RAYLEIGH WAVE IN A ROTATING INITIALLY STRESSED PIEZOELECTRIC HALF-SPACE

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ABSTRACT. The governing equations of an initially stressed rotating piezoelectric medium are solved for surface wave solutions. The appropriate solutions in the half-space of the medium satisfy the required boundary conditions to obtain the frequency equation of Rayleigh wave for charge free as well as electrically shorted cases. The non-dimensional speed of the Rayleigh wave is computed numerically for particular examples of Lithium niobate and PZT-5H ceramics. The effects of rotation and initial stress are observed graphically on the non-dimensional speed of the Rayleigh wave.
KEY WORDS: Piezoelectric; surface waves; rotation; electrically shorted; charge free.

1. Introduction

Materials having electromechanical coupling are known as electroelastic materials, whereas the materials having linear coupling between mechanical and electric fields are considered as piezoelectric materials. Wave propagation in piezoelectric media has various applications in the fields of aerospace engineering, mechanical engineering, civil engineering, and bioengineering. Surface

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wave propagation in a piezoelectric plate is used to achieve the time delay effect in acoustic devices. Surface wave propagation in piezoelectric solids has been used with great success by the telecommunication industry and used in wireless transmission and reception technology for colour television sets, cell phones and global positioning systems. Bluestein [1], Cheng and Sun [2], Abbudi and Barnett [3], Li [4], Yang [5], Fang et al. [6], Wang et al. [7], Wang [8], Hu et al. [9], Yang and Zhou [10] and Yang [11] have discussed various problems of surface wave propagation in piezoelectric media. Spaight and Koerber [12] calculated numerically the time-harmonic surface waves in a Lithium niobate half-space.

The effects of pre-stress, acceleration, rotation and temperature variation, etc., on wave speed or frequency provide information for the design of acoustic sensors [13]. Particularly, frequency shifts due to rotation have been used to make gyroscopes [14–16]. Schoenberg and Censor [17] studied the effect of rotation on plane wave propagation in an isotropic medium. They showed the propagation of three plane waves in a rotating isotropic medium. They found that the longitudinal or transverse wave can exist only if the direction of propagation and the axis of rotation are either parallel or perpendicular. Clarke and Burdness [18] considered the rotation of isotropic elastic half-space about an axis perpendicular to the plane of motion with small rotation rate. Clarke and Burdness [19] extended also their results for an arbitrary rotation rate. The effects of rotation and magnetic field do not increase the number of waves in an isotropic medium, but it affects their speeds significantly.

Sinha et al. [20] discussed the effect of biasing stresses on propagation of surface waves. Montanaro [21] studied the wave propagation along symmetry axes in linearly elastic media with an initial stress. Guz [22] developed a method to measure the biaxial homogeneous initial stress field in the elastic body using elastic waves. Propagation of surface acoustic waves in pre-stressed piezoelectric structures was studied by Chai and Wu [23], Jin et al. [24], Simionescu-Panait [25], Liu et al. [26, 27], Qian et al. [28, 29], Jin et al. [30], Su et al. [31] and Du et al. [32, 33].

Singh [34] studied the reflection of plane waves at a traction-free and electrically shorted/charge-free surface of a pre-stressed piezoelectric medium. In the present paper, we studied the effects of initial stresses and rotation on the speed of Rayleigh wave along the stress free surface of a piezoelectric solid half-space.

2. Formulation of the problem

Following Liu et al. [26, 27], the equations of motion, the charge equation and the boundary conditions for a pre-stressed piezoelectric medium are:

$$(1) \qquad (\sigma_{ij} + u_{j,k}\sigma_{ik}^0)_{,i} + \rho_0 f_i = \rho_0[\ddot{\vec{u}} + (\vec{\Omega} \times \vec{\Omega} \times \vec{u}) + (2\vec{\Omega} \times \dot{\vec{u}})]_{,j},$$

$$(2) \qquad\qquad\qquad\qquad D_{i,i} = 0.$$

$$(3) \qquad\qquad (\sigma_{ij} + u_{j,k}\sigma_{ik}^0)N_i = T_j, \quad D_i N_i = \sigma.$$

