

RAYLEIGH WAVE IN A ROTATING MAGNETO-THERMO-ELASTIC HALF-PLANE*

BALJEET SINGH

*Department of Mathematics, Post Graduate Government College,
Sector 11, Chandigarh -160011, India,
e-mail: bsinghgc11@gmail.com*

SANGEETA KUMARI, JAGDISH SINGH

*Department of Mathematics, Maharishi Dayanand University,
Rohtak-124001, Haryana, India*

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ABSTRACT. The governing equations of generalized magneto-thermoelasticity with hydrostatic initial stress and rotation are solved for surface wave solutions. The particular solutions are applied to the boundary conditions at a free surface of a half-space to obtain the frequency equation of Rayleigh wave. For numerical purpose, the frequency equation is approximated for small thermal coupling. The velocity of propagation and amplitude-attenuation factor of Rayleigh wave are computed numerically for a particular material. Effects of rotation, magnetic field, hydrostatic initial stress and relaxation time on the velocity of propagation and amplitude-attenuation factor are shown graphically.

KEY WORDS: Generalized thermoelasticity, Rayleigh wave, frequency equation, hydrostatic initial stress, relaxation time, magnetic field, rotation.

1. Introduction

Lord and Shulman [1] and Green and Lindsay [2] developed the generalized thermoelasticity after modifying the classical dynamical coupled theory of thermoelasticity. These theories consider heat propagation as a wave phenomenon rather than a diffusion phenomenon and predict a finite speed of heat propagation. Ignaczak and Ostoja-Starzewski [3] presented the analysis of above two theories in their book on “Thermoelasticity with Finite Wave

*Corresponding author e-mail: bsinghgc11@gmail.com

Speeds". The representative theories on the range of generalized thermoelasticity are reviewed by Hetnarski and Ignaczak [4].

Surface waves in elastic solids were first studied by Lord Rayleigh [5] for an isotropic elastic solid. Thermoelastic Rayleigh waves in semi-infinite isotropic solids are studied by Lockett [6], Deresiewicz [7], Nayfeh and Nemat-Nasser [8], Carroll [9], Agarwal [10], Dawn and Chakraborty [11], and many others with various additional parameters.

Initial stresses in solids have significant influence on the mechanical response of the material from an initially-stressed configuration and have applications in geophysics, engineering structures and in the behaviour of soft biological tissues. Initial stress arises from processes, such as manufacturing or growth, and is present in the absence of applied loads. Montanaro [12] formulated the isotropic thermoelasticity with hydrostatic initial stress. Singh et al. [13], Othman et al. [14], Singh [15], and many others have applied Montanaro [12] theory to study the plane harmonic waves in context of generalized thermoelasticity.

In the present paper, the governing equations given by Montanaro [12] are modified in context of Lord and Shulman and Green and Lindsay theories with uniform magnetic field and rotation. These equations are solved for the surface wave solutions, which satisfy the required boundary conditions at the free surface and we obtain the frequency equation for the Rayleigh wave in the half-space. The frequency equation is approximated and analysed numerically to observe the effects of hydrostatic initial stress, magnetic field, rotation and relaxation time on the velocity of propagation and amplitude -attenuation factor.

2. Basic equations

We consider a homogenous, isotropic thermoelastic half space with hydrostatic initial stress initially at uniform temperature T_0 . We take origin of the co-ordinate system (x, y, z) at any point on the plane horizontal surface and z -axis pointing vertically downward into the half-space, which is represented by $z \geq 0$. The surface $z = 0$ is assumed to be subjected to stress free, thermally insulated or isothermal boundary conditions. We choose x -axis in the direction of wave propagation so that all particles on a line parallel to y -axis and equally displaced. Therefore, all the field quantities will be independent of y -coordinate. The medium is assumed to be rotating at constant rate with constant angular velocity $\vec{\Omega} = (0, \Omega, 0)$ about y -axis and under constant primary magnetic field \vec{H}_0 acting on y -axis. Following Lord and Shulman [1],

Green and Lindsay [2], Montanaro [12], Schoenberg and Censor [16] and Roy Choudhari and Banerjee [17], the basic governing field equations for rotating magneto-thermoelastic solid with hydrostatic initial stress in the absence of body forces and heat sources are given by:

(i) The stress-strain-temperature relation:

$$(1) \quad \sigma_{ij} = -p(\delta_{ij} + \omega_{ij}) + \bar{\lambda}e_{pp}\delta_{ij} + 2\bar{\mu}e_{ij} - \frac{\alpha}{\kappa_T}(T + a\dot{T})\delta_{ij},$$

(ii) The displacement-strain relation:

$$(2) \quad e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),$$

(iii) The small rotation-displacement relation:

$$(3) \quad \omega_{ij} = \frac{1}{2}(u_{j,i} - u_{i,j}),$$

(iv) The modified Fourier's law:

$$(4) \quad h_i + a^* \dot{h}_i = K \frac{\partial T}{\partial x_i},$$

(v) The equation of motion:

$$(5) \quad \rho_o[\ddot{u}_i + (\vec{\Omega} \times \vec{\Omega} \times \vec{u})_i + (2\vec{\Omega} \times \dot{\vec{u}})_i] \\ = \left(\bar{\mu} - \frac{p}{2}\right) \frac{\partial^2 u_i}{\partial x_p \partial x_p} + \left(\bar{\lambda} + \bar{\mu} + \frac{p}{2}\right) \frac{\partial^2 u_p}{\partial x_i \partial x_p} - \frac{\alpha}{\kappa_T} \frac{\partial(T + a\dot{T})}{\partial x_i} + \bar{\sigma}_{ip,p},$$

