

FLUID MECHANICS

PROPAGATION OF CYLINDRICAL RAYLEIGH WAVES IN A TRANSVERSLY ISOTROPIC THERMOELASTIC DIFFUSIVE SOLID HALF-SPACE

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ABSTRACT. The propagation of cylindrical Rayleigh waves in a transversely isotropic thermoelastic diffusive solid half-space subjected to stress free, isothermal/insulated and impermeable or isoconcentrated boundary conditions is investigated in the framework of different theories of thermoelastic diffusion. The dispersion equation of cylindrical Rayleigh waves has been derived. The phase velocity and attenuation coefficients have been computed from the dispersion equation by using Muller's method. Some special cases of dispersion equation are also deduced.

KEY WORDS: Wave propagation, transverse isotropic, generalized thermoelastic diffusion, phase velocity, attenuation coefficient.

1. Introduction

The spontaneous movement of the particles from a high concentration region to a low concentration region is defined as Diffusion and it occurs in response to a concentration gradient expressed as the change in the concentration due to change in position.

Thermal diffusion utilizes the transfer of heat across a thin liquid or gas to accomplish isotope separation. Today, thermal diffusion remains a practical

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process to separate isotopes of noble gases (e.g. xenon) and other light isotopes (e.g. carbon) for research purposes. The thermodiffusion in elastic solids is due to coupling of fields of temperature, mass diffusion and that of strain in addition to heat and mass exchange with the environment.

Nowacki [1–4] developed the theory of thermoelastic diffusion by using coupled thermoelastic model. Sherief and Saleh [5] investigated the problem of a thermoelastic half-space in the context of the theory of generalized thermoelastic diffusion with one relaxation time.

Singh discussed the reflection phenomena of waves from free surface of a thermoelastic diffusion elastic solid one with one relaxation time in [6] and with two relaxation times in [7]. Aouadi studied in [8] the generalized thermoelastic diffusion problem with variable electrical and thermal conductivity. Aouadi studied also the interaction between the processes of elasticity, heat and diffusion in an infinitely long solid cylinder [9] and in an infinite elastic body with spherical cavity [10]. Gawinecki and Szymaniec [11] proved a theorem about global existence of the solution for a nonlinear parabolic thermoelastic diffusion problem. Gawinecki et al. [12] proved a theorem about existence, uniqueness and regularity of the solution for the same problem. Uniqueness and reciprocity theorems for the equations of generalized thermoelastic diffusion problem, in isotropic media, was proved by Sherief et al. [13] on the basis of the variational principle equations, under restrictive assumptions on the elastic coefficients.

Aouadi [14] proved this theorem in the Laplace transform domain due to the inherit complexity of the derivation of the variational principle equations, under the assumption that the functions of the problem are continuous and the inverse Laplace transform of each is also unique. Aouadi [15] derived the uniqueness and reciprocity theorems for the generalized problem in anisotropic media, under the restriction that the elastic, thermal conductivity and diffusion tensors are positively definite. Kumar and Kansal [16] developed the basic equation of anisotropic thermoelastic diffusion based on the Green-Lindsay model.

The present study is devoted to the propagation of cylindrical Rayleigh waves in a transversely isotropic thermoelastic diffusive solid half-space subjected to stress free, isothermal/insulated and impermeable or isoconcentrated boundary conditions in the framework of different theories of thermoelastic diffusion. The dispersion equation of cylindrical Rayleigh waves has been derived on applying boundary conditions. Muller's method has been used to solve the dispersion equation and to compute phase velocity and attenuation coefficient of cylindrical Rayleigh waves. Some special cases of dispersion equation are also deduced.

2. Basic Equations

The basic equations for homogeneous anisotropic generalized thermoelastic diffusion in the absence of body forces, heat and mass diffusion sources, following Aouadi [15] and Kumar et al. [16] are:

(i) Constitutive relations:

$$(1) \quad \sigma_{ij} = c_{ijkl}e_{kl} + a_{ij}(T + \tau_1\dot{T}) + b_{ij}(C + \tau^1\dot{C}),$$

$$(2) \quad \rho T_0 S = k + \rho C_E(T + \alpha\dot{T}) - a_{ij}T_0 e_{ij} + aT_0(C + \beta\dot{C}),$$

$$(3) \quad P = b_{ij}e_{ij} + b(C + \tau^1\dot{C}) - a(T + \tau_1\dot{T}),$$

(ii) Equations of motion:

$$(4) \quad \sigma_{ij,j} = \rho\ddot{u}_i,$$

(iii) Equation of heat conduction:

$$(5) \quad \rho C_E(\dot{T} + \tau_0\ddot{T}) - a_{ij}T_0(\dot{e}_{ij} + \varepsilon\tau_0\ddot{e}_{ij}) + aT_0(\dot{C} + \gamma\ddot{C}) = K_{ij}T_{,ij},$$

(iv) Equation of mass diffusion:

$$(6) \quad \alpha_{ij}^* P_{,ij} = \dot{C} + \varepsilon\tau^0\ddot{C},$$

where $c_{ijkl}(c_{ijkl} = c_{klij} = c_{jikl} = c_{ijlk})$ are elastic parameters. $a_{ij}(= a_{ji}), b_{ij}(= b_{ji})$ are tensors of thermal and diffusion moduli, respectively. ρ, C_E are, the density and specific heat at constant strain, respectively; a, b are, coefficients describing the measure of thermoelastic diffusion effects and of diffusion effects, respectively, $T = \Theta - T_0$ is small temperature increment, Θ is the absolute temperature of the medium, T_0 is the reference temperature assumed to be such that $|T/T_0| \ll 1$ and C is the concentration of the diffusive body in the elastic body. τ^0, τ^1 are diffusion relaxation times with $\tau^1 \geq \tau^0 \geq 0$ and τ_0, τ_1 are thermal relaxation times with $\tau_1 \geq \tau_0 \geq 0$. $\sigma_{ij}(= \sigma_{ji}), K_{ij}(= K_{ji}), e_{ij}(= e_{ji}) = (u_{i,j} + u_{j,i})/2$ are components of stress, thermal conductivity, and strain tensor, respectively. u_i are the components of displacement vector \vec{u} . $\alpha_{ij}^*(= \alpha_{ji}^*)$ are diffusion parameters. P, S are the chemical potential and

entropy per unit mass, respectively; k is a material constant. Here, $\alpha = \beta = \varepsilon = \gamma = k = \tau_0 = \tau^0 = \tau_1 = \tau^1 = 0$ relates for Coupled Thermoelasticity (CT) model, $\alpha = \beta = k = \tau_1 = \tau^1 = 0$, $\varepsilon = 1$, $\gamma = \tau_0$ for Lord-Shulman (L-S) model and $\alpha = \tau_0$, $\beta = \tau^0$, $\varepsilon = 0$, $\gamma = \tau_0$ for Green-Lindsay (G-L) model. The symbols “;” and “.” correspond to partial and time derivatives, respectively.