Here D_i, T_j and σ are respective increments of the electric displacement, the surface traction and the electric charge density due to a dynamic disturbance superposed on the initial state, u_i are the components of displacement vector, f_i is the body force per unit mass, σ_{ik}^0 is the Piola-Kirchhoff stress tensor referred to the initial state, ρ_0 is the mass density in natural (undeformed) configuration, N_i are the components of the unit outward vector normal to a surface element, $\vec{\Omega} \times \vec{\Omega} \times \vec{u}$ is the centripetal acceleration due to time varying motion only and $2\vec{\Omega} \times \dot{\vec{u}}$ is the Coriolis acceleration, and:

$$(4) \qquad \sigma_{ij} = c_{ijkl}^* u_{k,l} + e_{mij}^* \Phi_{,m}, \quad D_m = e_{mij}^* u_{i,j} - \epsilon_{mn}^* \Phi_{,n},$$

where c_{ijkl}^*, e_{mij}^* and ϵ_{mn}^* are the elastic, piezoelectric and dielectric constants, respectively, which are related to the initial displacement and the electric potential gradients. The Φ is the scalar electric potential. The components of the biased field can be determined from the field equations and the boundary conditions in the initial biasing state (Liu et al. [27]).

We consider a homogeneous, linear, transversely isotropic, piezoelectric solid half-space. The origin of the Cartesian co-ordinate system (x_1, x_2, x_3) is at the surface of the half-space $x_3 = 0$. The surface $x_3 = 0$ is subjected to the required boundary conditions. We assume, that the wave fields do not depend on x_2, but implicit dependence is there, so that the displacement u$_2$ is non-vanishing. The equations of motion for a linear transversely isotropic piezoelectric medium and without body forces can be written as:

(5) $\quad c_1^* u_{1,11} + c_2^* u_{3,33} + c_5^* u_{3,13} + (e_{15}^* + e_{31}^*)\Phi_{,13} = \rho(\ddot{u}_1 - \Omega^2 u_1 + 2\Omega\dot{u}_3),$

(6) $\quad c_4^* u_{3,11} + c_3^* u_{3,33} + c_5^* u_{1,13} + e_{15}^* \Phi_{,11} + e_{33}^* \Phi_{,33} = \rho(\ddot{u}_3 - \Omega^2 u_3 + 2\Omega\dot{u}_1),$

(7) $\quad (e_{15}^* + e_{31}^*)u_{1,13} + e_{15}^* u_{3,11} + e_{33} u_{3,33} - \varepsilon_{11}^* \Phi_{,11} - \varepsilon_{33}^* \Phi_{,33} = 0.$

where,

$$c_1^* = c_{11}^* + \sigma_{11}^0, \qquad c_2^* = c_{44}^* + \sigma_{33}^0, \qquad c_3^* = c_{33}^* + \sigma_{33}^0,$$

$$c_2^* = c_{44}^* + \sigma_{11}^0, \qquad c_5^* = c_{13}^* + c_{44}^*.$$

3. Surface wave solutions

The solutions of eqs. (5) to (7) are now sought in the following form:

(8) $\qquad (u_1, u_3, \Phi) = [\bar{u}_1(x_3), \bar{u}_3(x_3), \bar{\Phi}(x_3)] \cdot e^{i\xi(x_1 - ct)},$

where ξ is the wave number and c is the wave velocity. Substituting eq. (8) into eqs. (5) to (7), we obtain:

(9) $\qquad (A_1 D^4 + A_2 D^2 + +A_3 D + A_4)(\bar{u}_1, \bar{u}_3, \bar{\Phi}) = 0,$

where A_1, A_2, A_3 and A_4 are given in Appendix I.