(vi) The equation of heat conduction:

$$(6) \quad \rho_o c_v (1 + a^* \frac{\partial}{\partial t}) \frac{\partial T}{\partial t} + T_o (1 + \Delta a^* \frac{\partial}{\partial t}) \frac{\alpha}{\kappa_T} \frac{\partial^2 u_p}{\partial t \partial x_p} = K \frac{\partial^2 T}{\partial x_p \partial x_p},$$

(vii) Maxwell equation governing the electromagnetic field:

$$(7) \quad \nabla \times \vec{h} = \vec{j}, \quad \nabla \times \vec{E} = -\mu_e \frac{\partial \vec{h}}{\partial t}, \quad \nabla \cdot \vec{h} = 0, \quad \nabla \cdot \vec{E} = 0,$$

(viii) Maxwell stresses:

$$(8) \quad \bar{\sigma}_{ij} = \mu_e [H_i h_j + H_j h_i - (\vec{H} \cdot \vec{h}) \delta_{ij}],$$

where $T = \Theta - T_0$ is small temperature increment, Θ is the absolute temperature of the medium, T_0 is the reference uniform temperature of the body chosen such that $|T/T_0| \ll 1$, ρ_0 is the mass density, q_i is the heat conduction vector, K is the thermal conductivity, c_v is the specific heat at constant strain, $\bar{\lambda}$, $\bar{\mu}$ are the counterparts of Lamé parameters, α is the volume coefficient of thermal expansion, κ_T is the isothermal compressibility, σ_{ij} are the components of the stress tensor, u_i are the components of the displacement vector, e_{ij} are the components of the strain tensor, ω_{ij} are the components of the small rotation tensor, δ_{ij} is the Kronecker delta, a , a^* are the thermal relaxation times, p is the initial pressure, \vec{h} is the perturbed magnetic field over \vec{H}_0 , \vec{j} is the electric current density, μ_e is the magnetic permeability, $\vec{h} = \nabla \times (\vec{u} \times \vec{H}_0)$ and $\vec{H} = \vec{H}_0 + \vec{h}$. The above governing equations reduce for L-S (Lord-Shulman) theory when $a = 0$, $\Delta = 1$ and for G-L (Green-Lindsay) theory, when $\Delta = 0$. The two additional terms $\vec{\Omega} \times (\vec{\Omega} \times \vec{u})$ and $2(\vec{\Omega} \times \dot{\vec{u}})$ on the left hand side of equation (5) represent the time dependent part of the centripetal acceleration and the coriolis acceleration.

3. Problem formulation

For the Rayleigh type waves in the half space $z \geq 0$, using the Helmholtz's representation of displacement components in terms of function φ and ψ of x , z and t :

$$(9) \quad u_1 = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial z}, \quad u_2 = 0, \quad u_3 = \frac{\partial \varphi}{\partial z} + \frac{\partial \psi}{\partial x},$$

in equations (5) and (6), we obtain:

$$(10) \quad \frac{\partial^2 \varphi}{\partial t^2} - \Omega^2 \varphi - 2\Omega \frac{\partial \psi}{\partial t} = c_1^2 \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} \right) - \frac{\gamma}{\rho_0} (T + a\dot{T}),$$

$$(11) \quad \frac{\partial^2 \psi}{\partial t^2} - \Omega^2 \psi + 2\Omega \frac{\partial \varphi}{\partial t} = c_2^2 \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} \right),$$

$$(12) \quad \rho_0 c_v (\dot{T} + a^* \ddot{T}) + \gamma T_0 \left(1 + \Delta a^* \frac{\partial}{\partial t} \right) \frac{\partial}{\partial t} \left[\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} \right] \\ = K \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right),$$

where:

$$c_1^2 = \frac{\bar{\lambda} + 2\bar{\mu} + \mu_e H_0^2}{\rho_0}, \quad c_2^2 = \frac{\bar{\mu} - \frac{\rho}{2}}{\rho_0}, \quad \gamma = \frac{\alpha}{\kappa_T}.$$

Introducing the following non-dimensional quantities:

$$\begin{aligned} x' &= \frac{x}{(c_1/\omega^*)}, & z' &= \frac{z}{(c_1/\omega^*)}, & t' &= t\omega^*, \\ u'_1 &= \frac{u_1}{(c_1/\omega^*)}, & u'_3 &= \frac{u_3}{(c_1/\omega^*)}, & T' &= \frac{\gamma T}{\rho_0 c_1^2}, \\ \varphi' &= \frac{\varphi}{(c_1/\omega^*)^2}, & \psi' &= \frac{\psi}{(c_1/\omega^*)^2}, & a' &= a\omega^*, \quad a^{*'} = a^*\omega^*, \end{aligned}$$

where:

$$\omega^* = \frac{\rho_0 c_v c_1^2}{K},$$

in equations (9)–(12) and suppressing the primes, we obtain the equations in dimensionless form:

