1$, the fluid is Maxwell one.

And the last terms on the right-hand side of the concentration equation (4) and (5) signify the thermo diffusion and thermal diffusion effects, respectively. T is the temperature, C is the fluid concentration, D_m is the coefficient of mass diffusivity, α is the thermal diffusivity, k is the thermal conductivity, $\mu = \frac{m}{\rho}$, m is the flow consistency coefficient, ρ is the density, $K_T$is the thermal diffusion ratio, T_m is the mean fluid temperature, β is the volumetric expansion – coefficient due to temperature, λ^* is the volumetric expansion – coefficient due to concentration, g is the acceleration due to gravity, C_∞ is the uniform concentration of the fluid far away from the plate, T_∞ is the uniform temperature of the fluid far away from the plate, C_w is the uniform concentration of the fluid at the plate surface, T_w is the uniform temperature of the fluid at the plate, Q is the heat generation constant, λ is the coefficient of the relaxation time.

The appropriate boundary conditions are:

$$u = U, \quad T = T_w, \quad C = C_w \ \text{ at } \ y = 0, \tag{6}$$

$$u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \ \text{ as } \ y \to \infty, \ t > 0. \tag{7}$$

Where U (at the time $t = 0$ the plate is impulsively set into motion with the velocity U) is the plate characteristics velocity.

3 Method of solution

Introducing a dimensionless similarity variable,

$$\eta = \frac{Ay}{t^{\frac{1}{n+1}}}. \tag{8}$$

Where A is constant and t is the time, such that,

$$u = Uf(\eta), \tag{9}$$

and define the dimensionless quantities,

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad R_n = \frac{R(n+1)}{\mu t^{\frac{1-n}{n+1}}}, \quad Sc_n = \frac{\mu t^{\frac{1-n}{1+n}}}{(n+1)DA^2}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty},$$

$$H_n = \frac{Q(n+1)t}{\rho c}, \quad Sr_n = \frac{D_m K_T (n+1) A^2 (T_w - T_\infty)}{T_m \upsilon (C_w - C_\infty) t^{\frac{1-n}{n+1}}},$$

$$Dn = \frac{D_m K_T (n+1) A^2 (C_w - C_\infty)}{\rho c (T_w - T_\infty) t^{\frac{1-n}{n+1}}}, \quad Pr_n = \frac{\rho c \mu t^{\frac{1-n}{n+1}}}{(n+1)kA}, \tag{10}$$

$$Gr_n = \frac{g\beta (n+1)^2 t (T_w - T_\infty)}{U}, \quad Gc_n = \frac{g\lambda^* t (n+1)^2 (C_w - C_\infty)}{U},$$

Using (9)–(11) in equations (4)–(6), the momentum, energy and concentration equations become:

$$\mu n (n+1)^2 A^{n+1} U^{n-1} (-f')^{n-1} f'' + c(n+1)^2 f' - (2\lambda + \lambda n - n - 1)\eta f' + \lambda \eta^2 f'' - Gr_n \theta - Gc_n \phi = 0 \tag{11}$$

$$\mu \theta'' + Pr_n H_n \theta + Pr_n (\eta + c(n+1))\theta' + D_n Pr_n \phi'' = 0, \tag{12}$$

$$\mu \phi'' + Sc_n (\eta + c(n+1))\phi' - R_n Sc_n \phi + Sr_n Sc_n \theta'' = 0. \tag{13}$$

The new boundary conditions are:

$$f(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1, \quad f(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0. \tag{14}$$

Where, Gr_n is the thermal Grashof number, Gc_n is the solutal(????) Grashof number, Pr_n is the Prandlt number, Sc_n Schmidt number, H_n is the heat generation parameter, D_n is the Dufour number, Sr_n is the Soret number, R_n is the chemical reaction parameter, and the prime symbol denotes derivative with respect to η.

4. Numerical solution

The resulting coupled non linear ordinary differential equations (11), (12) and (13) together with the boundary conditions (14) are solved simultaneously using the method of the Runge-Kutta shooting method;

Let $x_1 = \eta$, $x_2 = f$, $x_3 = f'$, $x_4 = \theta$, $x_5 = \theta'$, $x_6 = \phi$ and $x_7 = \phi'$. Then we obtain the following system;

(15)