Applying the transformation:

$$(7) \quad x' = x \cos \phi + y \sin \phi, y' = -x \sin \phi + y \cos \phi, z' = z,$$

where ϕ is the angle of rotation in the $x - y$ plane and using the relation $x = r \cos \theta$, $y = r \sin \theta$ and $z = z$, in the equations (4)–(6), we obtain the basic equations for homogeneous, transversely isotropic, generalized thermodiffusive elastic solid in cylindrical coordinates, as:

$$(8) \quad \begin{aligned} \sigma_{rr,r} + \frac{1}{r}\sigma_{r\theta,\theta} + \sigma_{rz,z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} &= \rho\ddot{u}, \\ \sigma_{r\theta,r} + \frac{1}{r}\sigma_{\theta\theta,\theta} + \sigma_{\theta z,z} + \frac{2}{r}\sigma_{r\theta} &= \rho\ddot{v}, \\ \sigma_{rz,z} + \frac{1}{r}\sigma_{\theta z,\theta} + \sigma_{zz,z} + \frac{\sigma_{rz}}{r} &= \rho\ddot{w}, \end{aligned}$$

$$(9) \quad \begin{aligned} K_1 \nabla_1^2 T + K_3 T_{,zz} - \rho C_E (\dot{T} + \tau_0 \ddot{T}) \\ = T_0 \left(\frac{\partial}{\partial t} + \varepsilon \tau_0 \frac{\partial^2}{\partial t^2} \right) (a_1 (e_{rr} + e_{\theta\theta}) + a_3 e_{zz}) + a T_0 (\dot{C} + \gamma \ddot{C}), \end{aligned}$$

$$(10) \quad \begin{aligned} \alpha_1^* [b_1 \nabla_1^2 (e_{rr} + e_{\theta\theta}) + b_3 \nabla_1^2 e_{zz}] + \alpha_3^* [b_1 (e_{rr,zz} + e_{\theta\theta,zz}) + b_3 e_{zz,zz}] \\ + \alpha_1^* a \left[\left(1 + \tau_1 \frac{\partial}{\partial t} \right) \nabla_1^2 T \right] + \alpha_3^* a \left[\left(1 + \tau_1 \frac{\partial}{\partial t} \right) T_{,zz} \right] \\ - \alpha_1^* b \left[\left(1 + \tau^1 \frac{\partial}{\partial t} \right) \nabla_1^2 C \right] - \alpha_3^* b \left[\left(1 + \tau^1 \frac{\partial}{\partial t} \right) C_{,zz} \right] + \dot{C} + \varepsilon \tau^0 \ddot{C} = 0, \end{aligned}$$

where:

$$\begin{aligned} \sigma_{rr} &= c_{11} e_{rr} + c_{12} e_{\theta\theta} + c_{13} e_{zz} - a_1 (T + \tau_1 \dot{T}) - b_1 (C + \tau^1 \dot{C}), \\ \sigma_{r\theta} &= c_{66} e_{r\theta}, \end{aligned}$$

$$\begin{aligned}
 \sigma_{\theta\theta} &= c_{12}e_{rr} + c_{11}e_{\theta\theta} + c_{13}e_{zz} - a_1(T + \tau_1\dot{T}) - b_1(C + \tau^1\dot{C}), \\
 \sigma_{\theta z} &= c_{44}e_{\theta z}, \\
 \sigma_{zz} &= c_{13}e_{rr} + c_{13}e_{\theta\theta} + c_{33}e_{zz} - a_3(T + \tau_1\dot{T}) - b_3(C + \tau^1\dot{C}), \\
 (11) \quad \sigma_{rz} &= c_{44}e_{rz},
 \end{aligned}$$

$$\begin{aligned}
 e_{rr} &= \frac{\partial u}{\partial r}, \quad e_{\theta\theta} = \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{u}{r}, \quad e_{zz} = \frac{\partial w}{\partial z}, \quad e_{r\theta} = \frac{1}{2} \left(\frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r} \right), \\
 e_{rz} &= \frac{1}{2} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right), \quad e_{\theta z} = \frac{1}{2} \left(\frac{\partial v}{\partial z} + \frac{1}{r} \frac{\partial w}{\partial \theta} \right), \quad \nabla_1^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}, \\
 a_{ij} &= -a_i \delta_{ij}, \quad b_{ij} = -b_i \delta_{ij}, \quad \alpha_{ij}^* = \alpha_i^* \delta_{ij}, \quad K_{ij} = K_i \delta_{ij} \quad (i \text{ not summed}), \\
 c_{66} &= (c_{11} - c_{12})/2,
 \end{aligned}$$

$$\begin{aligned}
 a_1 &= (c_{11} + c_{12})\alpha_1 + c_{13}\alpha_3, \quad a_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3, \\
 (12) \quad b_1 &= (c_{11} + c_{12})\alpha_{1c} + c_{13}\alpha_{3c}, \quad b_3 = 2c_{13}\alpha_{1c} + c_{33}\alpha_{3c}.
 \end{aligned}$$

Here, α_t , α_{tc} ($t = 1, 2, 3$) are the coefficients of linear thermal and diffusion expansion, respectively. In the above equations (11)–(12), we use the contracting subscript notations $1 \rightarrow 11$, $2 \rightarrow 22$, $3 \rightarrow 33$, $4 \rightarrow 23$, $5 \rightarrow 13$, $6 \rightarrow 12$ to relate c_{ijklm} to c_{in} ($i, j, k, m = 1, 2, 3$ and $l, n = 1, 2, 3, 4, 5, 6$).

3. Formulation of the problem

We consider an infinite homogeneous transversely isotropic, thermoelastic diffusive half-space, initially at uniform temperature T_0 . The origin of the coordinate system (r, θ, z) is taken at any point on the plane horizontal surface. We take z -axis as the axis of symmetry and pointing vertically downward into the half space, which is thus represented by $z \geq 0$. The surface $z = 0$ is subjected to stress free, thermally insulated or isothermal and impermeable or isoconcentrated boundary conditions. We take $r - z$ plane as the plane of incidence. Therefore, the basic governing equations of homogeneous transversely isotropic, generalized thermoelastic diffusion in the absence of body forces, heat and diffusive mass sources for two dimensional problem, are:

$$(13) \quad c_{11} \left(u_{,rr} + \frac{1}{r} u_{,r} - \frac{1}{r^2} u \right) + (c_{13} + c_{44}) w_{,rz} + c_{44} u_{,zz}$$

$$-a_1(T + \tau_1 \dot{T})_{,r} - b_1(C + \tau^1 \dot{C})_{,r} = \rho \ddot{u},$$