$$(13) \quad u_1 = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial z}, \quad u_2 = 0, \quad u_3 = \frac{\partial \varphi}{\partial z} + \frac{\partial \psi}{\partial x},$$

$$(14) \quad \frac{\partial^2 \varphi}{\partial t^2} - \Omega^2 - 2\Omega \frac{\partial \psi}{\partial t} = \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} \right) - (T + a\dot{T}),$$

$$(15) \quad \frac{\partial^2 \psi}{\partial t^2} - \Omega^2 + 2\Omega \frac{\partial \varphi}{\partial t} = \frac{1}{v^2} \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} \right),$$

$$(16) \quad (\dot{T} + a^*\ddot{T}) + \varepsilon \left(1 + \Delta a^* \frac{\partial}{\partial t} \right) \frac{\partial}{\partial t} \left[\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} \right] = \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right),$$

where the thermoelastic coupling constant ε is given by

$$(17) \quad \varepsilon = \frac{\gamma^2 T_0}{\rho_0^2 c_v c_1^2} \quad \text{and} \quad v^2 = \frac{c_1^2}{c_2^2}.$$

4. Frequency equation and solutions

For thermoelastic surface waves in the half-space propagating in x -direction, the functions T , φ and ψ are taken in the following form:

$$(18) \quad \{T, \varphi, \psi\} = \{\hat{T}(z), \hat{\varphi}(z), \hat{\psi}(z)\} \exp i(\eta x - \chi t),$$

where η is the wave number and χ is the frequency.

Substituting equation (18) in the equations (14)–(16), we obtain:

$$(19) \quad \left[-\chi^2 - \Omega^2 + \frac{1}{v^2}(\eta^2 - D^2) \right] \hat{\psi}(z) = 2i\chi \Omega \hat{\varphi}(z),$$

$$(20) \quad \hat{T}(z) = \frac{1}{(1 - ia\chi)} [\chi^2 + \Omega^2 - \eta^2 - D^2] \hat{\varphi}(z) - 2i\chi \Omega \hat{\psi}(z),$$

$$(21) \quad (-i\chi - a^*\chi^2 + \eta^2 - D^2)\hat{T}(z) + i\chi\varepsilon(1 - i\Delta a^*\chi)(-\eta^2 + D^2)]\hat{\varphi}(z) = 0,$$

From equations (19) to (21), we have:

$$(22) \quad (D^6 - AD^4 + BD^2 - C)(\hat{\varphi}(z), \hat{\psi}(z), \hat{T}(z)) = 0,$$

where $D = \frac{d}{dz}$ and,

$$A = [-i\chi\varepsilon(1 - i\Delta a^*\chi)(1 - ia\chi) + (-i\chi - a^*\chi^2 + \eta^2) \\ - (\chi^2 + \Omega^2 - \eta^2) - (v^2(\chi^2 + \Omega^2) - \eta^2)],$$

$$B = -i\chi\varepsilon(1 - i\Delta a^*\chi)(1 - ia\chi)[v^2(-\chi^2 - \Omega^2) + 2\eta^2] - 4\chi^2 v^2 \Omega^2 \\ + (-\chi^2 - \Omega^2 + \eta^2)(v^2(-\chi^2 - \Omega^2) + \eta^2) \\ + (-i\chi - a^*\chi^2 + \eta^2)[(v^2(-\chi^2 - \Omega^2) + \eta^2) + (-\chi^2 - \Omega^2 + \eta^2)],$$

$$C = -i\chi\varepsilon(1 - i\Delta a^*\chi)(1 - ia\chi)(v^2(\chi^2 - \Omega^2) + \eta^2)\eta^2 \\ + (-i\chi - a^*\chi^2 + \eta^2)[(-\chi^2 - \Omega^2 + \eta^2)(v^2(-\chi^2 - \Omega^2) + \eta^2) - 4\chi^2 v^2 \Omega^2].$$

A most general solution of the equation (22) is:

$$\begin{aligned}
 \varphi(z) &= \sum_{i=1}^3 [A_i \exp(-m_i z) + A_i^* \exp(m_i z)] \exp i(\eta x - \chi t), \\
 \psi(z) &= \sum_{i=1}^3 [B_i \exp(-m_i z) + B_i^* \exp(m_i z)] \exp i(\eta x - \chi t), \\
 T(z) &= \sum_{i=1}^3 [C_i \exp(-m_i z) + C_i^* \exp(m_i z)] \exp i(\eta x - \chi t),
 \end{aligned}
 \tag{23}$$

where $A_i, B_i, C_i, A_i^*, B_i^*, C_i^*$ are constants and $m_i, (i = 1, 2, 3)$ are the roots of the equation:

$$(m^6 - Am^4 + Bm^2 - C) = 0,
 \tag{24}$$

The equation (24) is a cubic one in m^2 and hence roots are given by m_1^2, m_2^2, m_3^2 such that:

$$m_1^2 + m_2^2 + m_3^2 = A,
 \tag{25}$$

$$m_1^2 m_2^2 + m_2^2 m_3^2 + m_3^2 m_1^2 = B,
 \tag{26}$$

$$m_1^2 m_2^2 m_3^2 = C.
 \tag{27}$$

In general, the roots $m_i, (i = 1, 2, 3)$ are complex and here we are considering surface waves, without loss of generality, we can assume $\text{Re}(m_i) > 0$. We choose only that form of m_i which satisfies the radiation condition.