Equation (9) is written as:

(10) $\quad \left[\left(\dfrac{d^2}{dx_3^2} + \lambda_1^2 \xi^2\right)\left(\dfrac{d^2}{dx_3^2} + \lambda_2^2 \xi^2\right)\left(\dfrac{d^2}{dx_3^2} + \lambda_3^2 \xi^2\right)\right](\bar{u}_1, \bar{u}_3, \bar{\Phi}) = 0,$

where

(11) $\quad \lambda_1^2 + \lambda_2^2 + \lambda_3^2 = -\dfrac{A_2}{A_1}, \quad \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_2^1 = \dfrac{A_3}{A_1}, \quad \lambda_1^2\lambda_2^2\lambda_3^2 = -\dfrac{A_4}{A_1},$

The general solutions of equation (10) are:

$$\text{(12)} \quad \bar{u}_1 = [Ae^{-\iota\xi\lambda_1 x_3} + Be^{-\iota\xi\lambda_2 x_3} + Ce^{-\iota\xi\lambda_3 x_3} \\ + A^* e^{\iota\xi\lambda_1 x_3} + B^* e^{\iota\xi\lambda_2 x_3} + C^* e^{\iota\xi\lambda_3 x_3}]e^{i\xi(x_1-ct)},$$

$$\text{(13)} \quad \bar{u}_3 = [A'e^{-\iota\xi\lambda_1 x_3} + B'e^{-\iota\xi\lambda_2 x_3} + C'e^{-\iota\xi\lambda_3 x_3} \\ + A^{*\prime} e^{\iota\xi\lambda_1 x_3} + B^{*\prime} e^{\iota\xi\lambda_2 x_3} + C^{*\prime} e^{\iota\xi\lambda_3 x_3}]e^{i\xi(x_1-ct)},$$

$$\text{(14)} \quad \bar{\Phi} = [A''e^{-\iota\xi\lambda_1 x_3} + B''e^{-\iota\xi\lambda_2 x_3} + C''e^{-\iota\xi\lambda_3 x_3} \\ + A^{*\prime\prime} e^{\iota\xi\lambda_1 x_3} + B^{*\prime\prime} e^{\iota\xi\lambda_2 x_3} + C^{*\prime\prime} e^{\iota\xi\lambda_3 x_3}]e^{i\xi(x_1-ct)},$$

where A, B, C, A^*, B^*, C^*, A', B', C', $A^{*\prime}$, $B^{*\prime}$, $C^{*\prime}$, A'', B'', C'', $A^{*\prime\prime}$, $B^{*\prime\prime}$ and $C^{*\prime\prime}$ are constants. In general, the roots λ_j $(j = 1, 2, 3)$ are complex and as we are considering surface wave only. We can assume $\text{Re}(\lambda_j) > 0$. We choose only form of λ_j which satisfies the radiation condition. Hence, the solution is a superposition of a plane waves attenuating with depth. Now, the solution of the auxiliary equation (9) in the half-space $(x_3 > 0)$ which satisfies above condition is:

$$\text{(15)} \quad u_1 = [Ae^{-\iota\xi\lambda_1 x_3} + Be^{-\iota\xi\lambda_2 x_3} + Ce^{-\iota\xi\lambda_3 x_3}]e^{i\xi(x_1-ct)},$$

$$\text{(16)} \quad u_3 = [A'e^{-\iota\xi\lambda_1 x_3} + B'e^{-\iota\xi\lambda_2 x_3} + C'e^{-\iota\xi\lambda_3 x_3}]e^{i\xi(x_1-ct)},$$

$$\text{(17)} \quad \Phi = [A''e^{-\iota\xi\lambda_1 x_3} + B''e^{-\iota\xi\lambda_2 x_3} + C''e^{-\iota\xi\lambda_3 x_3}]e^{i\xi(x_1-ct)},$$

where A', A'', B', B'', C', C'' are related with A, B and C as:

(18) $A' = k'A$, $B' = k''B$, $C' = k'''C$, $A'' = k''''A$, $B'' = k'''''B$, $C'' = k''''''C$.

