

SURFACE WAVE SPEED OF FUNCTIONALLY GRADED MAGNETO-ELECTRO-ELASTIC MATERIALS WITH INITIAL STRESSES*

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ABSTRACT. The shear surface wave at the free traction surface of half-infinite functionally graded magneto-electro-elastic material with initial stress is investigated. The material parameters are assumed to vary exponentially along the thickness direction, only. The velocity equations of shear surface wave are derived on the electrically or magnetically open circuit and short circuit boundary conditions, based on the equations of motion of the graded magneto-electro-elastic material with the initial stresses and the free traction boundary conditions. The dispersive curves are obtained numerically and the influences of the initial stresses and the material gradient index on the dispersive curves are discussed. The investigation provides a basis for the development of new functionally graded magneto-electro-elastic surface wave devices.

KEY WORDS: Initial stress, graded material, magneto-electro-elastic material, surface wave, open and short circuit

1. Introduction

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Functionally graded materials have been known to possess extensive applications in many fields, such as aerospace, electronics, protection industries and so on. Much work has been performed on the research like thermomechanical response and fracture behaviour of functionally graded material since their appearance. A key feature of such analyses is that the determination of the current state of the material, as characterized by the distributions of stresses, strains, displacements and damage through the thickness, is a strong function of the material initial condition. While most of the foregoing analyses envision the initial state as the ‘stress-free’ condition at which the sintering, diffusion bonding or spray deposition of the material is accomplished. It is widely recognized that essentially all processing methods produce ‘intrinsic’ or ‘quench’ stresses, over and above the ‘thermal stresses’ induced during temperature excursions from the ‘stress-free’ processing temperature (or relaxation temperature) to service temperature [1]. The consideration of ‘initial mechanical state’ during the research of functionally graded materials is vital to the success of high performance design.

These initial stresses impose a pronounced influence on the propagation of waves. Biot [2] showed that the acoustic wave propagation under initial stress was fundamentally different from the stress free case and could not be represented by simply introducing the stress dependent elastic coefficients into the classical theory. In his treatment, he considered the fluid as a particular case of an elastic medium under initial stress with zero rigidity. Liu et al. [3], Jin et al. [4], Qian et al. [5] studied the effect of initial stress on the propagation behaviour of Love waves in an elastic substrate with a piezoelectric layer and a piezoelectric substrate with an elastic layer, respectively. Danoyan and Piliposian [6] studied the existence and behaviour of surface electro-elastic horizontal waves in the similar layered structure, when the shear bulk wave velocity in the elastic layer is greater than, or equal to that in the substrate. Recently, Qian et al. [7] discussed the effect of initial stress on Love waves in a piezoelectric structure carrying a functionally graded material layer. Gupta et al. [8] studied the torsion surface wave propagation in an initially stressed non-homogeneous layer over a non-homogeneous half-space.

Magneto-electro-elastic materials possess two functions of piezoelectric and piezomagnetic property, especially with the electromagnetic coupling effect while a single piezoelectric or piezomagnetic material does not have. The functionally gradient magneto-electro-elastic materials are widely used in surface acoustic wave devices. Thus, many people are of a great interest to the wave propagation research of magneto-electro-elastic materials. Li and Wei [9] investigated the piezoelectric and piezomagnetic effects and the influence

of short and open circuit on the surface wave speed in half-infinite magneto-electro-elastic material. Chen et al. [10] discussed the elastic wave propagation in magneto-electro-elastic multilayered plates. Melkumyan [11] derived the analytical solutions of shear surface waves in magneto-electro-elastic materials on many kinds of different boundary conditions. Liu et al. [12] researched the Love waves propagating in layered piezoelectric piezomagnetic structures. Wu et al. [13] studied the wave propagation in anisotropic nonhomogeneous magneto-electro-elastic plates. Pan et al. [14] acquired exact solution for functionally graded and layered magneto-electro-elastic plates, whose material parameters change linearly. Yang et al. [15] have discussed SH (shear horizontal) wave scattering on cracks in infinite magneto-electro-elastic materials. In all the above-mentioned researches, the initial stresses induced during the processing, were not taken into account.

In present contribution, we study the propagation behaviour of shear surface wave in a functionally graded magneto-electro-elastic half-space with initial stresses and discuss the effects of the initial stresses and material gradient index on the surface wave velocities. For convenience in the analysis, we assume that material properties change exponentially along the thickness direction. The speed equations of the shear surface wave are derived under the initial stresses on different electrically and magnetically boundary conditions. Some significant results have been obtained, which can provide some help for the design and production of surface wave devices composed of functionally graded magneto-electro-elastic materials.