$$\varphi(z), \quad \psi(z), \quad T(z) \rightarrow 0 \text{ as } z \rightarrow \infty
 \tag{28}$$

Due to the radiation condition (28), we have the following appropriate solution

in the half space $z \geq 0$,

$$(29) \quad \begin{aligned} \varphi(z) &= \sum_{i=1}^3 A_i \exp(-m_i z) \exp i(\eta x - \chi t), \\ \psi(z) &= \sum_{i=1}^3 B_i \exp(-m_i z) \exp i(\eta x - \chi t), \\ T(z) &= \sum_{i=1}^3 C_i \exp(-m_i z) \exp i(\eta x - \chi t), \end{aligned}$$

where $B_i = F_i A_i$, $C_i = F_i^* A_i$, and

$$F_i = \frac{2i\frac{\Omega}{\chi}}{\left(-1 - \frac{\Omega^2}{\chi^2}\right) + \left(1 - \frac{m_i^2}{\eta^2}\right) \frac{1}{c^2 v^2}}, \quad (i = 1, 2, 3)$$

$$F_i^* = \frac{\chi^2}{1 - ia\chi} \left[2i\frac{\Omega}{\chi} F_i + \left(1 + \frac{\Omega^2}{\chi^2}\right) - \frac{1}{c^2} \left(1 - \frac{m_i^2}{\eta^2}\right) \right], \quad (i = 1, 2, 3).$$

5. Boundary Conditions

The required boundary conditions at the free surface $z = 0$ are the vanishing of tangential stress, normal stress and the heat flux or temperature potential:

$$(30) \quad \sigma_{13} + \bar{\sigma}_{13} = 0, \sigma_{33} + \bar{\sigma}_{33} = -p, \frac{\partial T}{\partial z} + hT = 0.$$

With the help of equations (1)–(4) and (13), the above boundary conditions are written in non - dimensional form as:

$$(31) \quad (2 + p_1) \frac{\partial^2 \varphi}{\partial x \partial z} + \frac{\partial^2 \psi}{\partial x^2} - (1 + p_1) \frac{\partial^2 \psi}{\partial z^2} = 0,$$

$$(32) \quad \left(1 - \frac{2}{v^2} - p_2\right) \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} - (T + a\dot{T}) \left(\frac{2}{v^2} + p_2\right) \frac{\partial^2 \psi}{\partial x \partial z} = 0,$$

$$(33) \quad \frac{\partial T}{\partial z} + hT = 0,$$

where $p_1 = \frac{p}{\rho_0 c_2^2}$, $p_2 = \frac{p}{\rho_0 c_1^2}$.

The solution given by the equation (29) satisfies the boundary conditions (31) to (33) and we obtain the following frequency equation:

$$(34) \quad \begin{aligned} & m_1 F_1^*(s_2 r_3 - r_2 s_3) - m_2 F_2^*(s_1 r_3 - r_1 s_3) + m_3 F_3^*(s_1 r_2 - r_1 s_2) \\ & = h[F_1^*(s_2 r_3 - r_2 s_3) - F_2^*(s_1 r_3 - r_1 s_3) + F_3^*(s_1 r_2 - r_1 s_2)], \end{aligned}$$

where

$$s_i = i(2 + p_1)\eta m_i + \eta^2 F_i + (1 + p_1)F_i m_i^2, \quad (i = 1, 2, 3),$$

$$r_i = -\eta^2 \left(1 - \frac{2}{v^2} - p_2\right) + m_i^2 - \left[(1 - ia\chi) + i\left(\frac{2}{v^2} + p_2\right)\eta m_i\right] F_i^*,$$

($i = 1, 2, 3$).

6. Particular cases

(a) For thermally insulated case ($h \rightarrow 0$), the frequency equation (34) reduces to

$$(35) \quad m_1 F_1^*(s_2 r_3 - r_2 s_3) - m_2 F_2^*(s_1 r_3 - r_1 s_3) + m_3 F_3^*(s_1 r_2 - r_1 s_2) = 0,$$

(b) For isothermal case, ($h \rightarrow \infty$), the frequency equation (34) reduces to

$$(36) \quad F_1^*(s_2 r_3 - r_2 s_3) - F_2^*(s_1 r_3 - r_1 s_3) + F_3^*(s_1 r_2 - r_1 s_2) = 0,$$

7. Special cases

(a) In absence of rotation and after manipulation of symbols, the frequency equation (34) reduces to the following equation:

$$(37) \quad \begin{aligned} & [2 + (p_2 - c^2)v^2][2 - c^2v^2 + p_1(1 - c^2v^2)][\beta_1^2 + \beta_2^2 + \beta_1\beta_2 - 1 + c^2] \\ & - [4 + 2p_1 + (2p_2 + p_1p_2)v^2]\beta_1\beta_2\beta_3(\beta_1 + \beta_2) \\ & = -\frac{h}{\eta}[(\beta_1^2 + \beta_2^2)\{2 + (p_2 - c^2)v^2\}\{2 - c^2v^2 + p_1(1 - c^2v^2)\} \\ & \quad - \{4 + 2p_1 + (2p_2 + p_1p_2)v^2\}\beta_3(\beta_1^2\beta_2^2 + 1 - c^2)], \end{aligned}$$

which agrees with the equation (24) of Singh et al. [18].