The expressions for k', k'', k''', k'''', k''''' and k'''''' are given Appendix II. Then, we have the final form of the solutions as:

(19) $$u_1 = [Ae^{-\iota\xi\lambda_1 x_3} + Be^{-\iota\xi\lambda_2 x_3} + Ce^{-\iota\xi\lambda_3 x_3}]e^{i\xi(x_1 - ct)},$$

(20) $$u_3 = [k'Ae^{-\iota\xi\lambda_1 x_3} + k''Be^{-\iota\xi\lambda_2 x_3} + k'''Ce^{-\iota\xi\lambda_3 x_3}]e^{i\xi(x_1 - ct)},$$

(21) $$\Phi = [k''''Ae^{-\iota\xi\lambda_1 x_3} + k'''''Be^{-\iota\xi\lambda_2 x_3} + k''''''Ce^{-\iota\xi\lambda_3 x_3}]e^{i\xi(x_1 - ct)}.$$

4. Boundary conditions

The boundary conditions at stress free and electrically shorted/charge free surface are:

(22) $$\sigma_{33} + u_{3,k}\sigma_{3k}^0 = 0, \quad \sigma_{31} + u_{1,k}\sigma_{3k}^0 = 0,$$

(23) $$D_3 = 0 \text{ (Charge free surface)}$$

(24) $$\Phi = 0 \text{ (Electrically shorted surface)}$$

where, $\sigma_{33} = c_{33}^* u_{3,3} + e_{33}^* \Phi_{,3} + c_{11}^* u_{1,1}$, $\sigma_{31} = c_{44}^*(u_{3,1} + u_{1,3}) + e_{15}^* \Phi_{,1}$, $D_3 = e_{31}^* u_{1,1} + e_{33}^* u_{3,3} - \epsilon_{33}^* \Phi_{,3}$.

The solutions given by Eqs. (19) to (21) satisfy the boundary conditions (22) and (23) and we obtain the following homogeneous system of three equations in A, B and C:

(25) $$(\lambda_1 L_1 + c_{31}^*)A + (\lambda_2 L_2 + c_{31}^*)B + (\lambda_3 L_3 + c_{31}^*)C = 0,$$

(26) $[L_4 - (c_{44}^* + \sigma_{33}^0)\lambda_1]A + [L_5 - (c_{44}^* + \sigma_{33}^0)\lambda_2]B + [L_6 - (c_{44}^* + \sigma_{33}^0)\lambda_3]C = 0$

(27) $\qquad (e_{31}^* + \lambda_1 L_7)A + (e_{31}^* \lambda_2 L_8)B + (e_{31}^* + \lambda_3 L_9)C = 0,$

where,

$$L_1 = -(e_{33}^* + \sigma_{33}^0)k' - e_{33}^* k'''', \qquad L_2 = -(e_{33}^* + \sigma_{33}^0)k'' - e_{33}^* k''''',$$

$$L_3 = -(e_{33}^* + \sigma_{33}^0)k''' - e_{33}^* k'''''', \qquad L_4 = c_{44}^* k' + e_{15}^* k'''',$$

$$L_5 = c_{44}^* k'' + e_{15}^* k''''', \qquad L_6 = c_{44}^* k''' + e_{15}^* k'''''',$$

$$L_7 = -e_{33}^* k' + \epsilon_{33}^* k'''', \qquad L_8 = -e_{33}^* k'' + \epsilon_{33}^* k''''',$$

$$L_9 = -e_{33}^* k''' + \epsilon_{33}^* k''''''.$$

The homogeneous system of equations (25) to (27) having non-trivial solution if:

(28) $$\begin{bmatrix} \lambda_1 L_1 + c_{31}^* & \lambda_2 L_2 + c_{31}^* & \lambda_3 L_3 + c_{31}^* \\ L_4 - (c_{44}^* + \sigma_{33}^0)\lambda_1 & L_5 - (c_{44}^* + \sigma_{33}^0)\lambda_2 & L_6 - (c_{44}^* + \sigma_{33}^0)\lambda_3 \\ e_{31}^* + \lambda_1 L_7 & e_{31}^* + \lambda_2 L_8 & e_{31}^* + \lambda_3 L_9 \end{bmatrix} = 0.$$

Eq.(28) determines the speed of Rayleigh surface wave under the influences of the rotation and initial stresses in a piezoelectric solid half-space for charge free surface. Similarly, the solutions given by (19) to (21) satisfy the boundary conditions (22) and (24) and we obtain:

(29) $$\begin{bmatrix} \lambda_1 L_1 + c_{31}^* & \lambda_2 L_2 + c_{31}^* & \lambda_3 L_3 + c_{31}^* \\ L_4 - (c_{44}^* + \sigma_{33}^0)\lambda_1 & L_5 - (c_{44}^* + \sigma_{33}^0)\lambda_2 & L_6 - (c_{44}^* + \sigma_{33}^0)\lambda_3 \\ k'''' & k''''' & k'''''' \end{bmatrix} = 0.$$

Eq.(29) determines the speed of Rayleigh surface wave under the influences of the rotation and initial stresses in a piezoelectric solid half-space for electrically shorted surface.

5. Numerical examples

Lithium Niobate is one of the most versatile and well-developed active optical materials. This crystal finds applications in electro-optics, acousto-optics, non-linear optics and guided-wave optics. The following physical properties of Lithium Niobate (Weis and Gaylord [35]) and PZT-5H materials are

used for computing the non-dimensional speed of Rayleigh surface wave:

(a) Lithium Niobate:
$c_{11}^* = 2.03 \times 10^{11}$ N/m^2, $c_{33}^* = 2.424 \times 10^{11}$ N/m^2, $c_{44}^* = 0.595 \times 10^{11}$ N/m^2,
$c_{13}^* = 0.752 \times 10^1$ N/m^2, $e_{31}^* = 0.23$ C/m^2, $e_{33}^* = 1.33$ C/m^2, $e_{15}^* = 3.7$ C/m^2,
$\epsilon_{11}^* = 85.2$, $\epsilon_{33}^* = 28.7$, $\rho = 4.647 \times 10^3$ kg/m^3.

(b) PZT-5H Ceramics:
$c_{11}^* = 12.1 \times 10^{10}$ N/m^2, $c_{33}^* = 11.7 \times 10^{10}$ N/m^2, $c_{44}^* = 2.34 \times 10^{10}$ N/m^2,
$c_{13}^* = 8.41 \times 10^{10}$ N/m^2, $e_{31}^* = -6.5$ C/m^2, $e_{33}^* = 23.3$ C/m^2, $e_{15}^* = 17$ C/m^2,
$\epsilon_{11}^* = 3100$, $\epsilon_{33}^* = 3400$, $\rho = 7.7 \times 10^3$ kg/m^3.

For small piezoelectric coupling, the roots λ_1^2, λ_2^2 and λ_3^2 are approximated from equation (11) as:

$$\lambda_1^2 = \frac{\epsilon_{11}^*}{\epsilon_{33}^*}, \; \lambda_2^2 = \frac{c_1^*}{c_2^*} - \frac{\rho_0 \Omega^2}{c_2^*}\left(1 - \frac{\Omega^2}{\omega^2}\right), \quad \lambda_3^2 = \frac{c_4^*}{c_3^*} - \frac{\rho_0 c^2}{c_3^*}\left(1 - \frac{\Omega^2}{\omega^2}\right).$$

where, $\omega = \xi c$. The equations (28) and (29) are then approximated for the purpose of numerical computations. The approximated equations (28) and (29) are solved for non-dimensional speed by functional iteration method.

For the above values of material constants the non-dimensional speed $\frac{\rho_0 c^2}{c_2^*}$ is computed for the range $5 \leq \frac{\Omega}{\omega} \leq 20$ of rotation parameter and for the range $0 \leq \sigma_{11}^0 \leq 2$ Pa of initial parameter. The non-dimensional speed of Rayleigh wave is shown graphically in Figs 1 to 3.

In both materials for $\sigma_{11}^0 = 0.3$ Pa, the non-dimensional speed of Rayleigh wave decreases sharply along a curve with the increase in rotation parameter for both electrically shorted and charge free cases as shown in Fig. 1. With the increase in rotation parameter, the difference in variations of the speed for electrically shorted and a charge free case diminishes. From Fig. 1, the effect of rotation is observed on dimensional speed of Rayleigh wave at a fixed value of initial stress.