$$(14) \quad (c_{13} + c_{44}) \left(u_{,rz} + \frac{1}{r} u_{,z} \right) + c_{44} (w_{,rr} + \frac{1}{r} w_{,r}) + c_{33} w_{,zz} \\ - a_3 (T + \tau_1 \dot{T})_{,z} - b_3 (C + \tau^1 \dot{C})_{,z} = \rho \ddot{w},$$

$$(15) \quad K_1 (T_{,rr} + \frac{1}{r} T_{,r}) + K_3 T_{,zz} - \rho C_E (\dot{T} + \tau_0 \ddot{T}) \\ = T_0 \left(\frac{\partial}{\partial t} + \varepsilon \tau_0 \frac{\partial^2}{\partial t^2} \right) (a_1 (u_{,r} + \frac{1}{r} u) + a_3 w_{,z}) + a T_0 (\dot{C} + \gamma \ddot{C}),$$

$$(16) \quad \alpha_1^* \left[b_1 \left(u_{,rrr} + \frac{2}{r} u_{,rr} - \frac{1}{r^2} u_{,r} + \frac{1}{r^3} u \right) + b_3 \left(w_{,rrz} + \frac{1}{r} w_{,rz} \right) \right] \\ + \alpha_3^* \left[b_1 \left(u_{,rzz} + \frac{1}{r} u_{,zz} \right) + b_3 w_{,zzz} \right] + \alpha_1^* a \left[\left(1 + \tau_1 \frac{\partial}{\partial t} \right) \left(T_{,rr} + \frac{1}{r} T_{,r} \right) \right] \\ + \alpha_3^* a \left[\left(1 + \tau_1 \frac{\partial}{\partial t} \right) T_{,zz} \right] - \alpha_1^* b \left[\left(1 + \tau^1 \frac{\partial}{\partial t} \right) \left(C_{,rr} + \frac{1}{r} C_{,r} \right) \right] \\ - \alpha_3^* b \left[\left(1 + \tau^1 \frac{\partial}{\partial t} \right) C_{,zz} \right] + (\dot{C} + \varepsilon \tau^0 \ddot{C}) = 0.$$

We define the dimensionless quantities:

$$(17) \quad r' = \frac{\omega_1^* r}{v_1}, \quad z' = \frac{\omega_1^* z}{v_1}, \quad t' = \omega_1^* t, \quad u' = \frac{\omega_1^* u}{v_1}, \quad w' = \frac{\omega_1^* w}{v_1}, \\ T' = \frac{a_1 T}{\rho v_1^2}, \quad C' = \frac{b_1 C}{\rho v_1^2}, \quad \tau_0' = \omega_1^* \tau_0, \quad \tau_1' = \omega_1^* \tau_1, \quad \tau^{0'} = \omega_1^* \tau^0, \\ \tau^{1'} = \omega_1^* \tau^1, \quad h'_{1,2} = \frac{v_1 h_{1,2}}{\omega_1^*}, \quad \sigma'_{rz} = \frac{\sigma_{rz}}{a_1 T_0}, \quad \sigma'_{zz} = \frac{\sigma_{zz}}{a_1 T_0},$$

where:

$$\omega_1^* = \frac{\rho C_E v_1^2}{K_1}, \quad v_1^2 = \frac{c_{11}}{\rho}.$$

Upon introducing the quantities (17) in equations (13)-(16), after suppressing the primes, we obtain:

$$(18) \quad u_{,rr} + \frac{1}{r} u_{,r} - \frac{1}{r^2} u + \delta_1 u_{,zz} + \delta_2 w_{,rz} - \tau_t' T_r - \tau_c' C_r = \ddot{u},$$

$$(19) \quad \delta_1 \left(w_{,rr} + \frac{1}{r} w_{,r} \right) + \delta_2 \left(u_{,rz} + \frac{1}{r} u_{,z} \right) + \delta_4 w_{,zz} - p_1 \tau'_t T_{,z} - p_2 \tau'_c C_{,z} = \ddot{w},$$

$$(20) \quad T_{,rr} + \frac{1}{r} T_{,r} + p_3 T_{,zz} = \tau_t^0 \dot{T} + \zeta_1 \tau_c^0 \dot{C} + \zeta_2 \tau_e^0 \left(\dot{u}_{,r} + \frac{1}{r} \dot{u} + p_1 \dot{w}_{,z} \right),$$

$$(21) \quad q_1^* \left(u_{,rrr} + \frac{2}{r} u_{,rr} - \frac{1}{r^2} u_{,r} + \frac{1}{r^3} u \right) + q_7^* \left(u_{,rzz} + \frac{1}{r} u_{,zz} \right) \\ + q_2^* w_{,zzz} + q_8^* \left(w_{,rrz} + \frac{1}{r} w_{,rz} \right) + q_3^* \tau'_t \left[T_{,rr} + \frac{1}{r} T_{,r} \right] + q_4^* \tau'_t T_{,zz} \\ - q_5^* \tau'_c \left[C_{,rr} + \frac{1}{r} C_{,r} \right] - q_6^* \tau'_c C_{,zz} + \tau_f^0 \dot{C} = 0,$$

where

$$\delta_1 = \frac{c_{44}}{c_{11}}, \quad \delta_2 = \frac{c_{13} + c_{44}}{c_{11}}, \quad \delta_4 = \frac{c_{33}}{c_{11}}, \quad p_1 = \frac{a_3}{a_1}, \quad p_2 = \frac{b_3}{b_1}, \quad p_3 = \frac{K_3}{K_1}, \\ \zeta_1 = \frac{a T_0 v_1^2 a_1}{\omega_1^* K_1 b_1}, \quad \zeta_2 = \frac{a_1^2 T_0}{\rho K_1 \omega_1^*}, \quad q_1^* = \frac{\alpha_1^* \omega_1^* b_1^2}{\rho v_1^4}, \quad q_2^* = \frac{\alpha_3^* \omega_1^* b_1 b_3}{\rho v_1^4}, \quad q_3^* = \frac{\alpha_1^* \omega_1^* b_1 a}{a_1 v_1^2}, \\ q_4^* = \frac{\alpha_3^* \omega_1^* b_1 a}{a_1 v_1^2}, \quad q_5^* = \frac{\alpha_1^* \omega_1^* b}{v_1^2}, \quad q_6^* = \frac{\alpha_3^* \omega_1^* b}{v_1^2}, \quad q_7^* = \frac{\alpha_3^* \omega_1^* b_1^2}{\rho v_1^4}, \quad q_8^* = \frac{\alpha_1^* \omega_1^* b_1 b_3}{\rho v_1^4}, \\ \tau'_t = 1 + \tau_1 \frac{\partial}{\partial t}, \quad \tau'_c = 1 + \tau^1 \frac{\partial}{\partial t}, \quad \tau_t^0 = 1 + \tau_0 \frac{\partial}{\partial t}, \quad \tau_c^0 = 1 + \gamma \frac{\partial}{\partial t}, \\ \tau_e^0 = 1 + \varepsilon \tau_0 \frac{\partial}{\partial t}, \quad \tau_f^0 = 1 + \varepsilon \tau^0 \frac{\partial}{\partial t}.$$

4. Solution of the problem

We assume the solutions of the form:

$$(22) \quad (u, w, T, C) = (J_1(\xi r), W J_0(\xi r), S J_0(\xi r), R J_0(\xi r)) U e^{i(\xi m z - \omega t)},$$

where ω is the frequency and ξ is the complex wave number. J_0 and J_1 are, the Bessel functions of order zero and one, respectively. Here, U is the amplitude of displacement u and W , S and R , the amplitude ratios of displacement

w , temperature T and concentration C with respect to displacement u , are respectively.