(b) In absence of rotation, initial stress and magnetic field and after some manipulations of symbols, the frequency equation (34) reduces to the following equation:

$$(38) \quad [2 - c^2 v^2]^2 [\beta_1^2 + \beta_2^2 + \beta_1 \beta_2 - 1 + c^2] - 4\beta_1 \beta_2 \beta_3 (\beta_1 + \beta_2) \\ = -\frac{h}{\eta} [(\beta_1 + \beta_2) \{2 - c^2 v^2\}^2 - 4\beta_3 (\beta_1 \beta_2 + 1 - c^2)],$$

which agrees with the equation (20) of Dawn and Chakraborty [11].

(c) In the absence of rotation, initial stress, magnetic field and thermal parameters, the frequency equation (34) reduces to

$$(39) \quad (2 - c^2 v^2)^2 = 4\sqrt{(1 - c^2)}\sqrt{(1 - c^2 v^2)},$$

which is the frequency equation of Rayleigh wave for an isotropic elastic solid half space.

8. Numerical results and discussion

For small thermal coupling ($\varepsilon \ll 0$), we obtain the following approximated roots from equation (25) to (27) as:

$$(40) \quad \frac{m_1^2}{\eta^2} \approx 1 - \left(a^* + \frac{i}{\chi}\right) c^2,$$

$$(41) \quad \frac{m_2^2}{\eta^2} \approx 1 - \left(1 + \frac{\Omega^2}{\chi^2}\right) c^2,$$

$$(42) \quad \frac{m_3^2}{\eta^2} \approx 1 - \left(1 + \frac{\Omega^2}{\chi^2}\right) c^2 v^2,$$

Following Dawn and Chakraborty [11], we take:

$$(43) \quad c^2 = c^{*2} + \varepsilon(\xi_1 + i\xi_2),$$

where c^* is the classical Rayleigh wave velocity and ξ_1 and ξ_2 are two real values depending on the reduced frequency χ and a , a^* and the values ξ_1 and ξ_2 are computed numerically, when we put the values of c^2 in equation (35). Also, if

we put c^2 given by equation (43) in relation $\eta^2 = \frac{\chi^2}{c^2}$, then for small thermal coupling ($\varepsilon \ll 1$), we obtain the following relation analytically:

$$(44) \quad \eta = \frac{\chi}{c^*} \left(1 - \frac{\varepsilon\xi_1}{2c^{*2}} - i \frac{\varepsilon\xi_2}{2c^{*2}} \right).$$

Using these values of η in the relation $c = \frac{\chi}{\eta}$, we obtain the following relation:

$$(45) \quad c = \left(c^* + \frac{\varepsilon\xi_1}{2c^*} \right) + i \frac{\varepsilon\xi_2}{2c^*}.$$

Then the real part of c in relation (45) is called velocity of propagation and is equal to $c^* + \frac{\varepsilon\xi_1}{2c^*}$. The amplitude-attenuation factor $\exp \left[\frac{\varepsilon\chi\xi_2 x}{2c^{*3}} \right]$ with $\xi_2 < 0$ is obtained by putting η given in the equation (44) in $e^{i\text{Im}(\eta)x}$.

Substituting the value of c^2 given by relation (43) in equation (35) for the thermally insulated case and solving numerically, we obtain the numerical values of the velocity of propagation and amplitude-attenuation for the following physically constants of Aluminium:

$$E = 6.9 \times 10^{10} \text{ N.m}^{-2}, \quad \sigma = 0.33, \quad \rho_0 = 2700 \text{ Kg.m}^{-3},$$

$$c_v = 987.9 \text{ J.Kg}^{-1}.\text{K}^{-1}, \quad K = 205.85 \text{ J.m}^{-1}.\text{s}^{-1}.\text{K}^{-1},$$

$$\alpha = 0.01, \quad \kappa_T = 0.05, \quad \omega = 2\text{s}^{-1}, \quad T_0 = 293 \text{ K},$$

$$x = 0.01\text{m}, \quad a = 0.05 \text{ s}, \quad \varepsilon = 0.05, \quad \mu_e = 1, \quad c^* = 0.9554.$$

The generalized Lamé's constant $\bar{\lambda}$ and $\bar{\mu}$ are related as:

$$(46) \quad \bar{\lambda} = \frac{E\sigma}{\zeta(1+\sigma)(1-2\sigma)}, \quad \bar{\mu} = \frac{E}{2\zeta(1+\sigma)},$$

where ζ is the initial stress parameter, E is Young's modulus and σ is Poisson ratio. $\zeta = 1$ corresponds to the isotropic elastic medium.

With the help of equations (40)–(42), the frequency equation (35) for case of thermally insulated is approximated and the velocity of propagation and amplitude-attenuation factor of Rayleigh wave are computed against magnetic

field H_0 , initial stress parameter p , rotation parameter Ω , relaxation time parameter a^* and frequency χ as shown in Figs 1–5.