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \\ x_5' \\ x_6' \\ x_7' \end{pmatrix} = \begin{pmatrix} 1 \\ x_3 \\ \dfrac{Gr_n x_4 + Gc_n x_6 - c(n+1)^2 x_3 + (2\lambda + \lambda n - n - 1) x_1 x_3}{\upsilon n (n+1)^2 A^{n+1} U^{n-1} (-x_3)^{n-1} - \lambda x_1^2} \\ x_5 \\ -\dfrac{H_n Pr_n x_4}{\mu} - \dfrac{Pr_n (x_1 + c(n+1)) x_5}{\mu} - \dfrac{D_n Pr_n}{\mu} \left(\dfrac{-Sc_n Sr_n H_n Pr_n x_4 + Sc_n Sr_n Pr_n (x_1 + c(n+1)) x_5 + \mu Sc_n R_n x_6 - Sc_n \mu x_7 (x_1 + c(n+1))}{\mu - D_n Sc_n Sr_n Pr_n} \right) \\ x_7 \\ \dfrac{-Sc_n Sr_n H_n Pr_n x_4 + Sc_n Sr_n Pr_n (x_1 + c(n+1)) x_5 + \mu Sc_n R_n x_6 - Sc_n \mu x_7 (x_1 + c(n+1))}{\mu - D_n Sc_n Sr_n Pr_n} \end{pmatrix}$$

with the initial conditions;

$$\begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \\ x_5(0) \\ x_6(0) \\ x_7(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ \gamma \\ 1 \\ \Omega \\ 1 \\ \Gamma \end{pmatrix}. \tag{16}$$

Equation (15) together with the initial condition (16) is solved using Runge – Kutta shooting method. The values of γ, Ω and Γ are obtained such that the boundary conditions (14) are satisfied.

In a shooting method, the missing (unspecified) initial condition at the initial point of the interval is assumed, and the differential equation is then integrated numerically as an initial valued problem to the terminal point. The accuracy of the assumed missing initial condition is then checked by comparing the calculated value of the dependent variable at the terminal point with its given value there. If a difference exists, another value of the missing initial condition must be assumed and the process is repeated. This process is continued until the agreement between the calculated and the given condition at the terminal point is within the specified degree of accuracy.

In fact, the essence of this method is to reduce the boundary value problem to an initial value problem and then solved using the fourth order Runge–Kutta shooting technique to find $f'(0) = \gamma$, $\theta'(0) = \Omega$ and $\phi'(0) = \Gamma$. It is observed from (14) that the velocity, temperature and concentration decrease with increase in the value of η. Theoretically, the entry length for the fluid flow is given as $\eta \in [0, \infty]$, but it can be assumed that the flow length has a theoretical maximum. Using this approximation, the entry length of the fluid flow is taken as $\eta \in [0, 1]$.

The numerical results are presented graphically in Figs 1–13.

5. Skin friction, rate of heat and mass transfer

We will now calculate the physical quantities of engineering primary interests, which are the local wall shear stress, local surface heat flux and the local mass flux respectively from the following definitions:

$$\tau_w = \left(m \left(-\frac{\partial u}{\partial y} \right)^n \right)_{y=0}, \tag{17}$$