Upon using solutions (22) in equations (18)–(21), we obtain:

$$(23) \quad \xi^2(\delta_1 m^2 + 1) - \omega^2 + \iota \delta_2 \xi^2 m W - \xi[\tau_t^{11} S + \tau_c^{11} R] = 0,$$

$$(24) \quad -\iota \delta_2 \xi^2 m + [\xi^2(\delta_1 + \delta_4 m^2) - \omega^2] W + \iota \xi m [p_1 \tau_t^{11} S + p_2 \tau_c^{11} R] = 0,$$

$$(25) \quad \iota \xi m \zeta_2 \tau_e^{10} - \xi w p_1 \zeta_2 \tau_e^{10} m W + [\iota w \tau_t^{10} - (1 + p_3 m^2) \xi^2] S + \iota w \zeta_1 \tau_c^{10} R = 0,$$

$$(26) \quad \xi^2 [q_1^* + q_7^* m^2] + \iota \xi^3 m [q_8^* + q_2^* m^2] W \\ + \tau_t^{11} \xi^2 [q_3^* + q_4^* m^2] S - \tau_c^{11} \xi^2 [q_5^* + q_6^* m^2] R + \iota w \tau_f^{10} R = 0,$$

where

$$(27) \quad \tau_t^{11} = 1 - \iota w \tau_1, \quad \tau_c^{11} = 1 - \iota w \tau^1, \quad \tau_t^{10} = 1 - \iota w \tau_0, \\ \tau_c^{10} = 1 - \iota w \gamma, \quad \tau_e^{10} = 1 - \iota w \varepsilon \tau_0, \quad \tau_t^{11} = 1 - \iota w \varepsilon \tau_0,$$

The system of the equations (23)–(26) has a non-trivial solution if the determinant of the coefficients $1, W, S, R$ vanishes, which yields to the following polynomial characteristic equation:

$$(28) \quad m^8 + A^* m^6 + B^* m^4 + C^* m^2 + D^* = 0,$$

where the coefficients A^*, B^*, C^*, D^* are given in Appendix. Since, we are interested in surface waves only, so it is essential that motion is confined to free surface $z = 0$ of the half-space so that the characteristic roots m_p^2 ($p = 1, 2, 3, 4$) must satisfy the radiation conditions $Im(\xi m_p) \geq 0$. Then the formal expressions for displacements, temperature change and concentration can be written as:

$$(29) \quad u_1 = \sum_{p=1}^4 A_p J_1(\xi r) \exp[\iota(\xi m_p z - \omega t)],$$

$$(30) \quad (u_3, T, C) = \sum_{p=1}^4 (n_{1p}, n_{2p}, n_{3p}) A_p J_0(\xi r) \exp[\iota(\xi m_p z - \omega t)],$$

where A_p, B_p ($p = 1, 2, 3, 4$) are arbitrary constants. The coupling constants n_{1p}, n_{2p}, n_{3p} ($p = 1, 2, 3, 4$) are given in Appendix.

5. Boundary Conditions

The dimensionless boundary conditions at the free surface $z = 0$ are given by:

(i) Mechanical Conditions (stress-free surface):

$$(31) \quad \begin{aligned} \sigma_{zz} &= (\delta_2 - \delta_1) \left(u_{,r} + \frac{1}{r}u \right) + \delta_4 w_{,z} - p_1 \tau_t^1 T - p_2 \tau_c^1 C = 0, \\ \sigma_{rz} &= \delta_1 (w_{,r} + u_{,z}) = 0, \end{aligned}$$

(ii) Thermal Conditions:

$$(32) \quad \frac{\partial T}{\partial z} + h_1 T = 0,$$

(iii) Concentration Conditions:

$$(33) \quad \frac{\partial C}{\partial z} + h_2 C = 0,$$

Here, $h_1 \rightarrow 0$ corresponds to thermally insulated boundaries and $h_1 \rightarrow \infty$ refers to isothermal surfaces. Similarly, $h_2 \rightarrow 0$ corresponds to impermeable boundaries and $h_2 \rightarrow \infty$ refers to isoconcentrated surfaces.

6. Derivation of secular equations

Substituting the values of u_1, u_3, T and C from equations (29) and (30) in the boundary conditions (31)–(33), we obtain a system of four simultaneous linear equations as:

$$(34) \quad \sum_{p=1}^4 P_{1p} A_p = 0, \quad \sum_{p=1}^4 P_{2p} A_p = 0, \quad \sum_{p=1}^4 P_{3p} A_p = 0, \quad \sum_{p=1}^4 P_{4p} A_p = 0,$$

where

$$P_{1p} = \delta_2 - \delta_1 + i\delta_4 m_p n_{1p} - \frac{p_1 \tau_t^{11} n_{2p}}{\xi} - \frac{p_2 \tau_c^{11} n_{3p}}{\xi}, \quad P_{2p} = i m_p - n_{1p},$$

$$P_{3p} = (\iota\xi m_p + h_1)n_{2p}, \quad P_{4p} = (\iota\xi m_p + h_2)n_{3p}, \quad p = 1, 2, 3, 4.$$

The system of equations (34) has a non-trivial solution of system of equations if the determinant of the coefficients of amplitudes A_p ($p = 1, 2, 3, 4$) vanishes. This, after lengthy algebraic reductions leads to the secular equation obtained as:

$$(35) \quad P_{41}D_1 - P_{42}D_2 + P_{43}D_3 - P_{44}D_4 = 0,$$

where

$$D_1 = P_{12}(P_{23}P_{34} - P_{33}P_{24}) - P_{13}(P_{22}P_{34} - P_{32}P_{24}) + P_{14}(P_{22}P_{33} - P_{32}P_{23}),$$

$$D_2 = P_{11}(P_{23}P_{34} - P_{33}P_{24}) - P_{13}(P_{21}P_{34} - P_{31}P_{24}) + P_{14}(P_{21}P_{33} - P_{31}P_{23}),$$

$$D_3 = P_{11}(P_{22}P_{34} - P_{32}P_{24}) - P_{12}(P_{21}P_{34} - P_{31}P_{24}) + P_{14}(P_{21}P_{32} - P_{31}P_{22}),$$

$$D_4 = P_{11}(P_{22}P_{33} - P_{32}P_{23}) - P_{12}(P_{21}P_{33} - P_{31}P_{23}) + P_{13}(P_{21}P_{32} - P_{31}P_{22}).$$