The velocity of propagation and amplitude-attenuation are plotted against the magnetic field parameter H_0 in Fig. 1 when $p = 2 \times 10^{10} \text{ N.m}^{-2}$ (compression), $a^* = 0.2 \text{ s}$, $\chi = 0.1$ and $\Omega = 2, 4, 8$. For $\Omega = 2$, the velocity of propagation

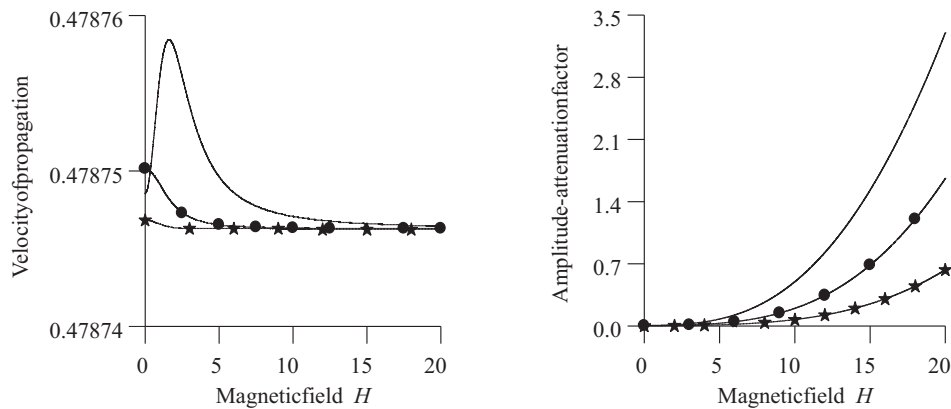


Fig. 1. Variations of velocity of propagation and amplitude-attenuation factor against the magnetic field H_0 . (Solid curve – $\chi = 0.1$, $p = 2$, $\Omega = 2$, $a^* = 0.2$; Solid curve with circle – $\chi = 0.1$, $p = 2$, $\Omega = 4$, $a^* = 0.2$; Solid curve with star – $\chi = 0.1$, $p = 2$, $\Omega = 8$, $a^* = 0.2$)

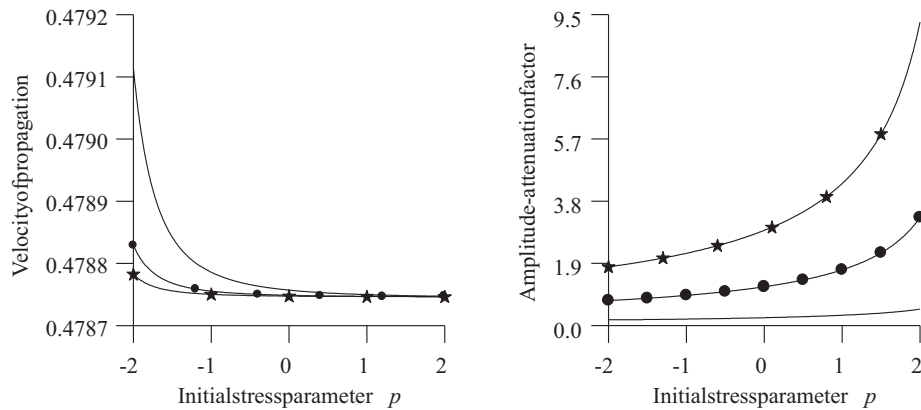


Fig. 2. Variations of velocity of propagation and amplitude-attenuation factor against the initial stress parameter p . (Solid curve – $\chi = 0.1$, $H_0 = 10$, $\Omega = 2$, $a^* = 0.2$; Solid curve with circle – $\chi = 0.1$, $H_0 = 20$, $\Omega = 2$, $a^* = 0.2$; Solid curve with star – $\chi = 0.1$, $H_0 = 30$, $\Omega = 2$, $a^* = 0.2$)

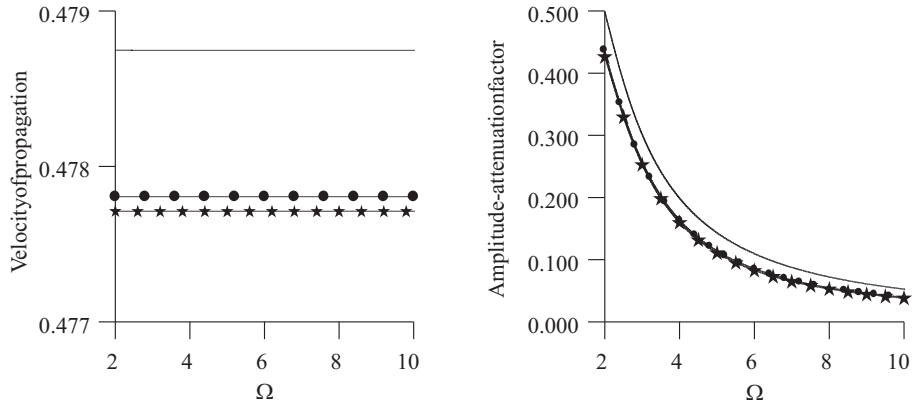


Fig. 3. Variations of velocity of propagation and amplitude-attenuation factor against the rotation parameter Ω . (Solid curve - $\chi = 0.1, p = 2, H_0 = 10, a^* = 0.2$; Solid curve with circle - $\chi = 0.1, p = 2, H_0 = 10, a^* = 0.02$; Solid curve with star - $\chi = 0.1, p = 2, H_0 = 10, a^* = 0.002$)

