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FLUID MECHANICS

RAYLEIGH WAVE IN A ROTATING NONLOCAL MAGNETOELASTIC HALF-PLANE*

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Abstract. This paper investigates the propagation of Rayleigh surface waves in a rotating semi-infinite solid medium, permeated by an initial magnetic field in the context of linear nonlocal elasticity. Frequency equations are derived and the combined effect of magnetic field and rotation on Rayleigh wave propagation, based on the linear theory of nonlocal elasticity has been studied. Effects of magnetic field, as well as rotation on Rayleigh wave propagation in a nonlocal medium, have also been analyzed in details as special cases. Numerical calculations, graphs and discussions presented in this paper lead us to some important conclusions. Fourier double integral transform technique has been applied to solve the problem.
Key words: Rayleigh wave, rotation, magneto-elasticity, nonlocal elasticity, attenuation exponent.

1. Introduction

Propagation of Rayleigh surface waves plays a very important role in the areas like engineering sciences, seismology, geophysics and geodynamics.

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Ivanov [1], Abd-Alla et al. [2], Shams and Ogden [3] discussed various problems on Rayleigh surface wave propagation in isotropic linear and nonlinear elastic half spaces. Acharya and Sengupta [4, 5] deduced and discussed micropolar thermoelastic Rayleigh wave velocity equation from two dimensional wave propagation, without stretch [4] and with stretch [5], in an infinite plate of finite thickness. Love [6] proved that Rayleigh surface wave, under the linear theory of classical elasticity, is non-dispersive in nature. But, experiments show that the atomic theory of lattices predicts otherwise. Gazis et al. [7], Eringen and Kim [8] treated such type of problems from the view point of lattice dynamics. The existence of dispersive character in such waves has been confirmed in the investigation made by Maradudin et al. [9]. However, no systematic studies of continuum theory exist, which may lead to similar conclusion in all such problems. Eringen [10], in his research paper indicated that a continuum approach to such problem has special advantages, due to many facts. Such a continuum approach leads us to the theory of nonlocal elasticity. The volume of literature on the subject of nonlocal continuum mechanics has been increasing gradually such as Inan and Eringen [11], Inan [12], Acharya and Mondal [13], Lazar et al. [14], Zhou and Wang [15], Chakraborty [16], due to its impressive agreements of the theoretical results with experimental studies. In this theory, the distant neighbours of a point have a role to play in the waves propagation. The stress at$(\vec{x}, t)$, in this case, depends on the strain at all other points$\{\vec{x}'\}$of the body, at time t. Eringen [11] in his above mentioned paper investigated Rayleigh surface waves with small wave length, under the linear nonlocal theory of elasticity and observed, that Rayleigh surface waves are definitely dispersive in nature, while the rate of amplitude attenuations of waves remains the same as in classical elasticity.

The study of elastic wave propagation in a rotating medium was initiated by Schoenberg and Censor [17]. Chandrasekharaiya [18], Sharma and Othman [19], Tomar and Ogden [20], Ogden and Singh [21] considered rotation in their problem under different situation. Effect of rotation on Rayleigh surface waves under the nonlocal theory of elasticity has been studied by Acharya and Mondal [22].

An increasing attention is being devoted to the interactions between magnetic and strain fields. Interactions between these two fields take place by means of Lorentz forces, which appear in the equations of motion, as well as by means of a term entering Ohm's law and describing the electric field, produced by the velocity of the material particle, moving in a magnetic field. Moreover, the earth is subject to its own magnetic field and the propagation of seismic waves on, or near the surface of the earth is affected by the presence of such magnetic field. Some of the most recent papers, published in this field, may be mentioned as Ezzat and Youssef [23], Bakshi et al. [24], Singh et al. [25], Shekhar and Pervez [26]. Hajdo and Eringen [27] investigated application of nonlocal theory to electromagnetic dispersion.

It is found during our review process that hardly any attention has been given to the effect of magnetic field on the propagation of Rayleigh waves based on the nonlocal theory of elasticity. In this research work, we investigate the combined effect of rotation and magnetic field on nonlocal Rayleigh surface waves in a semi-infinite medium. Effects of rotation and magnetic field on nonlocal Rayleigh waves have been studied and discussed in details as special cases. Numerical computations are performed in different cases and graphs are depicted to highlight the effect of rotation and magnetic field, as well as their combined effect on the propagation of Rayleigh waves in a nonlocal elastic medium. Some important observations have also been pointed out.