2. The velocity equation of shear surface wave

The constitutive equations of the magnetic-electric-elastic solid can be written as:

$$(1a) \quad \sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{kij}E_k - h_{kij}H_k$$

$$(1b) \quad D_i = e_{ikl}\varepsilon_{kl} + \kappa_{ik}E_k + \beta_{ik}H_k$$

$$(1c) \quad B_i = h_{ikl}\varepsilon_{kl} + \beta_{ik}E_k + \mu_{ik}H_k$$

where:

$$(2) \quad \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad E_i = -\phi_{,i}, \quad H_i = -\varphi_{,i}$$

in the quasi-static approximation. The quasi-static approximation is valid and provided that the variations of the electromagnetic field with time are enough

small. The frequency of electromagnetic field is the same with the frequency of the surface wave in the present considered problem. The frequency of the surface wave reflects the vibration frequency of mass point and it is usually very low. Moreover, the size of the surface acoustic wave device was several orders smaller in magnitude than the wavelength of the electromagnetic wave corresponding working frequency. So, the coupled electromagnetic field with the mechanical field can be assumed into the quasi-static electromagnetic field. The assumption is usually valid for surface acoustic wave at MHz and below. For the wave motion of small amplitude, the equations of motion of the magnetic-electric-elastic solid with initial stresses can be written as [5]:

$$(3) \quad \sigma_{ji,j} + (u_{i,k}\sigma_{kj}^0)_{,j} = \rho\ddot{u}_i \quad , \quad D_{i,i} = 0 \quad , \quad B_{i,i} = 0$$

In Eqs (1)–(3), σ_{ij} is the stress tensor; σ_{kj}^0 is the initial stress; ε_{ij} the strain tensor; E_i the electric field vector; H_i the magnetic field vector. The coefficients c_{ijkl} , κ_{ik} and μ_{ik} are the elastic constants, dielectric constants and the magnetic permittivity. The coefficients e_{kij} , h_{kij} and β_{ik} are the piezoelectric, piezomagnetic and electromagnetic constants. u_i is the mechanical displacement vector D_i the electric displacement vector, B_i the magnetic induction vector. ϕ , φ are the electric and magnetic potential, ρ the mass density.

For a transversely isotropic medium with x_3 -axis being the symmetric axis and the poling axis of the magneto-electro-elastic material, the constitutive equations can be written in term of components:

$$(4) \quad \begin{aligned} \sigma_{11} &= c_{11}\varepsilon_{11} + c_{12}\varepsilon_{22} + c_{13}\varepsilon_{33} - e_{31}E_3 - h_{31}H_3, \\ \sigma_{22} &= c_{12}\varepsilon_{11} + c_{11}\varepsilon_{22} + c_{13}\varepsilon_{33} - e_{31}E_3 - h_{31}H_3, \\ \sigma_{33} &= c_{13}\varepsilon_{11} + c_{13}\varepsilon_{22} + c_{33}\varepsilon_{33} - e_{33}E_3 - h_{33}H_3, \\ \sigma_{23} &= 2c_{44}\varepsilon_{23} - e_{15}E_2 - h_{15}H_2, \\ \sigma_{31} &= 2c_{44}\varepsilon_{31} - e_{15}E_1 - h_{15}H_1, \\ \sigma_{12} &= (c_{11} - c_{12})\varepsilon_{12}, \\ D_1 &= 2e_{15}\varepsilon_{31} + \kappa_{11}E_1 + \beta_{11}H_1, \\ D_2 &= 2e_{15}\varepsilon_{23} + \kappa_{11}E_2 + \beta_{11}H_2, \\ D_3 &= e_{31}\varepsilon_{11} + e_{31}\varepsilon_{22} + e_{33}\varepsilon_{33} + \kappa_{33}E_3 + \beta_{33}H_3, \\ B_1 &= 2h_{15}\varepsilon_{31} + \beta_{11}E_1 + \mu_{11}H_1, \\ B_2 &= 2h_{15}\varepsilon_{23} + \beta_{11}E_2 + \mu_{11}H_2, \\ B_3 &= h_{31}\varepsilon_{11} + h_{31}\varepsilon_{22} + h_{33}\varepsilon_{33} + \beta_{33}E_3 + \mu_{33}H_3. \end{aligned}$$

The material constants and the initial stresses are assumed to vary exponentially along the thickness direction, i.e.:

$$(5) \quad \begin{aligned} c_{ij}(x_2) &= c_{ij}^0 e^{kx_2}, & e_{ij}(x_2) &= e_{ij}^0 e^{kx_2}, \\ h_{ij}(x_2) &= h_{ij}^0 e^{kx_2}, & \kappa_{ij}(x_2) &= \kappa_{ij}^0 e^{kx_2}, \\ \beta_{ij}(x_2) &= \beta_{ij}^0 e^{kx_2}, & \mu_{ij}(x_2) &= \mu_{ij}^0 e^{kx_2}, \\ \rho &= \rho^0 e^{kx_2}, & \sigma_{ij}^0(x_2) &= \sigma_{ij0}^0 e^{kx_2}, \end{aligned}$$

where: $c_{ij}^0 = c_{ij}(0)$, $e_{ij}^0 = e_{ij}(0)$, $h_{ij}^0 = h_{ij}(0)$, $\kappa_{ij}^0 = \kappa_{ij}(0)$, $\beta_{ij}^0 = \beta_{ij}(0)$, $\mu_{ij}^0 = \mu_{ij}(0)$, $\rho^0 = \rho(0)$ and $\sigma_{ij0}^0 = \sigma_{ij}^0(0)$. k is the functional gradient index.