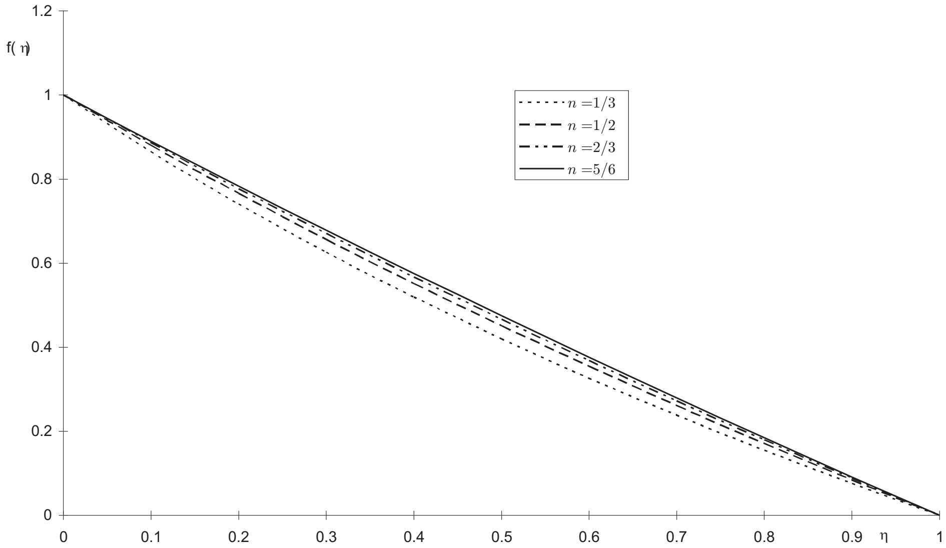

Fig. 1. Velocity profile for various values of the power law exponents, n

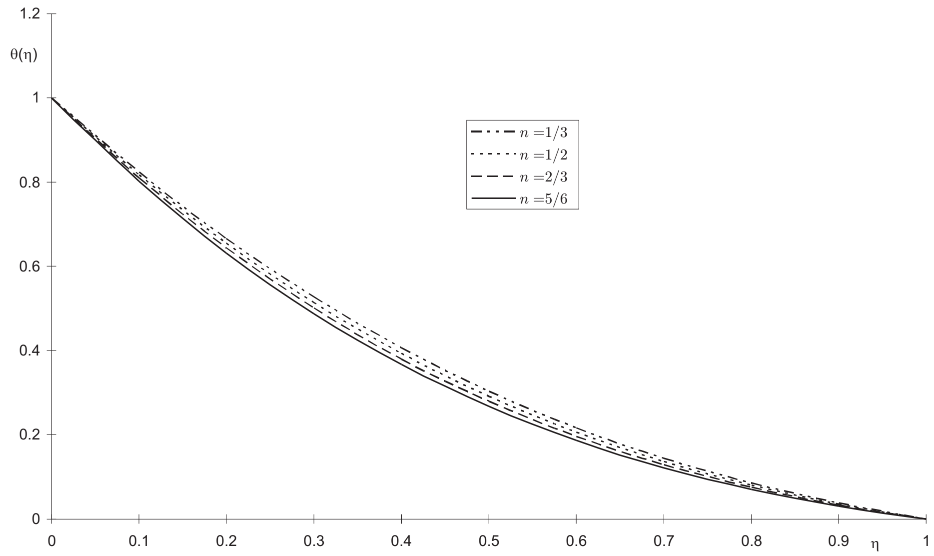

Fig. 2. Temperature profile for various values of the power law exponents, n

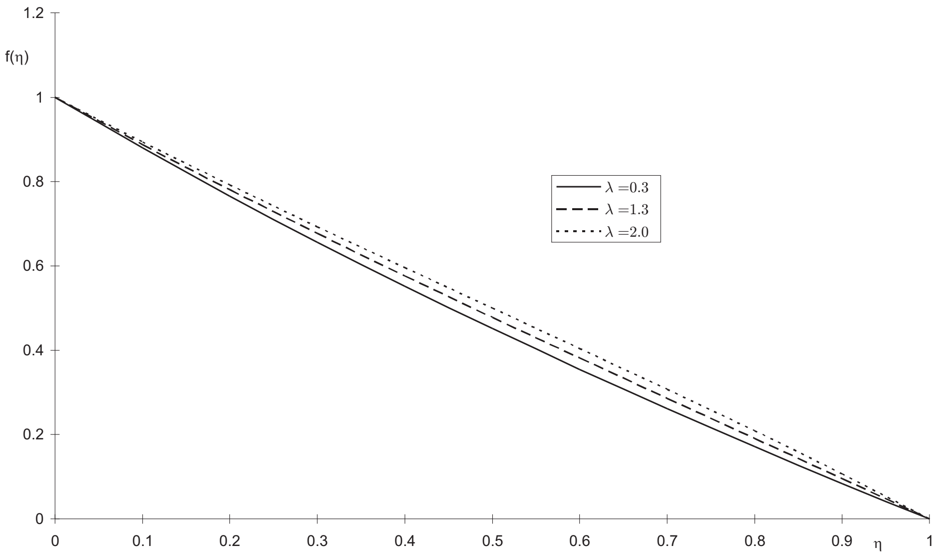

Fig. 3. Velocity profile for different values of the relaxation time parameter

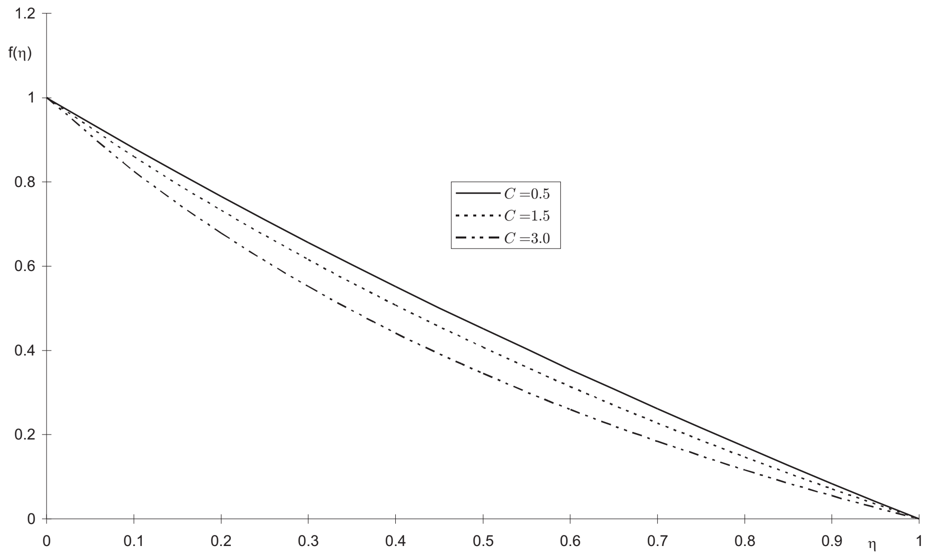