2. Basic equations

We introduce an elastic half space, occupying a region $x_2 \geq 0$ in the rectangular Cartesian coordinate system $Ox_1x_2x_3$, the origin O is situated at any point on the plane boundary $x_2 = 0$ and Ox_2 points vertically downwards into the bulk of the material medium, as shown in Fig. 1. The elastic medium is rotating with a uniform angular velocity $\mathbf{\Omega} = \Omega \mathbf{n}$, where Ω is the magnitude of the vector $\mathbf{\Omega}$ and $\mathbf{n}$ is a unit vector representing the direction of the axis of rotation. The displacement equation of motion in the rotating frame of reference has two additional terms (Schoenberg and Censor [17]): Centripetal acceleration, $\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u})$ due to time varying motion only and the Coriolis

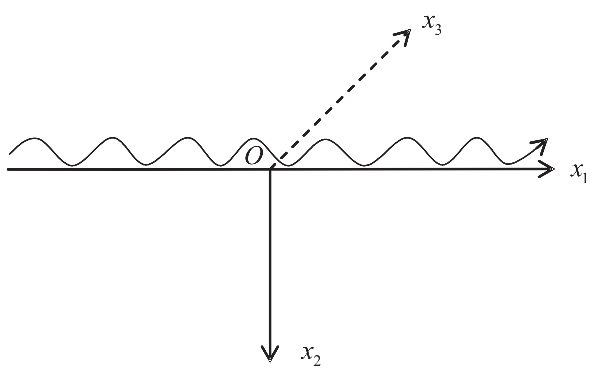

Fig. 1. Problem geometry

acceleration $2\mathbf{\Omega} \times \dot{\mathbf{u}}$, where $\mathbf{u}$ is the dynamic displacement vector and $\dot{\mathbf{u}}$ (dot indicates differentiation with respect to time) represents the particle velocity. Introduction of an initial magnetic field $\mathbf{H} = (0, 0, H_0)$ gives rise to an induced magnetic field $\mathbf{h} = (0, 0, h)$ and an induced electric field $\mathbf{E}$.

For a slowly moving homogeneous, electrically conducting elastic solid medium, the simplified linear equations of electrodynamics are (Ezzat and Youssef [23], Singh, et al. [25]):

$$\text{curl } \mathbf{h} = \mathbf{J}, \tag{1}$$

$$\text{curl } \mathbf{E} = -\mu_0 \dot{\mathbf{h}}, \tag{2}$$

$$\text{div } \mathbf{h} = 0, \tag{3}$$

$$\mathbf{E} = -\mu_0 \left(\dot{\mathbf{u}} \times \mathbf{H}\right), \tag{4}$$

where $\dot{\mathbf{u}}$ is the particle velocity of the medium, $\mathbf{J}$ is the current density vector, μ_0 is the magnetic permeability. Moreover, the deformation is supposed to be small.

Due to the application of an initial magnetic field $\mathbf{H}$, the relevant constitutive equations of linear nonlocal elasticity, the equations of motion and the stress tensor in a medium which rotates with a uniform angular velocity $\mathbf{\Omega}$, are given by Othman and Song [28], Acharya and Mondal [22], Eringen [10]:

$$\sigma_{ij,i} + \mu_0 \left(\mathbf{J} \times \mathbf{H}\right)_j = \rho \left[\ddot{\mathbf{u}}_j + \{\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u})\}_j + \left(2\mathbf{\Omega} \times \dot{\mathbf{u}}\right)_j\right], \tag{5}$$

$$\sigma_{ij} = \lambda u_{r,r}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + \int_V \left[\lambda' u_{r,r'}\delta_{ij} + \mu'\left(u_{i,j'} + u_{j,i'}\right)\right] dV\left(\overline{x}'\right), \tag{6}$$

where, σ_{ij}, ρ and u_i are stress tensor, mass density and displacement vector, respectively. $\mu_0 \left(\mathbf{J} \times \mathbf{H}\right)$ are the components of Lorentz force. λ, μ are Lame' elastic constants, λ', μ' are nonlocal elastic modulii, which depend on $\left|\overline{x} - \overline{x}'\right|$ for homogeneous solid:

$$\overline{u} = u_i,\ \sigma_{ij,i} \equiv \frac{\partial \sigma_{ij}}{\partial x_i},\ u_{i,j} \equiv \frac{\partial u_i\left(\overline{x}, t\right)}{\partial x_j},\ u_{i,j'} \equiv \frac{\partial u_i\left(\overline{x}', t\right)}{\partial x'_j},\ \dot{u}_i \equiv \frac{\partial u_i\left(\overline{x}, t\right)}{\partial t},$$

$$\ddot{u}_i \equiv \frac{\partial^2 u_i\left(\overline{x}, t\right)}{\partial t^2},\ \overline{x} = x_i \equiv \left(x_1, x_2, x_3\right),\ \overline{x}' = x'_i \equiv \left(x'_1, x'_2, x'_3\right),\ t = \text{time}.$$

Since the stress at $(\overline{x}, t)$ depends on the strain at all other points $\{\overline{x}'\}$ of the body at time t, introduction of the volume integral in (6) has been made which indicates the effect of distant neighbour of a point $\overline{x}$. This creates the basic difference between the classical elasticity and the nonlocal elasticity (Eringen [10]).