For the shear surface wave, the mechanical displacement components and the electric and magnetic potential are as following:

$$(6) \quad \begin{aligned} u(x_1, x_2) &= v(x_1, x_2) = 0, & w &= w(x_1, x_2, t), \\ \phi &= \phi(x_1, x_2, t), & \varphi &= \varphi(x_1, x_2, t). \end{aligned}$$

Substituting Eq.(2), Eq.(4)–Eq.(6) into Eq.(3), only consider the initial stresses σ_{11}^0 , σ_{22}^0 , we have the following equations of motion:

$$(7a) \quad c_{44}^0 \nabla^2 w + e_{15}^0 \nabla^2 \phi + h_{15}^0 \nabla^2 \varphi + k(c_{44}^0 w_{,2} + e_{15}^0 \phi_{,2} + h_{15}^0 \varphi_{,2}) = \rho^0 \ddot{w} - \sigma_1^0 w_{,11} - \sigma_2^0 w_{,22}$$

$$(7b) \quad e_{15}^0 \nabla^2 w - \kappa_{11}^0 \nabla^2 \phi - \beta_{11}^0 \nabla^2 \varphi + k(e_{15}^0 w_{,2} - \kappa_{11}^0 \phi_{,2} - \beta_{11}^0 \varphi_{,2}) = 0$$

$$(7c) \quad h_{15}^0 \nabla^2 w - \beta_{11}^0 \nabla^2 \phi - \mu_{11}^0 \nabla^2 \varphi + k(h_{15}^0 w_{,2} - \beta_{11}^0 \phi_{,2} - \mu_{11}^0 \varphi_{,2}) = 0$$

where: $\nabla^2 = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2$, $\sigma_1^0 = \sigma_{110}^0$, $\sigma_2^0 = \sigma_{220}^0$. Introduce the two functions:

$$(8) \quad \psi = \phi - mw, \quad \chi = \varphi - nw.$$

Substitution of Eq.(8) into Eq.(7) yields:

$$(9a) \quad \bar{c}_{44}^0 (\nabla^2 w + kw_{,2}) = \rho^0 \ddot{w} - \sigma_1^0 w_{,11} - \sigma_2^0 w_{,22},$$

$$(9b) \quad \nabla^2 \psi + k\psi_{,2} = 0,$$

$$(9c) \quad \nabla^2 \chi + k\chi_{,2} = 0.$$

In Eqs (8)–(9),

$$(10) \quad m = \frac{\mu_{11}^0 e_{15}^0 - \beta_{11}^0 h_{15}^0}{\kappa_{11}^0 \mu_{11}^0 - (\beta_{11}^0)^2}, \quad n = \frac{\kappa_{11}^0 h_{15}^0 - \beta_{11}^0 e_{15}^0}{\kappa_{11}^0 \mu_{11}^0 - (\beta_{11}^0)^2},$$

$$(11) \quad \bar{c}_{44}^0 = c_{44}^0 + \frac{[\mu_{11}^0 (e_{15}^0)^2 + \kappa_{11}^0 (h_{15}^0)^2 - 2\beta_{11}^0 e_{15}^0 h_{15}^0]}{[\kappa_{11}^0 \mu_{11}^0 - (\beta_{11}^0)^2]}.$$

Then, the stress tensor, electric displacement vector and the magnetic induction vector in Eqs (4) can be expressed in term of w , ψ and χ :

$$(12) \quad \begin{aligned} \sigma_{11} &= \sigma_{22} = \sigma_{33} = \sigma_{12} = 0, \quad D_3 = 0, \quad B_3 = 0, \\ \sigma_{23} &= e^{kx_2} (\bar{c}_{44}^0 w_{,2} + e_{15}^0 \psi_{,2} + h_{15}^0 \chi_{,2}), \\ \sigma_{13} &= e^{kx_2} (\bar{c}_{44}^0 w_{,1} + e_{15}^0 \psi_{,1} + h_{15}^0 \chi_{,1}), \\ D_1 &= e^{kx_2} (-\kappa_{11}^0 \psi_{,1} - \beta_{11}^0 \chi_{,1}), \\ D_2 &= e^{kx_2} (-\kappa_{11}^0 \psi_{,2} - \beta_{11}^0 \chi_{,2}), \\ B_1 &= e^{kx_2} (-\beta_{11}^0 \psi_{,1} - \mu_{11}^0 \chi_{,1}), \\ B_2 &= e^{kx_2} (-\beta_{11}^0 \psi_{,2} - \mu_{11}^0 \chi_{,2}). \end{aligned}$$

Let the area $x_2 \geq 0$ is the functional gradient magneto-electro-elastic material, marked as A, and the area $x_2 \leq 0$ is vacuum, marked as B, as shown in Fig. 1. Let ϕ_A and φ_A denote the electric and magnetic potential, D_A and B_A the electric displacement vector and the magnetic induction vector along x_2 in the region A. The stress σ_{23} is marked as σ_A .