7. Special Cases

1. If we take,

$$(36) \quad \begin{aligned} c_{11} = c_{22} = c_{33}, \quad c_{12} = c_{13}, \quad c_{44} = c_{66}, \quad a_1 = a_3 = \beta_1, \\ b_1 = b_2 = \beta_2, \quad K_1 = K_3 = K, \quad \alpha_1^* = \alpha_3^* = D, \end{aligned}$$

in the above analysis, we obtain the results for the case of cubic crystal materials.

2. If we have,

$$(37) \quad \begin{aligned} c_{11} = c_{33} = \lambda + 2\mu, \quad c_{12} = c_{13} = \lambda, \quad c_{44} = \mu, \\ a_1 = a_3 = \beta_1 = (3\lambda + 2\mu)\alpha_t, \quad b_1 = b_2 = \beta_2 = (3\lambda + 2\mu)\alpha_c, \\ K_1 = K_3 = K, \quad \alpha_1^* = \alpha_3^* = D, \end{aligned}$$

the above analysis is reduced to the case of isotropic materials.

8. Numerical results and discussion

We consider the stress free, thermally insulated and impermeable boundary conditions, for the purpose of numerical calculation. The numerical results for copper material (thermoelastic diffusive solid) are taken from Kumar et al. [16]. The values of relaxation times are taken as:

$$\tau_0 = 0.1 \text{ s}, \quad \tau_1 = 0.03 \text{ s}, \quad \tau^0 = 0.01 \text{ s}, \quad \tau^1 = 0.4 \text{ s}.$$

Figures 1 and 2 show a flow chart to compute the phase velocity and attenuation coefficient of cylindrical Rayleigh waves by using Muller's method. The equation (35) is a complex transcendental one in two unknowns ξ and ω . The equation (35) can be written as $F(\xi) = 0$ for a given value of ω . Muller's method is used to find an approximate root of $F(\xi) = 0$. The algorithm of Muller method's in order to find phase velocity and attenuation coefficient is as follows:

1. Decide initially three approximations say ξ_1 , ξ_2 and ξ_3 of the root, number of iterations (maxit) and two error bounds (eps1 and eps2).
2. Put $I = 1$.
3. If $I \leq \text{maxit}$, then compute $F(\xi_1)$, $F(\xi_2)$ and $F(\xi_3)$, otherwise write "Process fails to converge the root" and go to step 10.
4. Compute q^* , x^* , y^* and z^* by the following relations:

$$q^* = \frac{\xi_3 - \xi_2}{\xi_2 - \xi_1}, \quad x^* = q^*F(\xi_3) - q^*(1 + q^*)F(\xi_2) + q^{*2}F(\xi_1),$$

$$y^* = (2q^* + 1)F(\xi_3) - (1 + q^*)^2F(\xi_2) + q^{*2}F(\xi_1), \quad z^* = (1 + q^*)F(\xi_3).$$

5. If $x^* \neq 0$, then calculate discriminant (disc) = $y^{*2} - 4x^*z^*$, $z_1 = y^* + \sqrt{\text{disc}}$, $z_2 = y^* - \sqrt{\text{disc}}$.

6. If $|z_1| \leq |z_2|$, then compute:

$$\xi_4 = \xi_3 - \frac{2z^*(\xi_3 - \xi_2)}{z_2}$$

Otherwise compute:

$$\xi_4 = \xi_3 - \frac{2z^*(\xi_3 - \xi_2)}{z_1}.$$

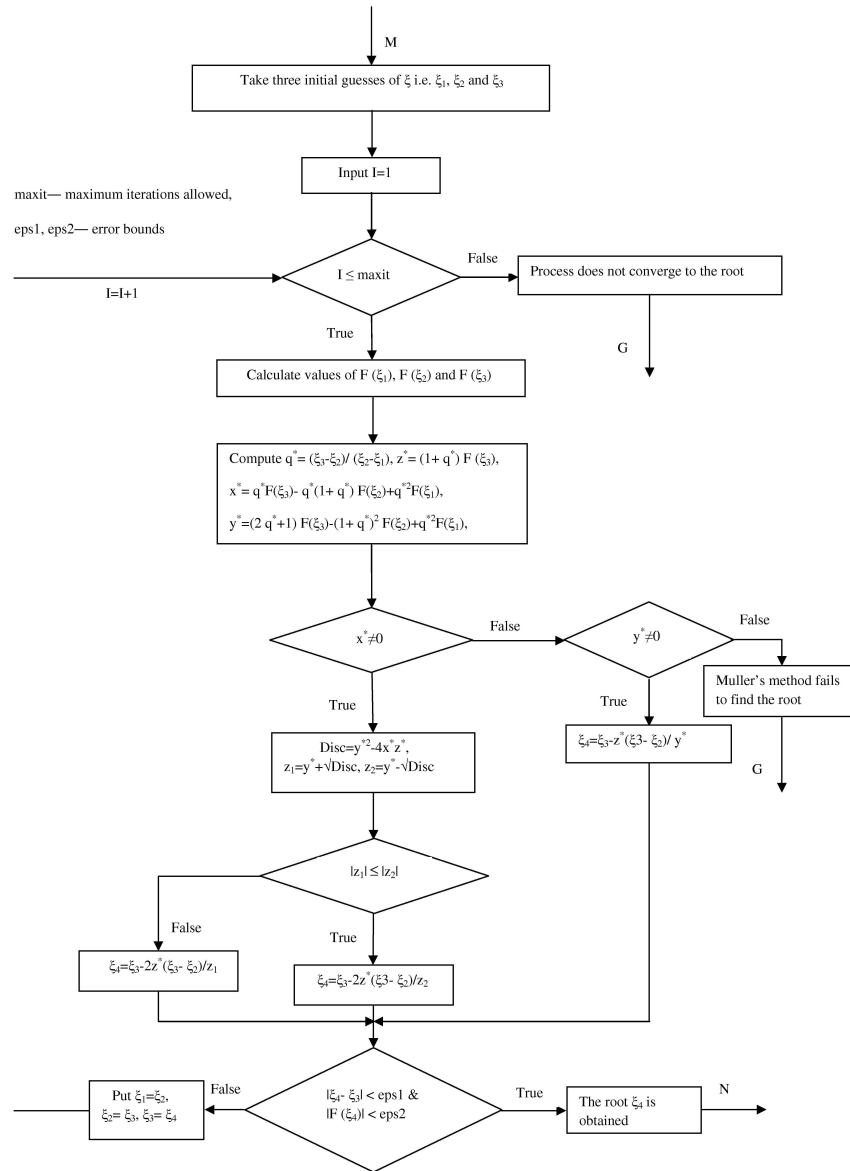


Fig. 1. Flow chart to find an approximate root of $F(\eta) = 0$

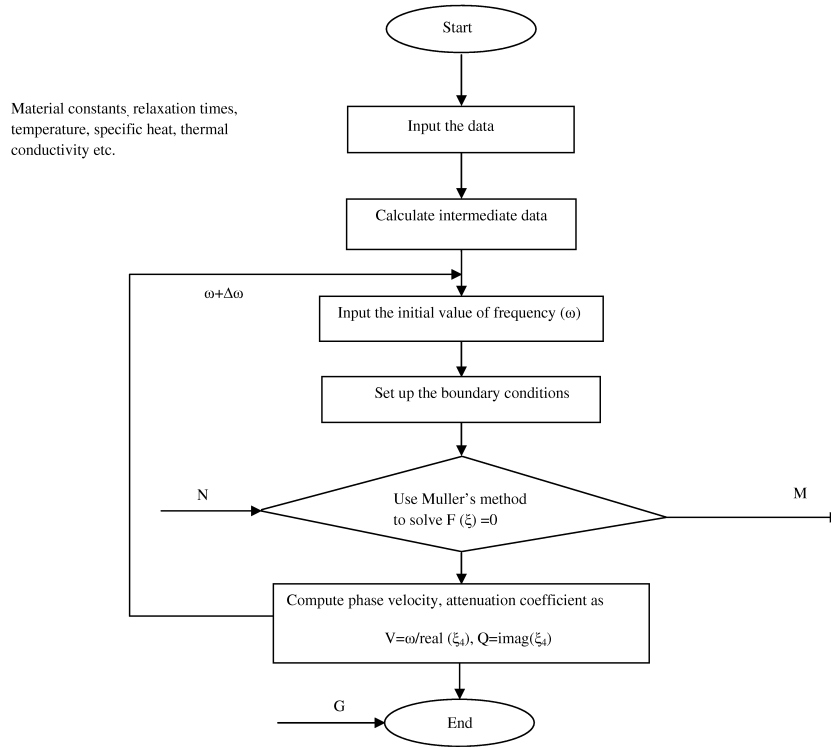


Fig. 2. Flow chart for calculating phase velocity, attenuation coefficient

7. If $|\xi_4 - \xi_3| < \text{eps1}$ and If $|F(\xi_4)| < \text{eps2}$, then root ξ_4 is obtained and go to step 9. Otherwise put $\xi_1 = \xi_2$, $\xi_2 = \xi_3$ and $\xi_3 = \xi_4$, $I = I + 1$ and go to step 3.

8. Otherwise if $x^* = 0$, then check whether $y^* \neq 0$ or $y^* = 0$. If $y^* \neq 0$, then calculate $\xi_4 = \xi_3 - \frac{z^*(\xi_3 - \xi_2)}{y^*}$, and go to step 7. If not, write "Muller's method fails to find the root" and go to step 10.

9. Compute phase velocity (V) and attenuation coefficient (Q) as:

$$V = \frac{\omega}{\text{real}(\xi_4)}, \quad Q = \text{Im}(\xi_4).$$

10. Stop the process.

The variations of phase velocity and attenuation coefficient with respect to frequency have been plotted in Figs 3 and 4. In all the figures, the solid line corresponds to CT theory of thermoelastic diffusion and small dash and long dash lines represent L-S and G-L theories of thermoelastic diffusion, respectively.

It is noticed in Fig. 3, that the values of phase velocity increase with the increase in the values of frequency in all three theories of thermoelastic diffusion, that is, CT, L-S and G-L. Corresponding to G-L theory, the values are larger in comparison to CT and L-S theories except in the initial range that is $0.1 \leq \omega \leq 0.4$. Figure 4 indicates that as the value of frequency increases, initially there is a sharp increase in the values of attenuation coefficient in CT, L-S and G-L theories, but finally the value decreases. Corresponding