Fig. 4. Velocity profile for the different values of the suction parameter, C

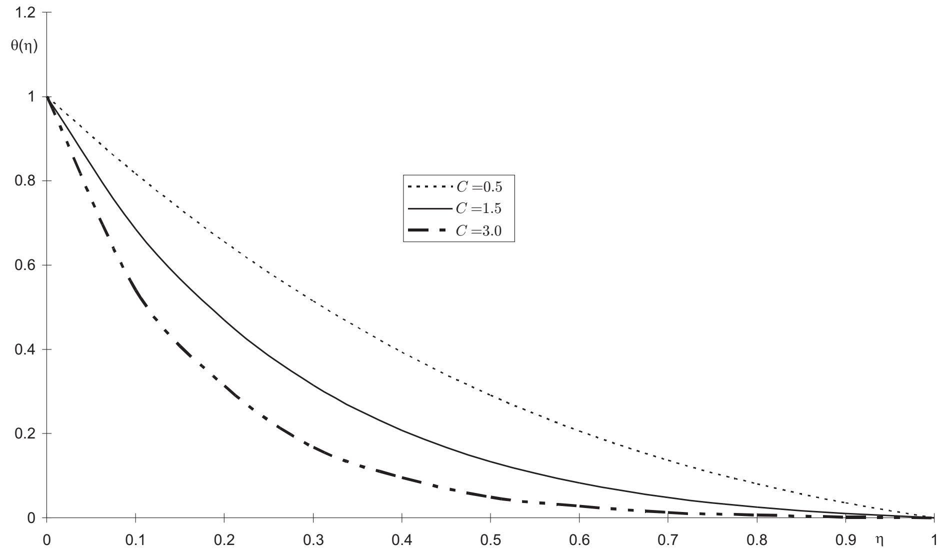

Fig. 5. Temperature profile for various values of the suction parameter, C

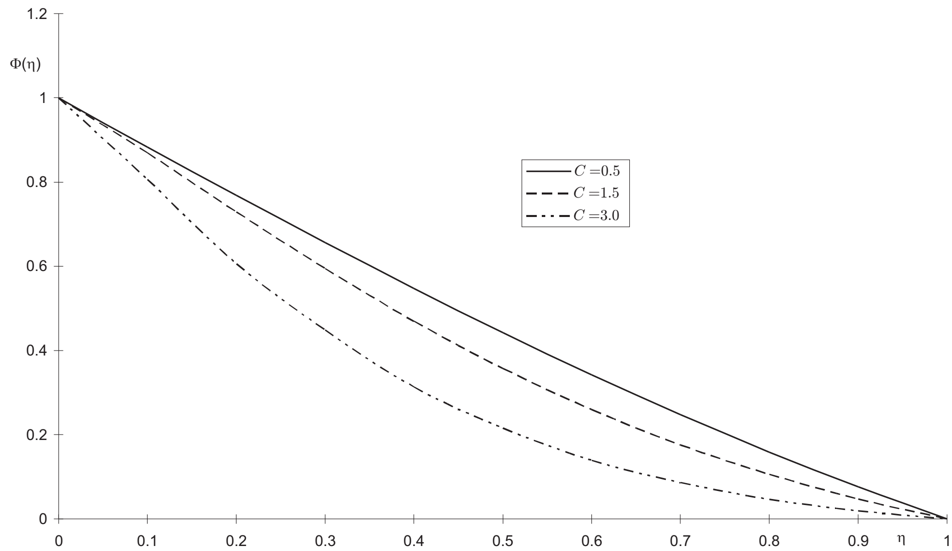

Fig. 6. Concentration profile for different values of the suction parameter, C

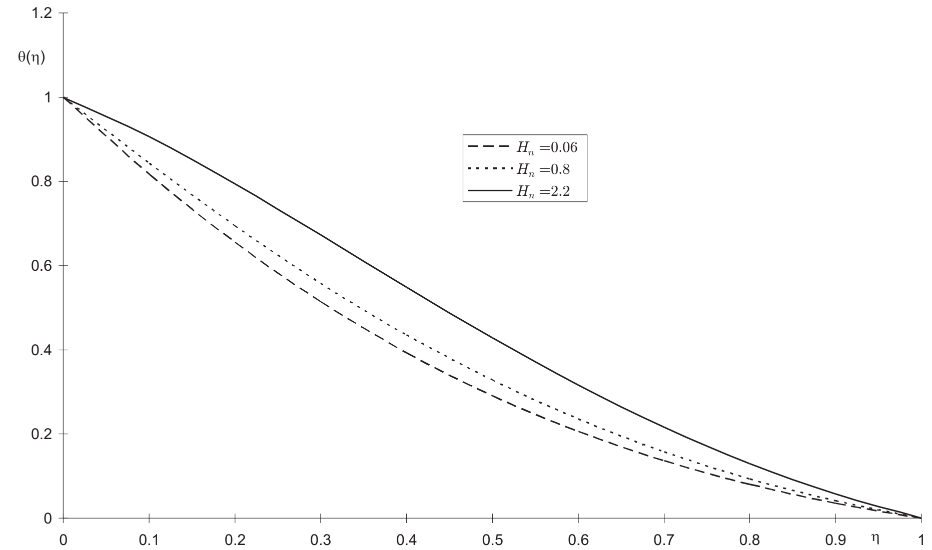

Fig. 7. Temperature profile for different values of the Heat generation parameter, H_n