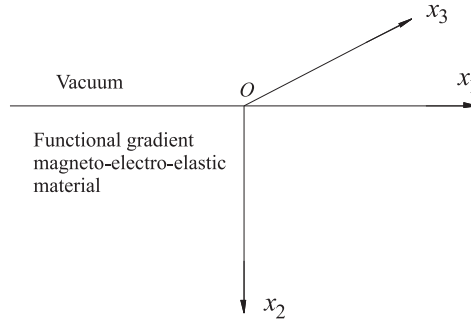


Fig. 1. Half-infinite functionally-graded magneto-electro-elastic material

For $x_2 \rightarrow +\infty$, $w = 0$, $\phi_A = 0$, $\varphi_A = 0$. The solution of Eq. (9) can be assumed as:

$$(13a) \quad w = A_1 e^{-\eta x_2} e^{i(\xi x_1 - \omega t)},$$

$$(13b) \quad \psi = A_2 e^{-\zeta x_2} e^{i(\xi x_1 - \omega t)},$$

$$(13c) \quad \chi = A_3 e^{-\zeta x_2} e^{i(\xi x_1 - \omega t)},$$

where: A_1 , A_2 and A_3 are unknown constants, ξ and ω the wave number and the angular frequency. Substitution of Eq.(13) into Eq.(12) yields:

$$(14) \quad \begin{aligned} \sigma_A &= -e^{kx_2} (\bar{c}_{44}^0 \eta A_1 e^{-\eta x_2} + e_{15}^0 \zeta A_2 e^{-\zeta x_2} + h_{15}^0 \zeta A_3 e^{-\zeta x_2}) e^{i(\xi x_1 - \omega t)}, \\ D_A &= e^{kx_2} (\kappa_{11}^0 \zeta A_2 e^{-\zeta x_2} + \beta_{11}^0 \zeta A_3 e^{-\zeta x_2}) e^{i(\xi x_1 - \omega t)}, \\ B_A &= e^{kx_2} (\beta_{11}^0 \zeta A_2 e^{-\zeta x_2} + \mu_{11}^0 \zeta A_3 e^{-\zeta x_2}) e^{i(\xi x_1 - \omega t)}, \\ \phi_A &= (A_2 e^{-\zeta x_2} + mA_1 e^{-\eta x_2}) e^{i(\xi x_1 - \omega t)}, \\ \varphi_A &= (A_3 e^{-\zeta x_2} + nA_1 e^{-\eta x_2}) e^{i(\xi x_1 - \omega t)} \end{aligned}$$

Substituting Eq.(13) into Eq.(9), for $v < v_A$, we have:

$$(15a) \quad \left(1 + \frac{\sigma_2^0}{\bar{c}_{44}^0}\right) \eta^2 - k\eta = \xi^2 \left(1 - \frac{v^2}{v_A^2} + \frac{\sigma_1^0}{\bar{c}_{44}^0}\right) > 0$$

$$(15b) \quad \zeta^2 - \zeta k = \xi^2$$

Then the solution of η , ζ can be obtained from Eq.(15) as:

$$(16a) \quad \eta = \frac{k + \sqrt{k^2 + 4 \left(1 + \frac{\sigma_2^0}{\bar{c}_{44}^0}\right) \left(1 - \frac{v^2}{v_A^2} + \frac{\sigma_1^0}{\bar{c}_{44}^0}\right) \xi^2}}{2 \left(1 + \frac{\sigma_2^0}{\bar{c}_{44}^0}\right)} > 0,$$

$$(16b) \quad \zeta = \frac{k + \sqrt{k^2 + 4\xi^2}}{2},$$

where: $v = \omega/\xi$ is the surface wave velocity and $v_A^2 = \bar{c}_{44}^0/\rho^0$.

In the vacuum area B, the electric potential ϕ_B and the magnetic potential φ_B satisfy Laplace's equations, i.e.:

$$(17) \quad \nabla^2 \phi_B = 0, \quad \nabla^2 \varphi_B = 0.$$

For $x_2 \rightarrow -\infty$, $\phi_B = 0$, $\varphi_B = 0$. The solution of Eq. (17) can be assumed to possess the following form:

$$(18a) \quad \phi_B = B_1 e^{\xi x_2} e^{i(\xi x_1 - \omega t)},$$

$$(18b) \quad \varphi_B = B_2 e^{\xi x_2} e^{i(\xi x_1 - \omega t)},$$

where: B_1 and B_2 are unknown constants. In vacuum, the electric displacement vector and the magnetic induction vector are expressed as, respectively:

$$(19a) \quad D_B = \kappa_0 E_B = -\kappa_0 \frac{\partial \phi_B}{\partial x_2} = -\kappa_0 B_1 e^{\xi x_2} \xi e^{i(\xi x_1 - \omega t)},$$

$$(19b) \quad B_B = \mu_0 H_B = -\mu_0 \frac{\partial \varphi_B}{\partial x_2} = -\mu_0 B_2 e^{\xi x_2} \xi e^{i(\xi x_1 - \omega t)},$$

where: κ_0 is the dielectric constant and μ_0 is the magnetic permittivity in vacuum.