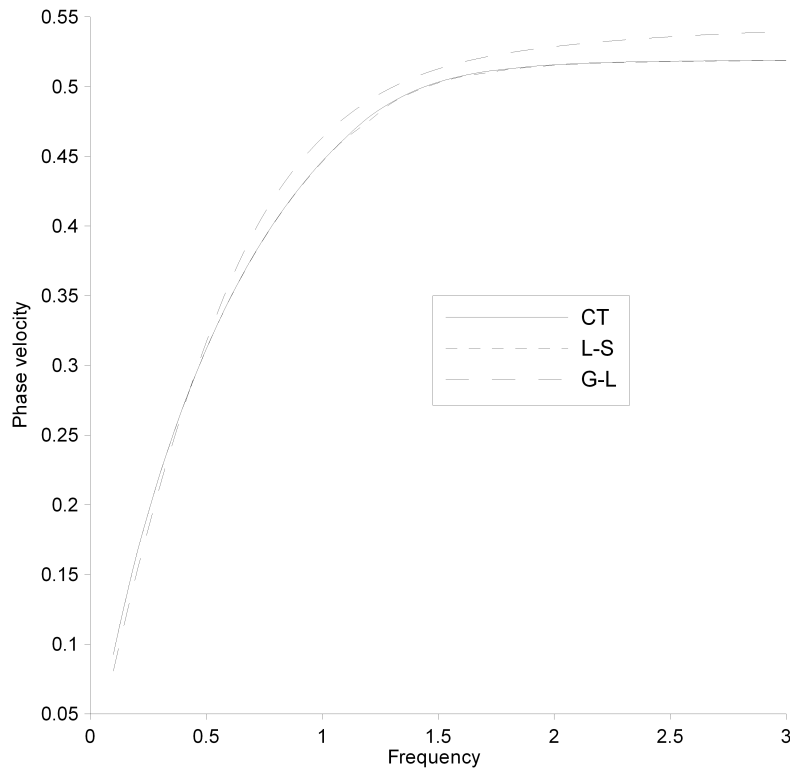


Fig.3. Variations of Phase velocity w.r.t. frequency

Fig. 3. Variations of Phase velocity w.r.t. frequency

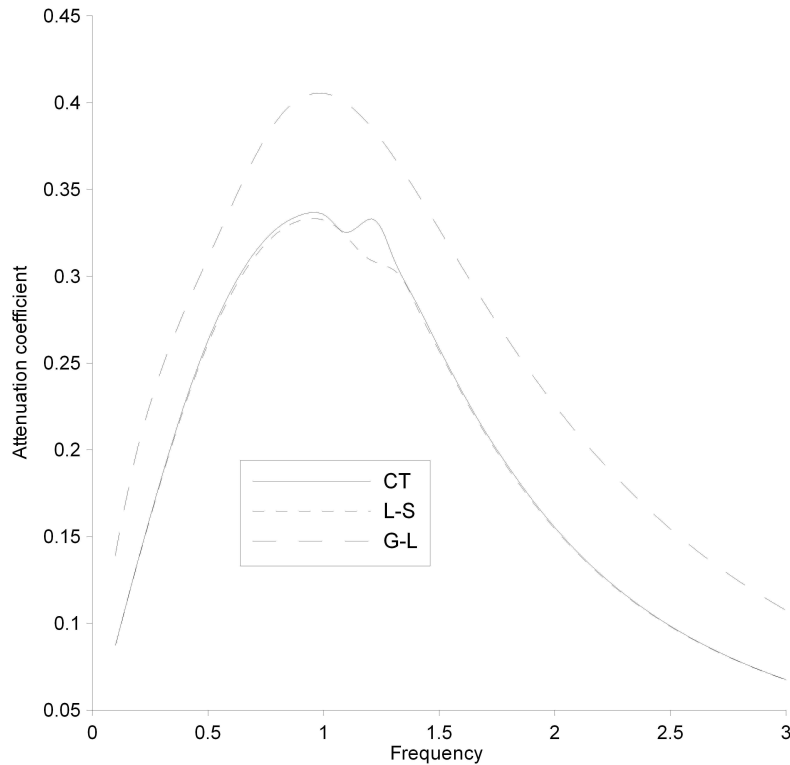


Fig.4. Variations of attenuation coefficient w.r.t. frequency

Fig. 3. Variations of attenuation coefficient w.r.t. frequency

to CT theory, the values of attenuation coefficient lie in between the values corresponding to L-S and G-L theories.

9. Conclusions

In the present paper, the propagation of cylindrical Rayleigh waves in a transversely isotropic thermoelastic diffusive solid half-space subjected to stress free, isothermal/insulated and impermeable or isoconcentrated boundary conditions is investigated in the framework of different theories of thermoelastic diffusion. Muller's method is used to compute the phase velocity and attenuation coefficient from the dispersion equation. It is found, that the values of phase velocity increase with the increase in the values of frequency, whereas initially the values attenuation coefficient increases, but finally decreases. The

values of phase velocity and attenuation coefficient are more in G-L theory in comparison to CT and L-S theories except in the initial range.

REFERENCES

- [1] NOWACKI, W. Dynamical Problems of Thermodiffusion in Solids-I, Bulletin of Polish Academy of Sciences Series, Science and Technology, **22** (1974(a)), 55–64.
- [2] NOWACKI, W. Dynamical Problems of Thermodiffusion in Solids-II, Bulletin of Polish Academy of Sciences Series, Science and Technology, **22** (1974(b)), 205–211.
- [3] NOWACKI, W. Dynamical Problems of Thermodiffusion in Solids-III, Bulletin of Polish Academy of Sciences Series, Science and Technology, **22** (1974(c)), 257–266.
- [4] NOWACKI, W. Dynamical Problems of Thermodiffusion in Solids. *Engineering Fracture Mechanics*, **8** (1976), 261–266.
- [5] SHERIEF, H. H., H. SALEH. A Half Space Problem in the Theory of Generalized Thermoelastic Diffusion. *International Journal of Solids and Structures*, **42** (2005), 4484–4493.
- [6] SINGH, B. Reflection of P and SV Waves from free Surface of an Elastic Solid with Generalized Thermodiffusion. *Journal of Earth and System and Sciences*, **114** (2005), No. 2, 159–168.
- [7] SINGH, B. Reflection of SV Waves from free Surface of an Elastic Solid in Generalized Thermodiffusion. *Journal of Sound and Vibration*, **291** (2006), Nos 3–5, 764–778.
- [8] AOUDI, M. Variable Electrical and Thermal Conductivity in the Theory of Generalized Thermodiffusion. *ZAMP*, **57** (2006(a)), No. 2, 351–367.
- [9] AOUDI, M. A Generalized Thermoelastic Diffusion Problem for an Infinitely Long Solid Cylinder. *International Journal of Mathematics and Mathematical Sciences*, (2006(b)), No. 13, Article ID 25976, 15 p.
- [10] AOUDI, M. A Problem for an Infinite Elastic Body with a Spherical Cavity in the Theory of Generalized Thermoelastic Diffusion. *International Journal of Solids and Structures*, **44** (2007), 5711–5722.
- [11] GAWINECKI, J. A., A. SZYMANIEC. Global Solution of the Cauchy Problem in Nonlinear Thermoelastic Diffusion in Solid Body, PAMM. Proc. Appl. Math. Mech., **1** (2002), 446–447.
- [12] GAWINECKI, J.A., P. KACPRZYK, P. BAR-YOSEPH. Initial Boundary Value Problem for Some Coupled Nonlinear Parabolic System of Partial Differential Equations Appearing in Thermoelastic Diffusion in Solid Body. *J. Anal. Appl.*, **19** (2000), 121–130.

- [13] SHERIEF, H. H., F. HAMZA, H. SALEH. The Theory of Generalized Thermoelastic Diffusion. *International Journal of Engineering Science*, **42** (2004), 591–608.
- [14] AOUADI, M. Uniqueness and Reciprocity Theorems in the Theory of Generalized Thermoelastic Diffusion. *Journal of Thermal Stresses*, **30** (2007), 665–678.
- [15] AOUADI, M. Generalized Theory of Thermoelastic Diffusion for Anisotropic Media. *Journal of Thermal Stresses*, **31** (2008), 270–285.
- [16] KUMAR, R., T. KANSAL. Propagation of Lamb Waves in Transversely Isotropic Thermoelastic Diffusive Plate. *International Journal of Solids and Structures*, **45** (2008), 5890–5913.