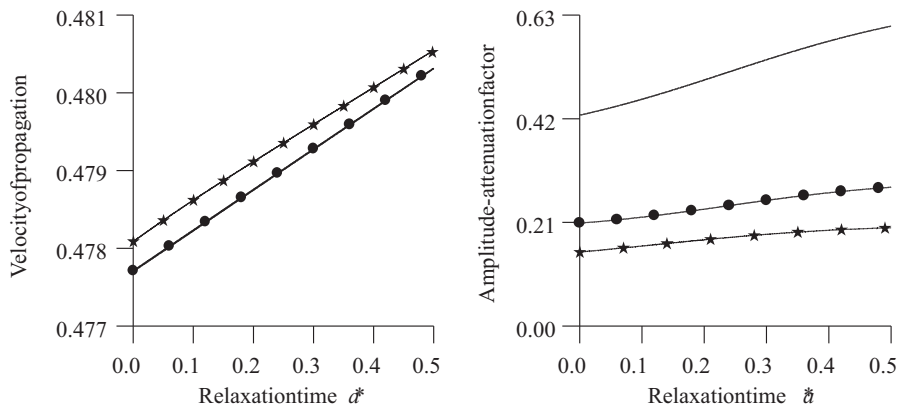


Fig. 4. Variations of velocity of propagation and amplitude-attenuation factor against the relaxation parameter a^* . (Solid curve - $\chi = 0.1, p = 2, \Omega = 2, H_0 = 10$; Solid curve with circle - $\chi = 0.1, p = 0, \Omega = 2, H_0 = 10$; Solid curve with star - $\chi = 0.1, p = -2, \Omega = 8, H_0 = 10$)

is $0.47874859 \times 10^3 \text{ m.s}^{-2}$ at $H_0 = 0$ and it changes slowly with the increase in the magnetic field. The comparison of curve corresponding $\Omega = 2$ with those for $\Omega = 4$ and 8 shows the effect of rotation on the velocity of propagation. The effect of rotation decreases at higher values of magnetic field. For $\Omega = 2$ the value of amplitude-attenuation is 0.0054348×10^3 . It increases with the increase

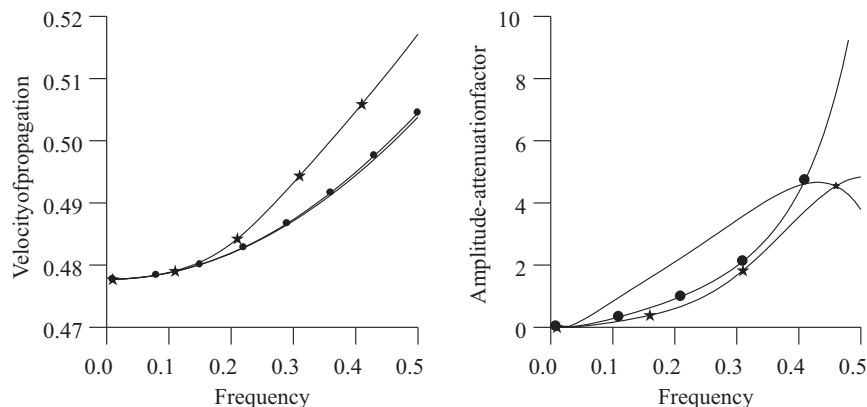


Fig. 5. Variations of velocity of propagation and amplitude-attenuation factor against the frequency χ . (Solid curve - $p = 2$, $\Omega = 2$, $a^* = 0.2$, $H_0 = 10$; Solid curve with circle - $p = 0$, $\Omega = 2$, $a^* = 0.2$, $H_0 = 10$; Solid curve with star - $p = -2$, $\Omega = 2$, $a^* = 0.2$, $H_0 = 10$)

in magnetic field. The comparison between different curves for $\Omega = 2, 4, 8$ shows that the effect of rotation increases with the increase in magnetic field.

The velocity of propagation and amplitude-attenuation factor are plotted against the initial stress parameter p in Fig. 2, when $\Omega = 2$, $a^* = 0.2$ s, $\chi = 0.1$ and $H_0 = 10, 20$ and 30 . For $H_0 = 10 \text{ A.m}^{-1}$, the velocity of propagation is $0.47911456 \times 10^3 \text{ m.s}^{-2}$ at $p = -2$. It decrease sharply with the increase in value of p . The comparison of curve corresponding $H_0 = 10$ with those for $H_0 = 20$ and 30 shows the effect of the magnetic field on the velocity of propagation. The effect of magnetic field decreases with the increase in initial stress p . The value of amplitude-attenuation is 0.1744×10^3 . It increases with the increase in the value of initial stress parameter. The comparison between different curves for $H_0 = 10, 20$ and 30 shows the effect of the magnetic field on the amplitude-attenuation. It is observed that the effect of magnetic field increases with the increase in the value of p .