3. Basic assumptions

For a two dimensional problem, one has to consider the domain of x_1 as $-\infty < x_1 < +\infty$ and that of x_2 as $0 < x_2 < \infty$. Moreover, it is assumed that everything is uniform in the x_3 direction. We consider here the possibility of a type of wave travelling in the direction of x_1 axis in such a manner, that the disturbance is largely confined in the neighbourhood of the boundary and at any instant all particles on any line parallel to Ox_3 possess equal displacement. Due to the first assumption, the wave is a surface one, which is an essential condition of Rayleigh wave and the second assumption induces that all partial derivatives with respect to x_3 are zero. In this case, the volume integral in (6) is reduced to a surface integral over x_1' and x_2' in their ranges.

4. Boundary conditions

We now describe the following boundary conditions to be satisfied for the considered problem:

a. Since the boundary surface $x_2 = 0$ is stress free, we have:

$$\sigma_{21} = \sigma_{22} = 0 \quad \text{for} \quad x_2 = 0. \tag{7}$$

b. Since the Rayleigh wave is a surface wave, we have:

$$u_1,\, u_2 \to 0 \quad \text{as} \quad x_2 \to \infty. \tag{8}$$

5. Problem formulation

The components of magnetic intensity vector in the medium are taken as:

$$H_{x_1} = 0, \quad H_{x_2} = 0, \quad H_{x_3} = H_0 + h\left(x_1, x_2, t\right). \tag{9}$$

The electric intensity vector is normal to magnetic intensity vector. Thus, it has components:

$$E_{x_1} = E_1, \quad E_{x_2} = E_2, \quad E_{x_3} = 0. \tag{10}$$

The current density vector $\mathbf{J}$ is parallel to $\mathbf{E}$. Thus:

$$J_{x_1} = J_1, \quad J_{x_2} = J_2, \quad J_{x_3} = 0. \tag{11}$$

Hence, the components of Lorentz forces for the present problem are:

$$\left.\begin{array}{l} \mu_0 (J \times H)_1 = \mu_0 H_0^2 (u_{1,11} + u_{2,12}) \\ \mu_0 (J \times H)_2 = \mu_0 H_0^2 (u_{1,21} + u_{2,22}) \\ \mu_0 (J \times H)_3 = 0 \end{array}\right\}. \tag{12}$$

Moreover, we set $\mathbf{\Omega} = (0, 0, \omega_3)$.
The dynamical equations of motion may be deduced from (5), as:

$$\sigma_{11,1} + \sigma_{21,2} + \mu_0 H_0^2 (u_{1,11} + u_{2,12}) = \rho \ddot{u}_1 - \rho \left(\omega_3^2 u_1 + 2\omega_3 \dot{u}_2\right), \tag{13}$$

$$\sigma_{12,1} + \sigma_{22,2} + \mu_0 H_0^2 (u_{1,21} + u_{2,22}) = \rho \ddot{u}_2 - \rho \left(\omega_3^2 u_2 - 2\omega_3 \dot{u}_1\right). \tag{14}$$

6. Problem solution

Following Eringen [10], we introduce Fourier double integral transform in the following form:

$$u_i (x_1, x_2, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \overline{u}_i (\xi, x_2, \omega) e^{-i(\xi x_1 + \omega t)} d\xi d\omega. \tag{15}$$

Applying the transform (15) in (5), (13) and (14) one obtains:

$$-i\xi\overline{\sigma}_{11} + \overline{\sigma}_{21,2} + \left\{\rho\left(\omega^2 + \omega_3^2\right) - \mu_0 H_0^2 \xi^2\right\} \overline{u}_1 - 2i\rho\omega\omega_3\overline{u}_2 - i\xi\mu_0 H_0^2 \overline{u}_{2,2} = 0, \tag{16}$$

$$-i\xi\overline{\sigma}_{12} + \overline{\sigma}_{22,2} + \rho\left(\omega^2 + \omega_3^2\right)\overline{u}_2 + 2i\rho\omega\omega_3\overline{u}_1 - i\xi\mu_0 H_0^2 \overline{u}_{1,2} + \mu_0 H_0^2 \overline{u}_{2,22} = 0, \tag{17}$$

where:

$$\overline{\sigma}_{11} = -i\xi (\lambda + 2\mu) \overline{u}_1 + \lambda \overline{u}_{2,2} + \int_0^{\infty} \left[-i\xi \left(\lambda' + 2\overline{\mu}'\right) \overline{u}_1 + \overline{\lambda}' \overline{u}_{2,2'}\right] dx_2', \tag{18}$$

$$\overline{\sigma}_{12} = \mu (\overline{u}_{1,2} - i\xi \overline{u}_2) + \int_0^{\infty} \overline{\mu}' \left(\overline{u}_{1,2'} - i\xi \overline{u}_2\right) dx_2', \tag{19}$$

$$\overline{\sigma}_{22} = -i\xi\lambda\overline{u}_1 + (\lambda + 2\mu)\,\overline{u}_{2,2} + \int_0^\infty \left[-i\xi\overline{\lambda}'\overline{u}_1 + \left(\overline{\lambda}' + 2\overline{\mu}'\right)\overline{u}_{2,2'}\right] dx_2'. \tag{20}$$