For $\frac{\Omega}{\omega} = 5$, the non-dimensional speed of Rayleigh wave for both materials decreases sharply in a straight line with the increase in initial stress parameter for both electrically shorted and charge free cases as shown in Fig. 2. From Fig. 2, for both materials it is observed that the non-dimensional speed is the same for electrically shorted and charge free cases at $\sigma_{11}^0 = 1$ Pa. It is also observed, that the non-dimensional speed of Rayleigh wave is affected by
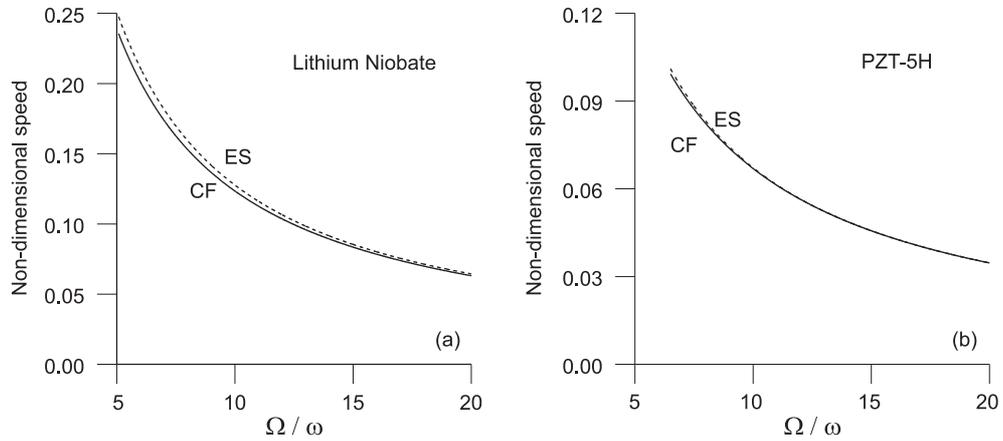
Fig. 1. The variations of non-dimensional speed $\dfrac{\rho_0 c^2}{c_2^*}$ against the rotation parameter $\dfrac{\Omega}{\omega}$ when $\sigma_{11}^0 = 0.3$ Pa for (a) ES – Electrically shorted case (b) CF – Charge free case
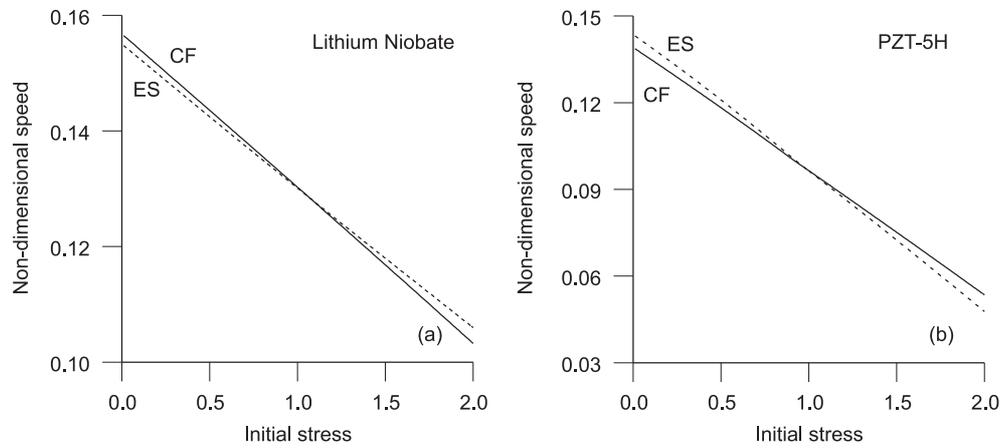
Fig. 2. The variations of non-dimensional speed $\dfrac{\rho_0 c^2}{c_2^*}$ against the initial stress parameter σ_{11}^0 when $\dfrac{\Omega}{\omega} = 5$ for (a) ES – Electrically shorted case (b) CF – Charge free case
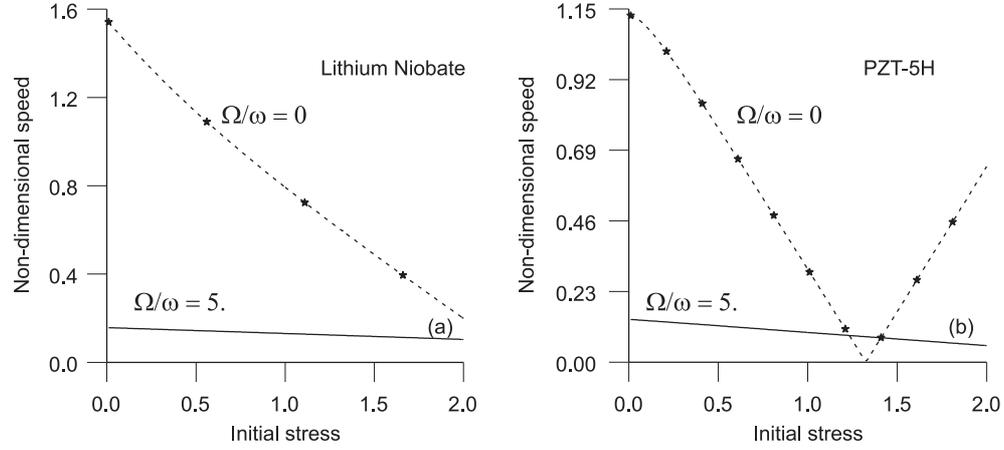
Fig. 3. The variations of non-dimensional speed $\dfrac{\rho_0 c^2}{c_2^*}$ against the initial stress parameter σ_{11}^0 when $\dfrac{\Omega}{\omega} = 0$ and 5 (charge free case only)