The mechanical traction-free and magnetically and electrically short circuit conditions at $x_2 = 0$, satisfy:

$$(20) \quad \sigma_A(x_1, 0, t) = 0, \quad \phi_A(x_1, 0, t) = 0, \quad \varphi_A(x_1, 0, t) = 0,$$

which results in the algebraic equations in the unknowns A_1 , A_2 , A_3 :

$$(21a) \quad \bar{c}_{44}^0 \eta A_1 + e_{15}^0 \zeta A_2 + h_{15}^0 \zeta A_3 = 0,$$

$$(21b) \quad A_2 + m A_1 = 0,$$

$$(21c) \quad A_3 + n A_1 = 0.$$

The nontrivial solution of A_1 , A_2 and A_3 exists only if the determinants of the coefficient matrix of Eq.(21) equals to zero, i.e.:

$$(22) \quad \begin{vmatrix} \bar{c}_{44}^0 \eta & e_{15}^0 \zeta & h_{15}^0 \zeta \\ m & 1 & 0 \\ n & 0 & 1 \end{vmatrix} = 0$$

from Eq.(22), we have:

$$(23) \quad \frac{\eta}{\zeta} = \frac{(m e_{15}^0 + n h_{15}^0)}{\bar{c}_{44}^0} = \frac{e_{15}^0 (\mu_{11}^0 e_{15}^0 - \beta_{11}^0 h_{15}^0) + h_{15}^0 (\kappa_{11}^0 h_{15}^0 - \beta_{11}^0 e_{15}^0)}{\bar{c}_{44}^0 [\kappa_{11}^0 \mu_{11}^0 - (\beta_{11}^0)^2]}.$$

Substituting Eq. (16) into Eq. (23) leads to the following wave velocity equation:

$$(24) \quad v^2 = v_A^2 \left(1 + \frac{\sigma_1^0}{\bar{c}_{44}^0} - \frac{\lambda^4 \left(1 + \frac{\sigma_2^0}{\bar{c}_{44}^0} \right) (k + \sqrt{k^2 + 4\xi^2})^2 - 2k\lambda^2(k + \sqrt{k^2 + 4\xi^2})}{4\xi^2} \right)$$

where:

$$(25) \quad \lambda^2 = \frac{e_{15}^0(\mu_{11}^0 e_{15}^0 - \beta_{11}^0 h_{15}^0) + h_{15}^0(\kappa_{11}^0 h_{15}^0 - \beta_{11}^0 e_{15}^0)}{\bar{c}_{44}^0[\kappa_{11}^0 \mu_{11}^0 - (\beta_{11}^0)^2]}$$

The mechanical traction-free and magnetically and electrically open circuit conditions at $x_2 = 0$ satisfy:

$$(26) \quad \begin{aligned} \sigma_A(x_1, 0, t) &= 0, \\ \phi_A(x_1, 0, t) &= \phi_B(x_1, 0, t), \quad \varphi_A(x_1, 0, t) = \varphi_B(x_1, 0, t), \\ D_A(x_1, 0, t) &= D_B(x_1, 0, t), \quad B_A(x_1, 0, t) = B_B(x_1, 0, t), \end{aligned}$$

Substitution of Eq. (14), Eq. (18) and Eq. (19) into Eq. (26), we can derive the algebraic equations in the unknowns A_1 , A_2 , A_3 , B_1 , and B_2 :

$$(27) \quad \begin{aligned} \bar{c}_{44}^0 \eta A_1 + e_{15}^0 \zeta A_2 + h_{15}^0 \zeta A_3 &= 0, \\ A_2 + m A_1 - B_1 &= 0, \\ A_3 + n A_1 - B_2 &= 0, \\ \kappa_{11}^0 \zeta A_2 + \beta_{11}^0 \zeta A_3 + \kappa_0 \xi B_1 &= 0, \\ \beta_{11}^0 \zeta A_2 + \mu_{11}^0 \zeta A_3 + \mu_0 \xi B_2 &= 0. \end{aligned}$$

The nontrivial solution exists only if the determinants of the coefficient matrix of Eq. (27) equals to zero, i.e.:

$$(28) \quad \begin{vmatrix} \bar{c}_{44}^0 \eta & e_{15}^0 \zeta & h_{15}^0 \zeta & 0 & 0 \\ m & 1 & 0 & -1 & 0 \\ n & 0 & 1 & 0 & -1 \\ 0 & \kappa_{11}^0 \zeta & \beta_{11}^0 \zeta & \kappa_0 \xi & 0 \\ 0 & \beta_{11}^0 \zeta & \mu_{11}^0 \zeta & 0 & \mu_0 \xi \end{vmatrix} = 0.$$

From Eq. (28) , we have:

$$(29) \quad \frac{\eta}{\zeta} = \frac{h_{15}^0(\zeta\kappa_{11}^0 + \kappa_0\xi)\mu_0n\xi + e_{15}^0(\zeta\mu_{11}^0 + \mu_0\xi)\kappa_0m\xi - h_{15}^0\zeta\beta_{11}^0\kappa_0m\xi - e_{15}^0\zeta\beta_{11}^0\mu_0n\xi}{\bar{c}_{44}^0[(\zeta\kappa_{11}^0 + \kappa_0\xi)(\zeta\mu_{11}^0 + \mu_0\xi) - \zeta^2(\beta_{11}^0)^2]}$$