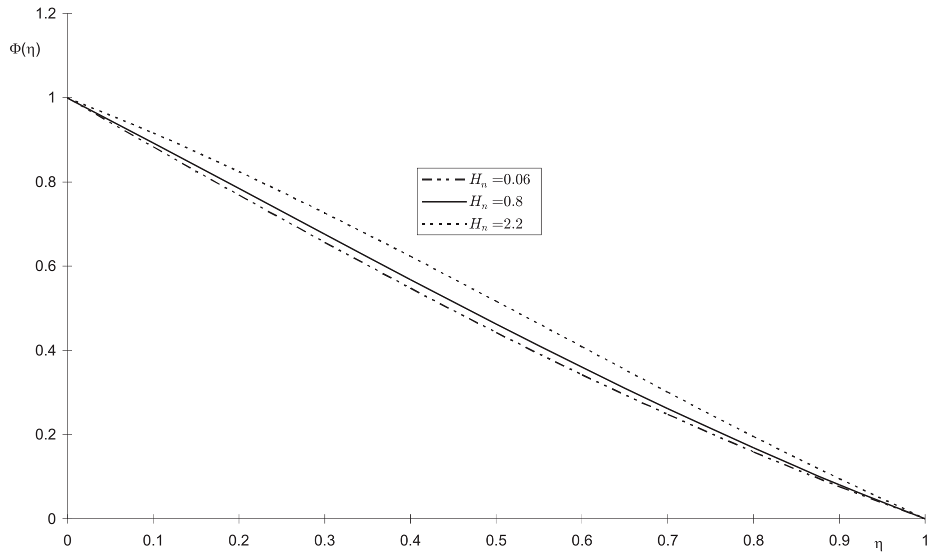

Fig. 8. Concentration profile for different values of the Heat generation parameter, H_n