Since λ' and μ' tend to zero rapidly as $\left|\overline{x}' - \overline{x}\right| \to \infty$, we may assume the expressions for λ' and μ' in the following forms (Eringen [10]):

$$\left.\begin{aligned} \overline{\lambda}' &= \overline{\lambda}\,(\xi)\,\delta\left(\left|x_2' - x_2\right|\right), \\ \overline{\mu}' &= \overline{\mu}\,(\xi)\,\delta\left(\left|x_2' - x_2\right|\right) \end{aligned}\right\}, \tag{21}$$

where, δ is the Dirac delta function.

Using (21), the stress components (18), (19) and (20) take the forms:

$$\overline{\sigma}_{11} = -i\xi\left\{\left(\lambda + \overline{\lambda}\right) + 2\left(\mu + \overline{\mu}\right)\right\}\overline{u}_1 + \left(\lambda + \overline{\lambda}\right)\overline{u}_{2,2}, \tag{22}$$

$$\overline{\sigma}_{12} = \left(\mu + \overline{\mu}\right)\left(\overline{u}_{1,2} - i\xi\overline{u}_2\right), \tag{23}$$

$$\overline{\sigma}_{22} = -i\xi\left(\lambda + \overline{\lambda}\right)\overline{u}_1 + \left\{\left(\lambda + \overline{\lambda}\right) + 2\left(\mu + \overline{\mu}\right)\right\}\overline{u}_{2,2}. \tag{24}$$

Replacing the expressions for $\overline{\sigma}_{11}, \overline{\sigma}_{12}$, and $\overline{\sigma}_{22}$ from (22), (23) and (24) in (16) and (17) and then substituting:

$$\overline{u}_k\left(\xi, x_2, \omega\right) = \overline{U}_k\left(\xi, \omega\right) e^{-\alpha x_2}, \tag{25}$$

equations (16) and (17) transform to:

$$\left[\alpha^2 - \frac{k^2}{h^2}\xi^2 + k^2\right]\overline{U}_1 + i\alpha\xi\left[\frac{k^2}{h^2} - 1 - \frac{2k^2\omega\omega_3}{\alpha\xi\left(\omega^2 + \omega_3^2\right)}\right]\overline{U}_2 = 0, \tag{26}$$

$$i\alpha\xi\left[\frac{k^2}{h^2} - 1 + \frac{2k^2\omega\omega_3}{\alpha\xi\left(\omega^2 + \omega_3^2\right)}\right]\overline{U}_1 + \left[\alpha^2\frac{k^2}{h^2} - \xi^2 + k^2\right]\overline{U}_2 = 0, \tag{27}$$

where:

$$\left.\begin{aligned} k^2 &= \frac{\rho\left(\omega^2 + \omega_3^2\right)}{\left(\mu + \overline{\mu}\right)}, \\ h^2 &= \frac{\rho\left(\omega^2 + \omega_3^2\right)}{\left(\lambda + \overline{\lambda}\right) + 2\left(\mu + \overline{\mu}\right) + \mu_0 H_0^2} \end{aligned}\right\}. \tag{28}$$

Elimination of $\overline{U}_1$ and $\overline{U}_2$from (26) and (27) leads to a quadratic equation in α^2, its roots are given by:

$$\left.\begin{array}{l}\alpha_1^2+\alpha_2^2=2\xi^2-h^2-k^2,\\ \alpha_1^2\alpha_2^2=\xi^4-\xi^2k^2-\xi^2h^2+k^2h^2-\dfrac{4k^2h^2\rho^2\omega^2\omega_3^2}{\left(\omega^2+\omega_3^2\right)^2}\end{array}\right\}. \tag{29}$$

Since $\overline{u}_1,\overline{u}_2\to 0$ as $x_2\to\infty$, the expressions for $\overline{u}_1,\overline{u}_2$ may be taken as:

$$\left.\begin{array}{l}\overline{u}_1=e^{-\alpha_1x_2}\overline{U}_{11}+e^{-\alpha_2x_2}\overline{U}_{12},\\ \overline{u}_2=\gamma_1e^{-\alpha_1x_2}\overline{U}_{11}+\gamma_2e^{-\alpha_2x_2}\overline{U}_{12}\end{array}\right\}, \tag{30}$$

where:

$$\gamma_j=\frac{-i\alpha_j\left[\frac{\xi}{h^2}-\frac{\xi}{k^2}+\frac{2\omega\omega_3}{\alpha_j\left(\omega^2+\omega_3^2\right)}\right]}{\left[\frac{\alpha_j^2}{h^2}-\frac{\xi^2}{k^2}+1\right]}. \tag{31}$$