Substituting Eq. (16) into Eq. (29) leads to the following wave velocity equation:

$$(30) \quad v^2 = v_A^2 \left(1 + \frac{\sigma_1^0}{\bar{c}_{44}^0} - \frac{\tau^4 \left(1 + \frac{\sigma_2^0}{\bar{c}_{44}^0} \right) \left(k + \sqrt{k^2 + 4\xi^2} \right)^2 - 2k\tau^2 \left(k + \sqrt{k^2 + 4\xi^2} \right)}{4\xi^2} \right),$$

where:

$$(31) \quad \tau^2 = \frac{h_{15}^0(\zeta\kappa_{11}^0 + \kappa_0\xi)\mu_0n\xi + e_{15}^0(\zeta\mu_{11}^0 + \mu_0\xi)\kappa_0m\xi - h_{15}^0\zeta\beta_{11}^0\kappa_0m\xi - e_{15}^0\zeta\beta_{11}^0\mu_0n\xi}{\bar{c}_{44}^0[(\zeta\kappa_{11}^0 + \kappa_0\xi)(\zeta\mu_{11}^0 + \mu_0\xi) - \zeta^2(\beta_{11}^0)^2]}.$$

3. Numerical results and discussions

Consider the piezoelectric material BaTiO₃ and the piezomagnetic material CoFe₂O₄. Their material constants are listed in Table 1. The dielectric constant and the magnetic permeability in vacuum are $\kappa_0 = 8.854 \times 10^{-12} \text{C}^2 \cdot \text{N}^{-1} \cdot \text{m}^{-1}$ and $\mu_0 = 4\pi \times 10^{-7} \text{Ns}^2 \cdot \text{C}^{-2}$. In the numerical examples, the wave speed of shear surface wave at different gradient index and different initial stresses are computed and the results are shown graphically. The influence of the gradient index and the initial stress are discussed based on the numerical results.

Table 1. Material parameters of BaTiO₃ and CoFe₂O₄ [12]

Materials	$c_{44}^0 /$ (10 ⁹ N.m ⁻²)	$\rho^0 /$ (10 ³ kg.m ⁻³)	$\kappa_{11}^0 /$ (10 ⁻⁹ C ² .N ⁻¹ .m ⁻¹)	$\mu_{11}^0 /$ (10 ⁻⁶ Ns ² .C ⁻²)	$e_{15}^0 /$ (C.m ⁻²)	$h_{15}^0 /$ (N.A ⁻¹ .m ⁻¹)
BaTiO ₃	44.0	5.7	9.82	5	11.4	0
CoFe ₂ O ₄	45.3	5.3	0.08	157	0	550

First, the surface wave speed at different gradient index and the fixed initial stresses σ_1^0 is computed and the results are shown in Fig. 2 and Fig. 3. It can be seen that the shear surface wave is non-dispersive for the half infinite uniform medium but dispersive for the half-infinite gradient medium. The

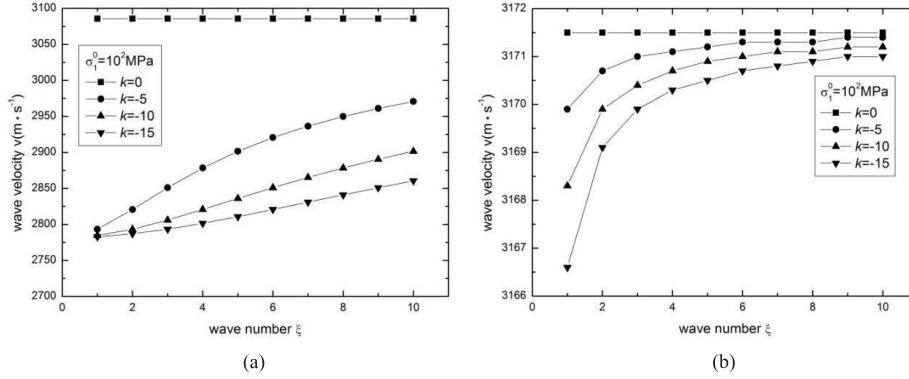


Fig. 2. The surface wave velocities of the piezoelectric material (BaTiO_3) with fixed initial stress σ_1^0 . (a) short circuit condition; (b) open circuit condition

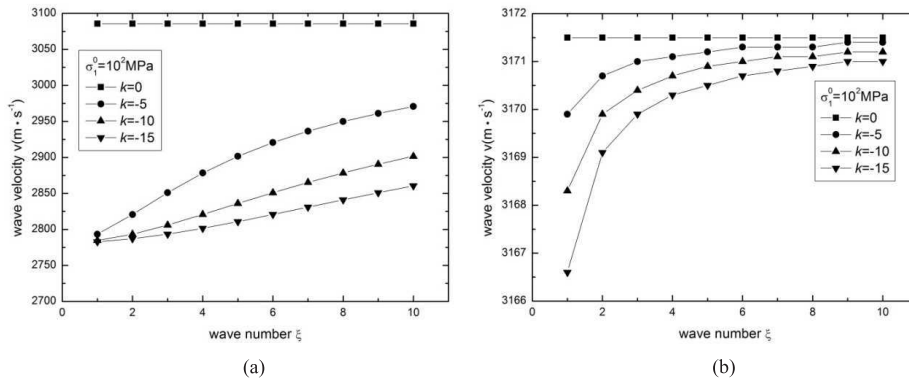


Fig. 3. The surface wave velocities of the piezomagnetic material (CoFe_2O_4) with fixed initial stress σ_1^0 . (a) short circuit condition; (b) open circuit condition

surface wave speed decreases gradually with the increase of the absolute value of gradient index for the piezoelectric and piezomagnetic medium, whether the boundary condition is a short circuit or open circuit. Furthermore, the influence of the gradient index on the surface wave speed is more evident at the low frequency, than at high frequency. Similar computations are performed for the fixed initial stresses σ_2^0 and similar trend is observed. It is found that the surface wave speed is more sensitive to the gradient index under electrically or magnetically short circuit condition, than under electrically or magnetically open circuit condition, for both piezoelectric and piezomagnetic materials under