Appendix

$$\begin{aligned}
f_1 &= p_3 \tau_c^{11} (q_6^* \delta_4 - q_2^* p_2), \\
f_2 &= \xi^2 \tau_c^{11} (q_6^* + p_3 q_5^*) - \omega (q_6^* \tau_c^{11} \tau_t^{10} + \zeta_1 \tau_c^{10} \tau_t^{11} q_4^* + p_3 \tau_f^{10}), \\
f_3 &= \xi^4 \tau_c^{11} q_5^* - \omega^2 \tau_t^{10} \tau_f^{10} - \omega \xi^2 (q_5^* \tau_c^{11} \tau_t^{10} + \zeta_1 \tau_c^{10} \tau_t^{11} q_3^* + \tau_f^{10}), \\
f_4 &= \omega p_1 \tau_t^{11} \tau_f^{10} - \tau_t^{11} \tau_c^{11} \xi^2 (p_1 q_5^* - p_2 q_3^*), \\
f_5 &= \omega (p_1 \tau_t^{11} \zeta_1 \tau_c^{10} - p_2 \tau_c^{11} \tau_t^{10}) + p_2 \tau_c^{11} \xi^2, \\
f_6 &= -\omega p_1 \zeta_2 \tau_e^{10} \tau_t^{11} \tau_c^{11} (q_6^* p_1 - q_4^* p_2) + (\delta_1 - \omega^2) q_6^* p_3 \tau_c^{11} + \delta_4 f_2 - q_2^* f_5 \\
&\quad - q_8^* p_2 p_3 \tau_c^{11} \xi^2, \\
f_7 &= (\delta_1 - \omega^2) f_2 + \delta_4 f_3 + \omega p_1 \zeta_2 \tau_e^{10} f_4 - q_8^* f_5 \xi^2, \\
f_8 &= (\delta_1 - \omega^2) f_3, \quad f_9 = -\omega p_3 \tau_c^{11} (q_6^* \delta_2 - q_7^* p_2), \\
f_{10} &= -\iota \xi \delta_2 f_2 - \xi \omega \zeta_2 \tau_e^{10} \tau_t^{11} \tau_c^{11} (q_6^* p_1 - q_4^* p_2) + \iota \xi^3 p_2 p_3 q_1^* \tau_c^{11} + \iota \xi q_7^* f_5, \\
f_{11} &= -\iota \xi \delta_2 f_3 + \xi \omega \zeta_2 \tau_e^{10} f_4 + \iota \xi^3 q_1^* f_5, \\
f_{12} &= \iota \xi \omega \zeta_2 \tau_e^{10} \tau_c^{11} (q_6^* \delta_4 - q_2^* \delta_2) + p_1 (q_7^* p_2 - q_6^* \delta_2) - \iota \xi \omega \zeta_1 \tau_c^{10} (q_2^* - q_7^* \delta_4), \\
f_{13} &= \iota \xi \omega \zeta_2 \tau_e^{10} \tau_c^{11} (\xi^2 (-q_8^* p_2 + q_5^* \delta_4 + p_1 (q_7^* p_2 - q_5^* \delta_2)) - q_6^* (-\delta_1 \xi^2 + \omega^2) \\
&\quad + \omega \tau_f^{10} (p_1 \delta_2 - \delta_4)) - \iota \xi \omega \zeta_1 \tau_c^{10} (\xi^2 (q_8^* - q_1^* \delta_4) + q_7^* (-\delta_1 \xi^2 + \omega^2)),
\end{aligned}$$

$$f_{14} = \iota\xi\omega(-\delta_1\xi^2 + \omega^2)(-\xi^2\zeta_1\tau_e^{10}q_1^* + \zeta_2\tau_e^{10}(\iota\omega\tau_f^{10} - \tau_e^{11}\xi^2q_5^*)),$$

$$f_{15} = \xi p_3(q_2^* - q_7^*\delta_4),$$

$$f_{16} = \iota\xi\omega p_1\zeta_2\tau_e^{10}\tau_t^{11}(-q_2^* + q_7^*p_1) + p_3\xi(\xi^2(q_8^* - q_1^*\delta_4) + q_7^*(-\delta_1\xi^2 + \omega^2)) \\ - \xi(\iota\omega\tau_t^{10} - \xi^2)(q_2^* - q_7^*\delta_4) + \iota\xi\omega\zeta_2\tau_e^{10}\tau_t^{11}q_4^*(p_1\delta_2 - \delta_4),$$

$$f_{17} = \iota\xi^3\omega\zeta_2\tau_e^{10}\tau_t^{11}p_1(-q_8^* + q_1^*p_1) + p_3\xi^3q_1^*(-\delta_1\xi^2 + \omega^2) \\ - \xi(\iota\omega\tau_t^{10} - \xi^2)(\xi^2(q_8^* - q_1^*\delta_4) + q_7^*(-\delta_1\xi^2 + \omega^2)) \\ + \iota\xi\omega\zeta_2\tau_e^{10}\tau_t^{11}(\xi^2q_3^*(p_1\delta_2 - \delta_4) + q_4^*(-\delta_1\xi^2 + \omega^2)),$$

$$f_{18} = \xi^3(-\delta_1\xi^2 + \omega^2)(-q_1^*(\iota\omega\tau_t^{10} - \xi^2) + \iota\omega\zeta_2\tau_e^{10}\tau_t^{11}q_3^*),$$

$$f_{19} = f_1(\xi^2 - \omega^2) + \delta_1f_6 - \iota\xi\delta_2f_9 + \xi\tau_c^{11}f_{15},$$

$$f_{20} = f_6(\xi^2 - \omega^2) + \delta_1f_7 - \iota\xi\delta_2f_{10} - \xi\tau_t^{11}f_{12} + \xi\tau_c^{11}f_{16},$$

$$f_{21} = f_7(\xi^2 - \omega^2) + \delta_1f_8 - \iota\xi\delta_2f_{11} + \xi\tau_c^{11}f_{17} - \xi\tau_t^{11}f_{13},$$

$$f_{22} = f_8(\xi^2 - \omega^2) + \xi\tau_c^{11}f_{18} - \xi\tau_t^{11}f_{14},$$

$$n_{1p} = -\frac{\xi^5m_p^5f_9 + \xi^3m_p^3f_{10} + \xi m_p f_{11}}{\xi^6m_p^6f_1 + \xi^4m_p^4f_6 + \xi^2m_p^2f_7 + f_8},$$

$$n_{2p} = \frac{\xi^4m_p^4f_{12} + \xi^2m_p^2f_{13} + f_{14}}{\xi^6m_p^6f_1 + \xi^4m_p^4f_6 + \xi^2m_p^2f_7 + f_8},$$

$$n_{3p} = -\frac{\xi^6m_p^6f_{15} + \xi^4m_p^4f_{16} + \xi^2m_p^2f_{17} + f_{18}}{\xi^6m_p^6f_1 + \xi^4m_p^4f_6 + \xi^2m_p^2f_7 + f_8}, \quad p = 1, 2, 3, 4,$$

$$A^* = \frac{f_{19}}{\delta_1f_1\xi^2}, \quad B^* = \frac{f_{20}}{\delta_1f_1\xi^4}, \quad C^* = \frac{f_{21}}{\delta_1f_1\xi^6}, \quad D^* = \frac{f_{22}}{\delta_1f_1\xi^8}.$$