Using (30) in (22), (23), (24) and then applying boundary conditions (7), one obtains the following:

$$\left.\begin{array}{l}\left(\alpha_1+i\xi\gamma_1\right)\overline{U}_{11}+\left(\alpha_2+i\xi\gamma_2\right)\overline{U}_{12}=0,\\ \left[i\xi\left\{\dfrac{k^2-2h^2}{k^2h^2}-\dfrac{\mu_0H_0^2}{\left(\mu+\overline{\mu}\right)k^2}\right\}+\left\{\dfrac{1}{h^2}-\dfrac{\mu_0H_0^2}{\left(\mu+\overline{\mu}\right)k^2}\right\}\alpha_1\gamma_1\right]\overline{U}_{11}+\\ \left[i\xi\left\{\dfrac{k^2-2h^2}{k^2h^2}-\dfrac{\mu_0H_0^2}{\left(\mu+\overline{\mu}\right)k^2}\right\}+\left\{\dfrac{1}{h^2}-\dfrac{\mu_0H_0^2}{\left(\mu+\overline{\mu}\right)k^2}\right\}\alpha_2\gamma_2\right]\overline{U}_{12}=0\end{array}\right\}. \tag{32}$$

Elimination of $\overline{U}_{11}$ and $\overline{U}_{12}$from (32) leads to the following equation:

(33)

$$\begin{array}{l}\left[i\xi\left\{\dfrac{k^2-2h^2}{k^2h^2}-\dfrac{\mu_0H_0^2}{\left(\mu+\overline{\mu}\right)k^2}\right\}+\left\{\dfrac{1}{h^2}-\dfrac{\mu_0H_0^2}{\left(\mu+\overline{\mu}\right)k^2}\right\}\alpha_2\gamma_2\right]\left(\alpha_1+i\xi\gamma_1\right)-\\ \left[i\xi\left\{\dfrac{k^2-2h^2}{k^2h^2}-\dfrac{\mu_0H_0^2}{\left(\mu+\overline{\mu}\right)k^2}\right\}+\left\{\dfrac{1}{h^2}-\dfrac{\mu_0H_0^2}{\left(\mu+\overline{\mu}\right)k^2}\right\}\alpha_1\gamma_1\right]\left(\alpha_2+i\xi\gamma_2\right)=0.\end{array}$$

Equation (33) gives the frequency equation for Rayleigh waves in a medium, which rotates with a uniform angular velocity $(0,0,\omega_3)$ under the theory of nonlocal elasticity, due to the application of initial magnetic field $(0,0,H_0)$.

In the absence of magnetic field, the above frequency equation for Rayleigh waves in a rotating medium under the theory of nonlocal elasticity may be presented as:

$$\left[i\xi\frac{k^2-2h^2}{k^2h^2}+\frac{\alpha_2\gamma_2}{h^2}\right](\alpha_1+i\xi\gamma_1)-\left[i\xi\frac{k^2-2h^2}{k^2h^2}+\frac{\alpha_1\gamma_1}{h^2}\right](\alpha_2+i\xi\gamma_2)=0, \tag{34}$$

where:

$$k^2=\frac{\rho\left(\omega^2+\omega_3^2\right)}{(\mu+\overline{\mu})},\ h^2=\frac{\rho\left(\omega^2+\omega_3^2\right)}{\left(\lambda+\overline{\lambda}\right)+2\left(\mu+\overline{\mu}\right)}. \tag{35}$$

The equation (34) represents the frequency equation for Rayleigh waves in a rotating medium, based on the theory of nonlocal elasticity and is in perfect agreement with the result obtained by Acharya and Mandal [22].

In the absence of rotation, the frequency equation for Rayleigh waves in magneto elastic nonlocal medium is given by the same equation (33), but the expressions for $k^2, h^2, \gamma_j, \alpha_1^2, \alpha_2^2$ are revised as:

$$\left.\begin{aligned} &k^2=\frac{\rho\omega^2}{(\mu+\overline{\mu})},\ h^2=\frac{\rho\omega^2}{\left(\lambda+\overline{\lambda}\right)+2\left(\mu+\overline{\mu}\right)+\mu_0H_0^2},\\ &\gamma_j=\frac{-i\alpha_j\left[\frac{\xi}{h^2}-\frac{\xi}{k^2}\right]}{\left[\frac{\alpha_j^2}{h^2}-\frac{\xi^2}{k^2}+1\right]},\ \alpha_1^2=\xi^2-h^2,\ \ \alpha_2^2=\xi^2-k^2 \end{aligned}\right\}. \tag{36}$$