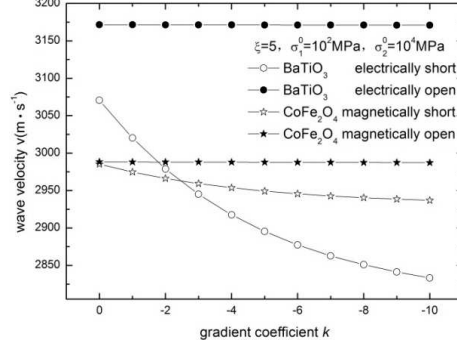


Fig. 4. The surface wave velocities at different gradient index and electrically and magnetically surface boundary for the piezoelectric and piezomagnetic material

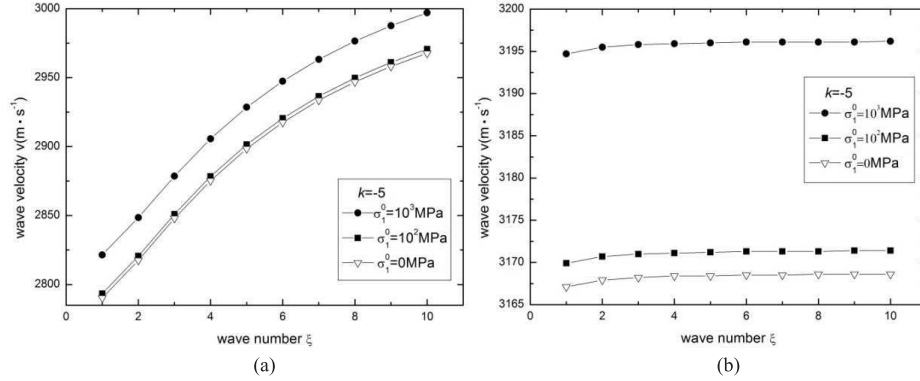


Fig. 5. The surface wave velocities of the piezoelectric material (BaTiO_3) with the fixed gradient index. (a) short circuit condition; (b) open circuit condition

short and open circuit conditions. This fact is shown in Fig. 4. In particular, the surface wave speed is the most sensitive to the gradient index for the piezoelectric material under the electrically short circuit condition.

The influence of the initial stress on the surface wave speed under the fixed gradient index are shown in Fig. 5 – Fig. 8. It is found, that the initial stresses σ_1^0 makes the surface wave speed increasing, while the initial stress σ_2^0 makes the surface wave speed decreasing. Furthermore, the effect of initial stress σ_1^0 is the same at different frequency, and the effect of initial stress σ_2^0 is more evident at high frequency, than at low frequency. Namely, the effect of initial stress σ_1^0 is frequency-independent, while the effect of initial stress σ_2^0

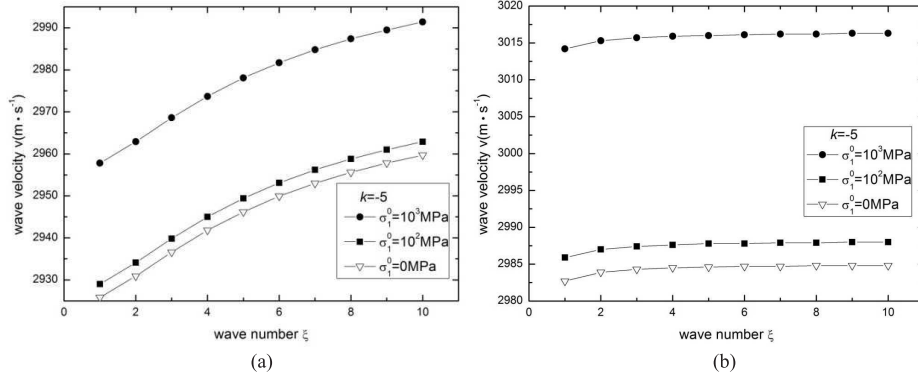


Fig. 6. The surface wave velocities of the piezomagnetic material (CoFe_2O_4) with fixed gradient index. (a) short circuit condition; (b) open circuit condition