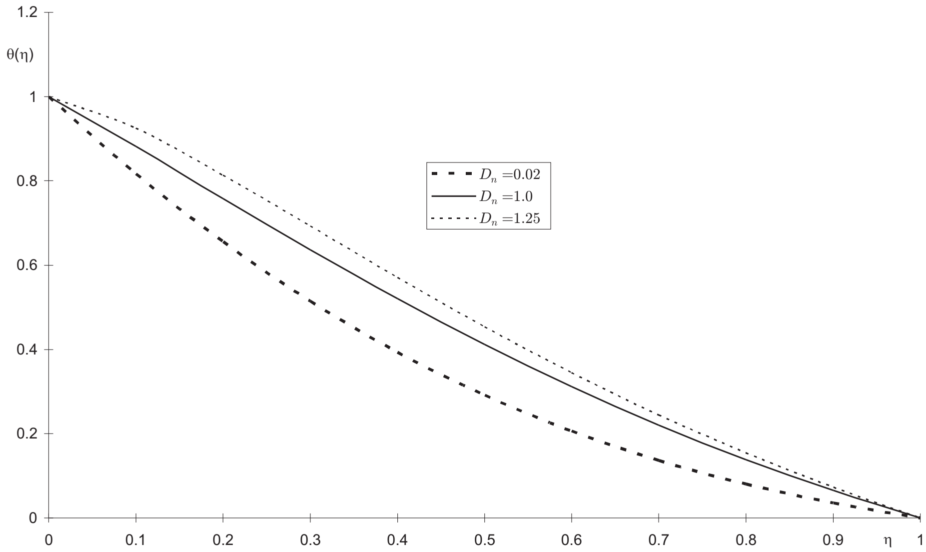

Fig. 9. Temperature profile for different values of the Dufour number, D_n

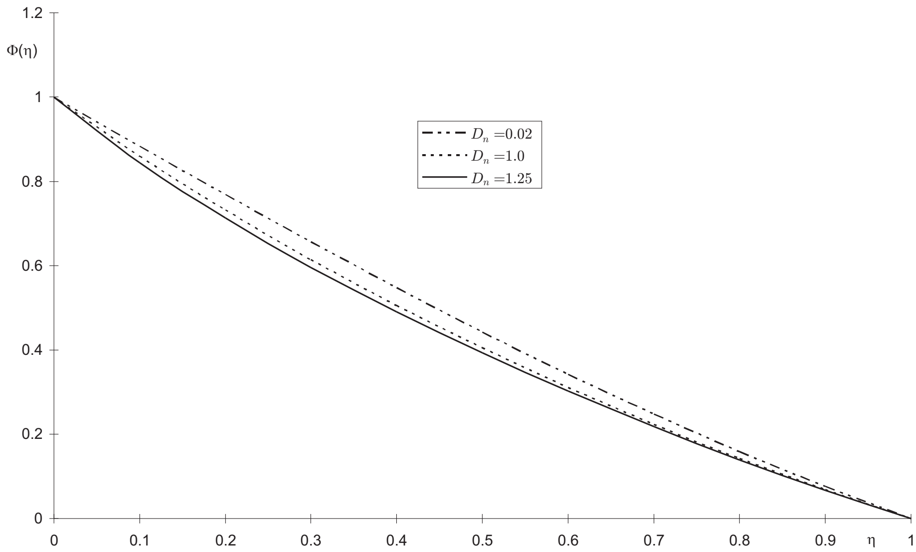

Fig. 10. Concentration profile for different values of the Dufour number, D_n

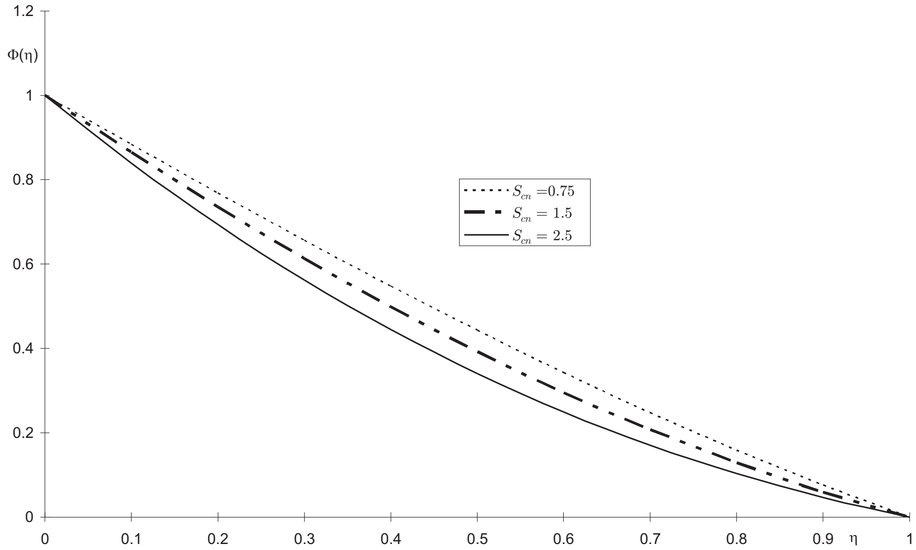

Fig. 11. Concentration profile for different values of the Schmidt number, S_{cn}