In the absence of rotation and magnetic field, the frequency equation for Rayleigh waves, under the theory of nonlocal elasticity, may be presented as:

$$\left(\frac{k^2}{\xi^2}-2\right)^4=16\left(1-\frac{h^2}{\xi^2}\right)\left(1-\frac{k^2}{\xi^2}\right), \tag{37}$$

where:

$$\left.\begin{aligned} &k^2=\frac{\rho\omega^2}{(\mu+\overline{\mu})},\ h^2=\frac{\rho\omega^2}{\left(\lambda+\overline{\lambda}\right)+2\left(\mu+\overline{\mu}\right)},\\ &\gamma_j=\frac{-i\alpha_j\left[\frac{\xi}{h^2}-\frac{\xi}{k^2}\right]}{\left[\frac{\alpha_j^2}{h^2}-\frac{\xi^2}{k^2}+1\right]},\ \alpha_1^2=\xi^2-h^2,\ \alpha_2^2=\xi^2-k^2 \end{aligned}\right\}. \tag{38}$$

This equation is in complete agreement with that obtained by Eringen [10].

The frequency equation for classical Rayleigh waves may also be deduced from (33) as special case. The form of this equation is the same, as in (37), but the expressions for k^2 and h^2 are given by:

$$k^2 = \frac{\rho\omega^2}{\mu} \quad \text{and} \quad h^2 = \frac{\rho\omega^2}{\lambda + 2\mu}.$$

For numerical calculations and discussions, following assumptions are performed:

1. The media is made of Poisson's solid i.e. $\lambda = \mu$,
2. Following Eringen [10], the concept of Poisson's material is extended to include $\overline{\lambda} = \overline{\mu}$,
3. Making parallel assumptions as Chandrasekharaiya [18] ($\omega_3 = \omega$), we take $\omega_3 = R\omega$,
4. We also set $\mu_0 H_0^2 = M(\mu + \overline{\mu})$,

where R and M are defined as rotational parameter and Alfven wave velocity parameter, respectively.

On the basis of the above assumptions and in view of (28), (29) and (31), the frequency equation may be transformed to a four degree equation in k^2/ξ^2. Thus, using (28), one obtains the wave velocity ($c = \omega/\xi$) for Rayleigh waves propagated in a nonlocal rotating magneto elastic solid, in the following form:

$$c = \sqrt{\frac{(k^2/\xi^2)}{1 + R^2}} \sqrt{\frac{\mu}{\rho}} \left(1 + \frac{\overline{\mu}}{\mu}\right)^{1/2}, \tag{39}$$

where: $\overline{\mu}$ is a function of ξ.

Equation (39) may be considered as a more generalized formula for finding Rayleigh wave velocity in the sense, that it includes the effects of magnetic field, rotation of the body and nonlocal character of the medium. Rayleigh wave velocity, in presence of magnetic field, Rayleigh wave velocity for a rotating body in a linearly elastic nonlocal medium, as well as for classical elasticity may be deduced from (39) as special cases, by making required modifications. Rayleigh wave velocity for nonlocal, non-rotating elastic solid in the absence of magnetic field may also be deduced from (39), as:

$$c = 0.9194 \sqrt{\frac{\mu}{\rho}} \left(1 + \frac{\overline{\mu}}{\mu}\right)^{1/2}, \tag{40}$$

or:

$$\frac{c}{c_r} = \left(1 + \frac{\overline{\mu}\,(\xi)}{\mu}\right)^{1/2},$$

where: $c_r = 0.9194\sqrt{\mu/\rho}$ =classical Rayleigh wave velocity for Poisson's solid. In contrast with the classical situation, as pointed out by Eringen, Rayleigh wave velocity in nonlocal solids is definitely dispersive in nature, due to the presence of $\overline{\mu}$, which is a function of ξ in the equation (40). Such a dispersive nature in a nonlocal medium persists whether the body rotates or not and irrespective of the presence or absence of magnetic field.

Using equation (29), the attenuation exponents α_1 and α_2 under the above assumptions may be obtained from the following formulas:

$$\left.\begin{aligned} &\frac{\alpha_1^2}{\xi^2} + \frac{\alpha_2^2}{\xi^2} = 2 - \frac{k^2}{\xi^2}\left(\frac{M+4}{M+3}\right) \\ &\frac{\alpha_1^2}{\xi^2} \times \frac{\alpha_2^2}{\xi^2} = 1 - \frac{k^2}{\xi^2}\left(\frac{M+4}{M+3}\right) + \left(\frac{k^2}{\xi^2}\right)^2 \frac{1}{(M+3)}\left(\frac{1-R^2}{1+R^2}\right)^2 \end{aligned}\right\}, \tag{41}$$

where: k^2/ξ^2may be found by solving equation (33). If we further assume: $R = 1$ (i.e. $\omega_3 = \omega$, Chandrasekharaiya[18]), then $\dfrac{\alpha_1^2}{\xi^2} = 1$ and $\dfrac{\alpha_2^2}{\xi^2} = 1 - \dfrac{k^2}{\xi^2}\left(\dfrac{M+4}{M+3}\right)$, which shows that one of the attenuation exponents, α_1^2/ξ^2 is independent of magnetic field. Moreover, in the absence of magnetic field one obtains $k^2/\xi^2 = 0.6667$ and hence, $\alpha_1^2/\xi^2 = 1$ and $\alpha_2^2/\xi^2 = 0.1111$. This result is in perfect agreement with the corresponding result obtained by Acharya and Mondal [22]. For a nonlocal, non-rotating elastic solid in the absence of magnetic field attenuation exponents are given by $\alpha_1^2/\xi^2 = 0.7182$ and $\alpha_2^2/\xi^2 = 0.1547$, which are the same as linear classical elasticity.