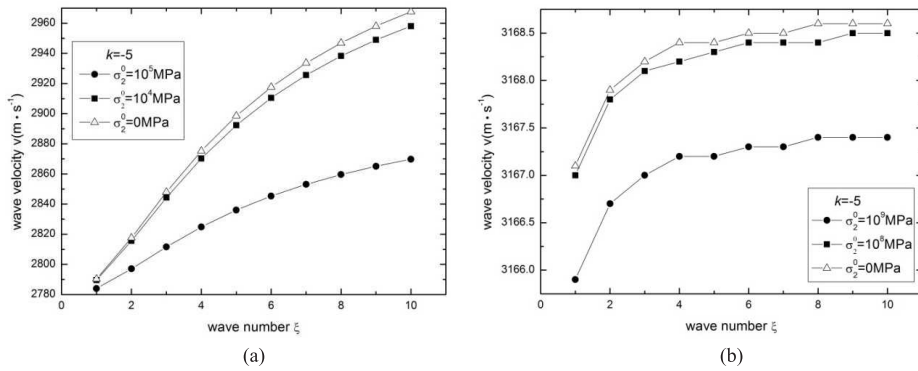


Fig. 7. The surface wave velocities of the piezoelectric material (BaTiO_3) with fixed gradient index. (a) short circuit condition; (b) open circuit condition

is frequency-dependent. Although the existence of initial stress σ_1^0 and σ_2^0 can make the surface wave speed changed, but this change is evident only when the value of σ_1^0 and σ_2^0 approach the value of c_{44} . This trend is shown in Fig. 9. The comparison of the effect of σ_1^0 and σ_2^0 shows that the effects of σ_1^0 is more evident than σ_2^0 . Besides, the piezoelectric material under short circuit condition, the effect of σ_2^0 is very small in the other cases. Therefore, if the initial stress is used to enhance the surface wave speed, the imposing σ_1^0 along the direction parallel to the free surface is better than imposing σ_2^0 along the direction normal to the free surface.

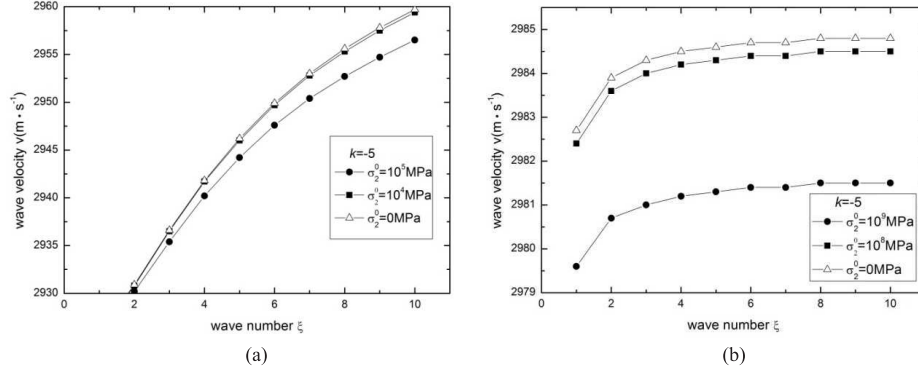


Fig. 8. The surface wave velocities of the piezomagnetic material (CoFe_2O_4) with fixed gradient index. (a) short circuit condition; (b) open circuit condition

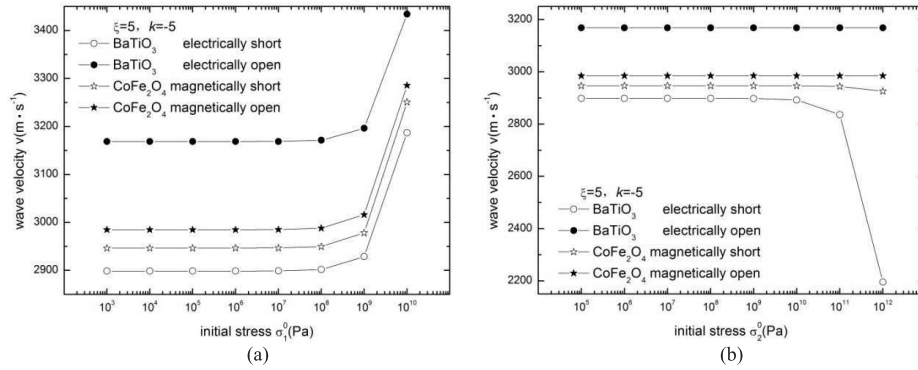


Fig. 9. The surface wave velocities of the piezoelectric and piezomagnetic material at different initial stress and electrically or magnetically short and open conditions. (a) at different initial stress σ_1^0 ; (b) at different initial stress σ_2^0

4. Conclusion

The shear surface wave can exist at the free-traction surface of half-infinite magnetic-electric-elastic medium. It is non-dispersive for half-infinite uniform medium but dispersive for the half-infinite graded medium. The surface wave speed decreases gradually with the increase of the absolute value of gradient index, whether the boundary condition is short circuit or open circuit. The surface wave speed is more sensitive to the gradient index under the short circuit condition, than under the open circuit condition. The initial stress has evident influence on the surface wave speed. In general, the existence of the

initial stress parallel to the surface has more evident influence than the initial stress perpendicular to the surface. Furthermore, the existence of the initial stress parallel to the surface makes the surface wave speed increasing but the existence of the initial stress perpendicular to the surface makes the surface wave speed decreasing. However, only when the initial stress approaches to the magnitude of elastic constants the effects of initial stress on the surface wave speed are pronounced.

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