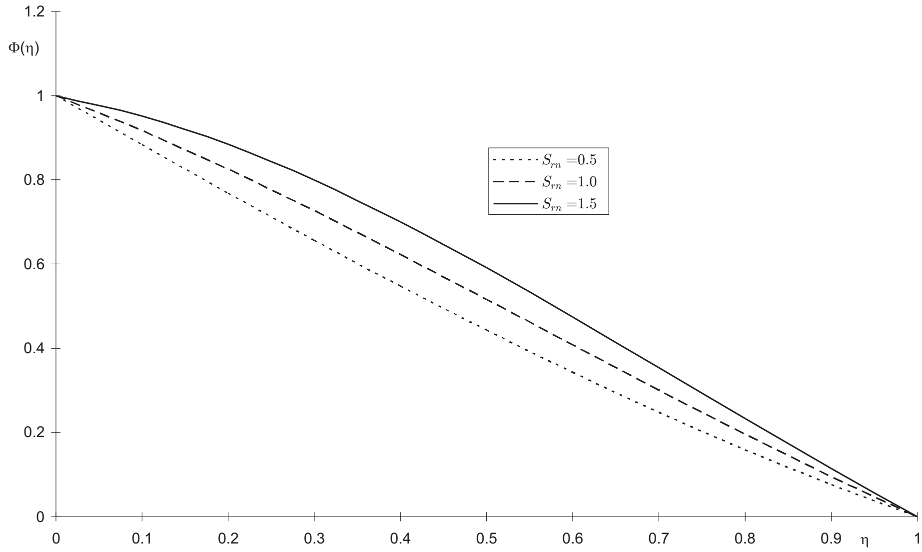

Fig. 12. Concentration profile for different values of the Soret number, S_{rn}

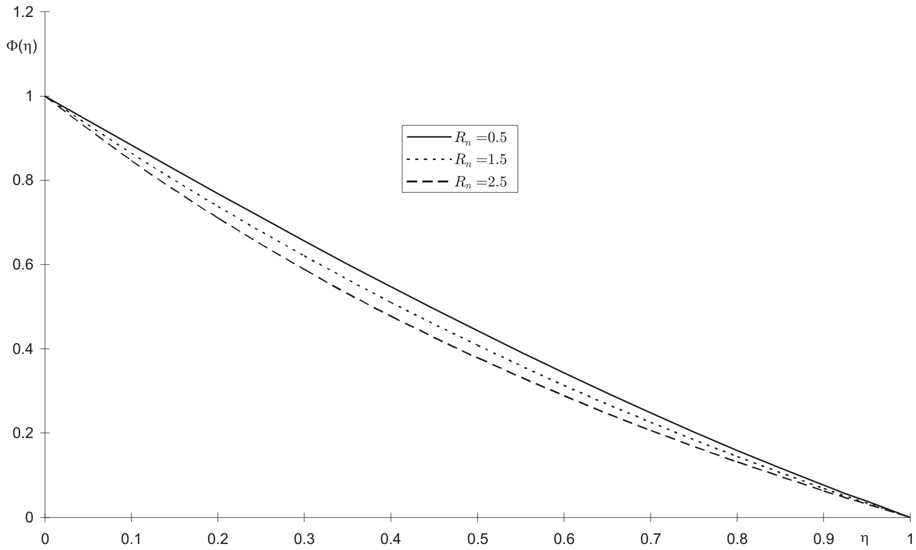

Fig. 13. Concentration for various values of Chemical reaction parameter, R_n

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{18}$$

$$M_w = -D_m\left(\frac{\partial C}{\partial y}\right)_{y=0}. \tag{19}$$

Where m is the flow index, k is the thermal conductivity and D_m is the diffusivity. And the dimensionless skin friction coefficient C_f, Nusselt number, Nu, and the Sherwood number, Sh, are given by:

$$\frac{1}{2}C_f Re_n = \left(-f'(0)\right)^n, \tag{20}$$

$$Nu = \frac{t^{\frac{1}{n+1}} q_w}{k(T_w - T_\infty)} = -\theta'(0), \tag{21}$$

$$Sh = \frac{t^{\frac{1}{n+1}} M_w}{D_m(C_w - C_\infty)} = -\phi'(0). \tag{22}$$

Where,

$$Re_n = \frac{t^{-\frac{n}{n+1}} \rho A^n U^{n-1}}{m}, \tag{23}$$

is the generalized Reynolds number for power law fluid.

These dimensionless values of the local skin friction coefficient, local Nusselt number and local Sherwood number are obtained from the process of numerical computations and are presented in Tables 1–8.

6. Discussion of result

The combined effect of the suction, thermo diffusion, thermal diffusion, chemical reaction on the convection heat and mass transfer in power law fluid with non constant relaxation time has been studied for a pseudo plastic power law fluid, ($n < 1$). The numerical computation was obtained for values of the power law exponents, relaxation, suction, heat generation, thermo diffusion, (Dufour number), thermal diffusion (Soret number) and chemical reaction parameters and Schimdt number; $n = 0.6$, $c = 0.5$, $G = 0.9$, $H_n = 0.06$, $Pr_n = 0.75$, $D_n = 0.02$, $Sc_n = 0.75$, $Sr_n = 0.5$, $R_n = 0.5$.

These values are then varied to observe their effects on the heat and mass transfer problem. The numerical result is presented as the temperature profiles 1–13 and Tables 1–8.

Figure 1 shows the velocity profile for different values of the power law exponents, it is clear from the figure that the velocity of the fluid flow increases as the power law exponent increases. This shows that the force which tends to oppose the fluid flow reduces as the power law exponent increases. Figure 2 depicts the temperature profile for various values of the power law exponents, the temperature of the fluid increases with the power law exponent. Figure 3 shows that the velocity of the fluid flow increases with the coefficient of the relaxation time.

Figures 4, 5 and 6 show the effect of the suction parameter on the velocity, temperature and concentration profiles. It is clearly shown that the velocity of the fluid flow decreases with increase in the suction parameter, the temperature and concentration of the fluid increase with increase in the suction parameter. Figures 7 and 8 show that temperature and concentration of the fluid decrease with increase in the heat generation parameter.

Figures 9 and 10 show respectively that the lower the value of the thermo diffusion parameter, the higher the temperature of the fluid and the higher the value of the thermo diffusion parameter, the higher the concentration of the fluid.

Figures 11, 12 and 13 clearly show that the concentration of the fluid increases with increase in the Schmidt number, thermal diffusion parameter and the chemical reaction parameter respectively. Tables 1 – 8 present the numerical results for the Nusselt number, Sherwood number and Skin friction.

Table 1. Numerical result for different power law exponents

n	Nu	Sh	$\frac{1}{2}C_f Re_n$
$\frac{1}{3}$	1.8474	1.1763	1.1243
$\frac{1}{2}$	1.9357	1.1782	1.1100
$\frac{2}{3}$	2.0260	1.1797	1.0995
$\frac{5}{6}$	2.1180	1.1809	1.0918

Table 2. Effect of the Relaxation parameter

λ	Nu	Sh	$\frac{1}{2}C_f Re_n$
0.3	1.9357	1.1782	1.1100
0.8	1.9357	1.1782	1.0924
1.3	1.9357	1.1782	1.0737
1.8	1.9357	1.1782	1.0537
2.0	1.9357	1.1782	1.0453

Table 1 shows that the skin friction decreases with increase in the power law exponents, while the Nusselt number and Sherwood number increase with increase in the power law exponent. Thus, with increase in the power law exponent of the pseudo plastic power law fluid, the rate of heat and mass transfer increase.