7. Parametric study and discussion

To highlight the effect of magnetic field, rotation and their combined effect in the case of Rayleigh wave propagation in a semi-infinite solid medium based on the nonlocal theory of elasticity, a numerical study is performed. One of the major parameters for this study is $k^2/\xi^2\,(= V)$, which plays a very important role for the evaluation of the wave velocity ratio, as well as ratios of attenuation exponents. To show the variations of V, different type of graphs are plotted based on numerical calculations, using equation (33), in Figs 2 and 3. The numerical calculations are performed, using commercially avail-

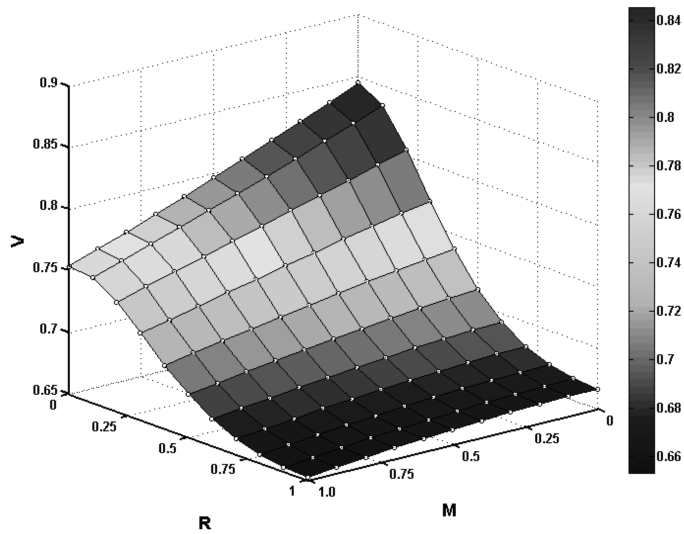

Fig. 2. Response of V against Alfven wave velocity parameter M and rotational parameter R

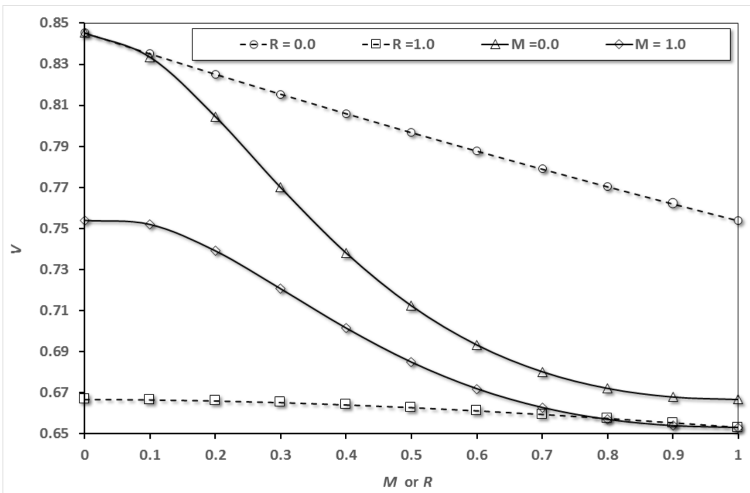

Fig. 3. Variation of V against the Alfven wave velocity parameter M, or rotational parameter R

able software MATLAB. Variations of square of attenuation exponent ratios $\alpha_1^2/\xi^2\,(=A_1)$ and $\alpha_2^2/\xi^2\,(=A_2)$ to be called attenuation exponent, henceforth, have been demonstrated through Figs 4 to 7. Though the figures are self-explanatory, brief descriptions are presented to show the behaviour and trends of plots for the propagation of Rayleigh waves in rotating semi-infinite solid media, permeated by an initial magnetic field in the context of non-local elasticity.