the presence of initial stress in medium at given rotation rate.

We consider the charge free case only in order to visualize the effect of rotation at different values of initial stress parameter. The non dimensional speed $\dfrac{\rho_0 c^2}{c_2^*}$ of the Rayleigh wave is computed for the range $0 \leq \sigma_{11}^0 \leq 2$ Pa; when $\dfrac{\Omega}{\omega} = 0$ and 5. For both materials, in Fig. 3, the effect of rotation is observed significantly on non-dimensional speed at different initial parameters.

6. Conclusion

The frequency equation of the Rayleigh wave at stress-free charge free/ electrically shorted surface of an initially stressed rotating piezoelectric medium is obtained. The non-dimensional speed of the Rayleigh wave is computed for two different piezoelectric materials (Lithium Niobate and PZT-5H ceramics). The effects of rotation and initial stress parameters are observed significantly on the non-dimensional speed of the Rayleigh wave for both piezoelectric materials. The present theoretical expressions and numerical simulation may be helpful in further experimental works on surface wave propagation in piezoelectric materials.

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Appendix I

The expressions for A_1, A_2, A_3 and A_4 are given as:

$$A_1 = -\frac{\epsilon_{33}^*}{e_{33}^*} - \frac{e_{33}^*}{c_3^*},$$

$$A_2 = \frac{\epsilon_{11}^*}{e_{33}^*} + \frac{\epsilon_{33}^*}{e_{33}^*}(a_1 + a_2) + 2\frac{e_{15}^*}{c_3^*} + \frac{e_{33}^*}{c_3^*}a_1 - \frac{\epsilon_{33}^* c_5^{*2}}{e_{33}^* c_2^* c_3^*} - \frac{(e_{15}^* + e_{31}^*)c_5^*}{e_{33}^* c_2^*}$$

$$- \frac{(e_{15}^* + e_{31}^*)c_5^*}{c_3^* c_2^*} + \frac{(e_{15}^* + e_{31}^*)^2}{e_{33}^* c_2^*},$$