From Table 2, the skin friction decreases with increase in the coefficient of the relaxation time. The rate of heat and mass transfer are not affected by the coefficient of the relaxation.

Table 3 shows that the skin friction, the Sherwood number and the Nusselt number increase with increase in suction parameter. Hence, rate of heat and mass transfer increases with increase in the suction parameter.

Table 4 shows that the skin friction increases with increases in the heat generation parameter, while the Nusselt and Sherwood numbers decrease with

Table 3. Effect of the Suction parameter

c	Nu	Sh	$\frac{1}{2}C_f Re_n$
0.5	1.9357	1.1782	1.1100
1.5	3.7793	1.1824	1.2145
2.0	4.8100	1.1966	1.2709
3.0	6.9585	1.2882	1.3914

Table 4. Effect of the Heat generation parameter

H_n	Nu	Sh	$\frac{1}{2}C_f Re_n$
0.06	1.9357	1.1782	1.1100
0.3	1.8330	1.1429	1.1105
0.8	1.6040	1.0649	1.1119
1.5	1.2429	0.9430	1.1142
2.2	0.8182	0.8012	1.1170

Table 5. Effect of the thermo diffusion parameter (Dufour number)

D_n	Nu	Sh	$\frac{1}{2}C_f Re_n$
0.02	1.9357	1.1782	1.1100
0.5	1.7034	1.2674	1.1109
1.0	1.1186	1.4882	1.1125
1.25	0.4009	1.7568	1.1138

increases in the heat generation parameter. Thus, rate of heat and mass transfer decreases with increase in the heat generation parameter.

Table 5 shows that with increase in the thermo diffusion parameter the skin friction and the Sherwood number increase, but the Nusselt number decreases with increase in the thermo diffusion parameter. Therefore, increase in the thermo diffusion parameter results in decrease in the rate of heat and increase in rate of mass transfer.

Table 6. Effect of the Schmidt number

Sc_n	Nu	Sh	$\frac{1}{2}C_f Re_n$
0.75	1.9357	1.1782	1.1100
1.5	1.9285	1.3874	1.1086
2.0	1.9236	1.5380	1.1077
2.5	1.9184	1.6945	1.1070

Table 7. Effect of the thermal diffusion parameter (Soret number)

Sr_n	Nu	Sh	$\frac{1}{2}C_f Re_n$
0.25	1.9293	1.3717	1.1089
0.5	1.9357	1.1782	1.1100
1.0	1.9489	0.7838	1.1121
1.5	1.9622	0.3792	1.1144

Table 8. Effect of the Chemical reaction

R_n	Nu	Sh	$\frac{1}{2}C_f Re_n$
0.5	1.9357	1.1782	1.1100
1.0	1.9318	1.2998	1.1094
1.5	1.9281	1.4159	1.1088
2.0	1.9245	1.5271	1.1083
2.5	1.9211	1.6339	1.1079

It is clear from Table 6 that the skin friction and the Nusselt number decrease with increase the Schmidt number, while the Sherwood number increases with increase the Schmidt number. And this shows that the rate of heat transfer decreases with increase the Schmidt number and the rate of mass transfer increases with increase the Schmidt number.

In Table 7, the skin friction and the Nusselt number increase with increase in the thermal diffusion parameter, while the Sherwood decreases with increase in the thermal diffusion parameter. Hence, the rate of heat transfer increases with the thermal diffusion and the rate of mass transfer decreases with increase in the thermal diffusion parameter.

Table 8 shows that the skin friction and the Nusselt number decrease as the chemical reaction parameter increases, but the Sherwood number increases with the chemical reaction parameter. Thus, the rate of heat transfer decreases with the increase chemical reaction parameter and the rate of mass transfer increases with increase in the chemical reaction parameter.

7. Conclusions

In this paper, the combined effects of the suction, chemical reaction, thermo diffusion and thermal diffusion on the convection heat and mass transfer flow in a power law fluid with a non constant relaxation time past a vertical porous plate have been investigated. And the results of the investigation reveal that:

- With increase in the power law exponent of the pseudo plastic power law fluid, the rate of heat and mass transfer increase.
- The rate of heat and mass transfer increases with increase in the suction parameter.
- The rate of heat and mass transfer decreases with increase in the heat generation parameter.
- Increase in the thermo diffusion parameter results in decrease in the rate of heat and increase in rate of mass transfer.
- The rate of heat transfer decreases with increase the Schmidt number and the rate of mass transfer increases with increase the Schmidt number.
- The rate of heat transfer increases with the thermal diffusion and the rate of mass transfer decreases with increase in the thermal diffusion parameter.
- The rate of heat transfer decreases with the increase chemical reaction parameter and the rate of mass transfer increases with increase in the chemical reaction parameter.

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