The velocity of propagation and amplitude-attenuation factor are plotted against the rotation parameter Ω in Fig. 3 when $p = 2$, $H_0 = 10$, $\chi = 0.1$ and $a^* = 0.2, 0.02$ and 0.002 s. For $a^* = 0.2$, the velocity of propagation is $0.4787471 \times 10^3 \text{ m.s}^{-2}$ at $\Omega = 2$ and it decrease very slowly with the increase in value of rotation parameter Ω . The comparison of curve corresponding $a^* = 0.2$ with those for $a^* = 0.02$ and 0.002 shows the effect of relaxation time on the velocity of propagation. The effect of relaxation time remains almost same at each value of Ω . For $a^* = 0.2$, the value of amplitude-attenuation is

0.498344×10^3 . It decreases with the increase in the value of rotation parameter Ω . The comparison between different curves for $a^* = 0.2, 0.02$ and 0.002 shows the effect of relaxation time on the amplitude-attenuation. However, the values of amplitude-attenuation for $a^* = 0.02$ and 0.002 coincide in Fig. 3 at each value of Ω .

The velocity of propagation and the amplitude-attenuation factor are plotted against the relaxation time parameter a^* in Fig. 4 when $\Omega = 2, H_0 = 10, \chi = 0.1$ and $p = -2, 0, 2$. For both $p = 0$ (without initial stress) and $p = 2$ (compression), the velocity of propagation is $0.4777 \times 10^3 \text{ m.s}^{-2}$ at $a^* = 0$ and it increases linearly with the increase in the value of a^* . The comparison of curves corresponding $p = 0$ and $p = 2$ with those for $p = -2$ (tension) shows the effect of initial stress parameter on the velocity of propagation. For $p = 2$, the value of amplitude-attenuation is 0.42705417×10^3 . It increases with the increase in the value of relaxation parameter a^* . The comparison between different curves for $p = -2, 0, 2$ shows the effect of the initial stress parameter on the amplitude-attenuation. It is observed, that the effect of rotation increases with the increase in the value of a^* . Here, is worthy to note that the curves corresponding to the velocity of propagation in Fig. 4 are similar for $p = 2$ and $p = 0$.

The velocity of propagation and the amplitude-attenuation factor are plotted against the frequency χ in Fig. 5, when $\Omega = 2, H_0 = 10, a^* = 0.2$ and $p = -2, 0, 2$. For both $p = -2, 0$ and 2 , the velocity of propagation is $0.4777 \times 10^3 \text{ m.s}^{-2}$ at $\chi = 0.01$ and it increases with the increase in the values of χ . The velocity of propagation for $p = 2$ is slightly less than that for $p = 0$ and the velocity of propagation for $p = -2$ is higher than for $p = 0$. The comparison of curves corresponding $p = -2, 0, 2$ shows the effect of the initial stress parameter on the velocity of propagation. The effect of the initial stress increases with the increase in values of frequency. For $p = -2, 0, 2$, the value of amplitude-attenuation is 0.02×10^3 at $\chi = 0.01$. It increases with the increase in the value of frequency. The comparison between different curves for $p = -2, 0, 2$ shows the effect of initial stress on the amplitude-attenuation. It is observed, that the effect of initial stress is different at each value of frequency.

From the above discussion, it is also observed that:

(i) For fixed values of frequency, relaxation time and initial stress (compression), the effect of rotation on velocity of propagation of Rayleigh wave is observed maximum at low magnetic field and it is observed minimum at high values of magnetic field. The effect of rotation on the amplitude-attenuation is reverse to that on the velocity of propagation.

(ii) For fixed values of frequency, relaxation time and rotation parame-

ters, the effect of magnetic field on velocity of propagation of Rayleigh wave is observed maximum during tensional initial stress and it is observed minimum for compression initial stress. The effect of magnetic field on the amplitude-attenuation is less for tensional initial stress as compared to that for compression initial stress.

(iii) For fixed values of frequency, initial stress and magnetic field parameters, the effect of relaxation time on the velocity of propagation and amplitude-attenuation is observed approximately the same at each value of rotation parameter.

(iv) For fixed values of frequency, magnetic field and rotation parameters, the effect of initial stress on the velocity of propagation of Rayleigh wave is observed more at lower values of relaxation time as compared to that at higher values of relaxation time. The effect of initial stress on the amplitude-attenuation is less for lower values of relaxation time as compared to that for higher values of relaxation time.

(v) For fixed values of magnetic field, rotation and relaxation time parameters, the effect of initial stress on the velocity of propagation of Rayleigh wave is observed minimum at very low frequency and it is observed maximum at higher values of frequency. The effect of initial stress on the amplitude-attenuation is observed minimum at low frequency as compared to that at higher values of frequency.

Numerical computations show that the dependence of velocity of propagation and the amplitude-attenuation on the initial stress (compression or tensional), magnetic field, relaxation time, frequency and the rotation. The velocity of Rayleigh wave reduces due to the presence of these parameters. The numerical results based on a particular material (aluminium) indicate that the effect of rotation, initial stress, magnetic field and the density on Rayleigh wave velocity are very pronounced and have significant applications in the fields of engineering structures, mining engineering, geophysics, earthquake science, nuclear devices, etc.

9. Conclusion

The frequency equation of Rayleigh wave in a rotating magneto-thermo-elastic half-space with hydrostatic initial stress is obtained. The frequency equation is approximated for small thermal coupling and the expressions for the velocity of propagation and amplitude-attenuation factors are obtained and computed numerically for the case of aluminium material. The velocity of propagation and the amplitude-attenuation factor are significantly influenced by

frequency, hydrostatic initial stress, rotation and magnetic field and relaxation time parameters.

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