$$A_3 = \frac{\epsilon_{11}^*}{e_{33}^*}(a_1 + a_2) - \frac{e_{15}^{*2}}{e_{33}^* c_5^*} - 2\frac{e_{15}^*}{c_3^*}a_1 - \frac{e_{33}^*}{c_3^*}a_1 a_2 + \frac{\epsilon_{11}^* c_5^{*2}}{e_{33}^* c_2^* c_3^*} + 2\frac{(e_{15}^* + e_{31}^*)c_5^* e_{15}^*}{e_{33}^* c_2^* c_3^*}$$

$$- \frac{(e_{15}^* + e_{31}^*)^2}{e_{33}^* c_2^*}a_2 + \frac{4\rho_0^2 c^2 \epsilon_{33}^* \Omega^2}{e_{33}^* c_2^* c_3^* \omega^2},$$

$$A_4 = \frac{\epsilon_{11}^*}{e_{33}^*}a_1 a_2 + \frac{e_{15}^{*2}}{e_{33}^* c_3^*}a_1 - \frac{4\rho_0^2 c^4 \epsilon_{11}^* \Omega^2}{e_{33}^* c_2^* c_3^* \omega^2},$$

where $a_1 = \dfrac{c_1^*}{c_2^*} - \dfrac{\rho_0 \Omega^2}{c_2^*}\left(1 - \dfrac{\Omega^2}{\omega^2}\right)$, $a_2 = \dfrac{c_4^*}{c_3^*} - \dfrac{\rho_0 \Omega^2}{c_3^*}\left(1 - \dfrac{\Omega^2}{\omega^2}\right).$

Appendix II

The expression for k', k'', k''', k'''', k''''' and k'''''' are given as:

$$k' = \frac{k_1(k_4' + k_7')}{k_4' + k_7' + k_1' k_7 - k_1' k_4}, \quad k'' = \frac{k_2(k_5' + k_8')}{k_5' + k_8' + k_2' k_8 - k_2' k_5},$$

$$k''' = \frac{k_3(k_6' + k_9')}{k_6' + k_9' + k_3' k_9 - k_3' k_6}, \quad k'''' = \frac{k_1(k_7 - k_4)}{k_4' + k_7' + k_1' k_7 - k_1' k_4},$$

$$k''''' = \frac{k_2(k_8 - k_5)}{k_8' + k_5' + k_2' k_8 - k_2' k_5}, \quad k'''''' = \frac{k_3(k_9 - k_6)}{k_9' + k_6' + k_3' k_9 - k_3' k_6}$$

where

$$k_1 = \frac{c_2^* \lambda_i^2 + c_1^* - \rho_0 c^2 - \rho_0 c^2 \left(\dfrac{\Omega}{\omega}\right)^2}{c_5^8 \lambda_i - 2\iota \rho_0 c^2 \dfrac{\Omega}{\omega}}, \qquad (i = 1, 2, 3)$$

$$k_i' = -\frac{\lambda_i(e_{15}^* + e_{31}^*)}{c_5^* \lambda_i - 2\iota \rho_0 c^2 \dfrac{\Omega}{\omega}}, \qquad (i = 1, 2, 3)$$

$$k_j = \frac{c_3^* \lambda_i^2 + c_4^* - \rho_0 c^2 \left(\dfrac{\Omega}{\omega}\right)^2}{c_5^* \lambda_i + 2\iota \rho_0 c^2 \dfrac{\Omega}{\omega}}, \qquad (i = 1, 2, 3), \quad (j = 4, 5, 6),$$

$$k_j' = \frac{e_{31}^* \lambda_i^2 + e_{15}^*}{c_5^* \lambda_i + 2\iota \rho_0 c^2 \dfrac{\Omega}{\omega}}, \qquad (i = 1, 2, 3), \quad (j = 4, 5, 6),$$

$$k_m = \frac{e_{33}^* \lambda_i^2 - e_{15}^*}{\lambda_i(e_{15}^* + e_{31}^*)}, \qquad (i = 1, 2, 3), \quad (m = 7, 8, 9),$$

$$k_m' = -\frac{\epsilon_{33}^* \lambda_i^2 + \epsilon_{11}^*}{\lambda_i(e_{15}^* + e_{31}^*)}, \qquad (i = 1, 2, 3), \quad (m = 7, 8, 9).$$