Figure 2 exhibits surface plot of variation of V for different values of M and R. It is noted that V attains its maximum value of 0.85 in the absence of Alfven wave velocity parameter and rotation parameter (i.e. $M = R = 0.0$). Subsequently, for any increment of M and/or R, the wave velocity ratio V, continuously decreases to reach its minimum value of 0.65, when both M and R attain their maximum value 1.0. It is also observed, that the rate of decrement of V varies differently for variations of M and R. To get an idea of change rate in wave velocity ratio V, with respect to M and R, graphs are plotted in Fig. 3. The dotted lines in Fig. 3 show the variations of Vagainst Alfven wave velocity parameter M for $R = 0$ and $R = 1$. The solid lines depict the variation of V against R for $M = 0$ and $M = 1$. It is observed that in the absence of rotation ($R = 0$), V continuously decreases from its peak value due to the increment in M. V decreases slowly for $R = 1$. For a particular value of Alfven wave velocity parameter M, V is diminished, due to rotation. Such diminutions become smaller as M increases. From this figure, it is observed that V quickly drops down to its smaller values in both the cases of $M = 0$ and $M = 1$, as R increases. For a particular value of rotational parameter R, the value of Vdecreases as M increases.

Figure 4 shows the response of attenuation exponents A_1 or A_2 against Alfven wave velocity parameter M for $R = 0$ and $R = 1$. For any particular value of M in its range, A_1 diminishes due to rotation. Such diminution almost remains same throughout the range of magnetic field. It is observed, that the attenuation exponents A_1 (for $R = 0, 1$) and A_2 (for $R = 0$) increase slowly with the increase of magnetic field, while hardly any variation in A_2 is observed when $R = 1$. For a particular value of M, effect of rotation is to diminish the value of A_1, while A_2 increases with rotation.

Figure 5 signifies the effect of rotational parameter R on A_1 (for $M = 0$, $M = 1$) and A_2 (for $M = 0$, $M = 1$). It is observed, that A_1 slowly decreases as R increases irrespective of the presence or absence of the magnetic field. For a particular value of R, effect of magnetic field is to increase the value of A_1. The effect of magnetic field also causes increment in A_2 on the range of $0 \leq R < 0.7$ (approx.), but there is hardly any change in A_2 on the range of $0.7 \leq R \leq 1$. Thus, we conclude that magnet has practically got no effect on A_2 on the range of $R > 0.7$.

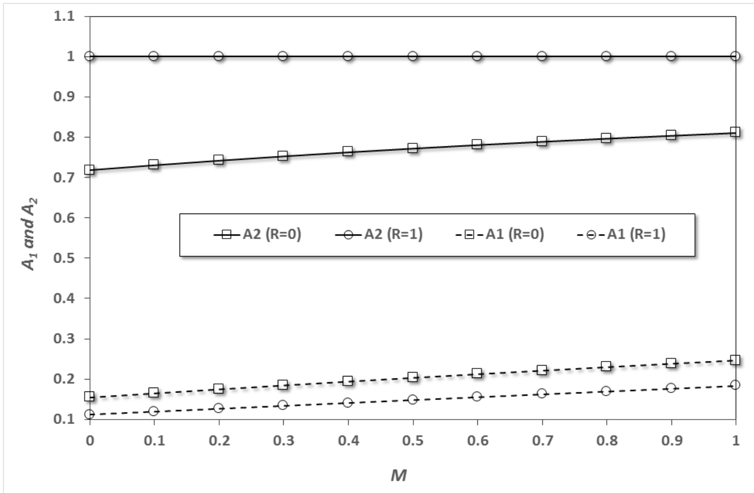

Fig. 4. Variation of attenuation exponents A_1 and A_2 against Alfven wave velocity parameter M

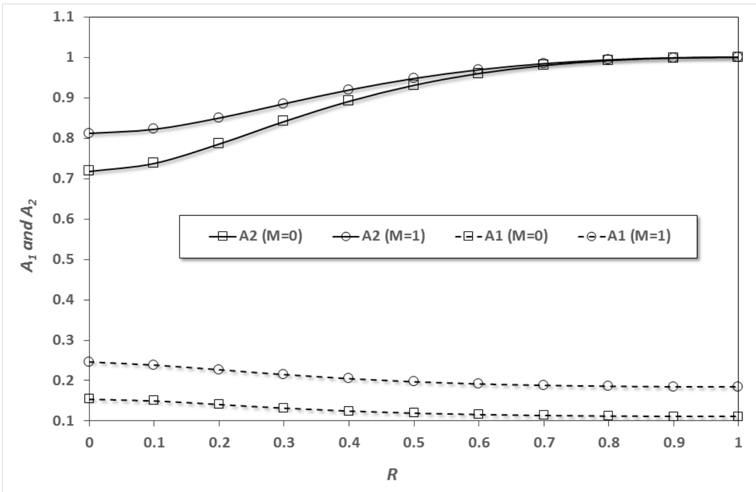

Fig. 5. Variation of attenuation exponents A_1 and A_2 against rotational parameter R

In order to analyze the combined effect of magnetic field (M) and rotation (R) on the attenuation exponents A_1 and A_2, two surfaces are plotted in Figs 6 and 7, respectively. The observations made earlier in Figs 4 and 5 may also be made from Figs 6 and 7 as special cases.

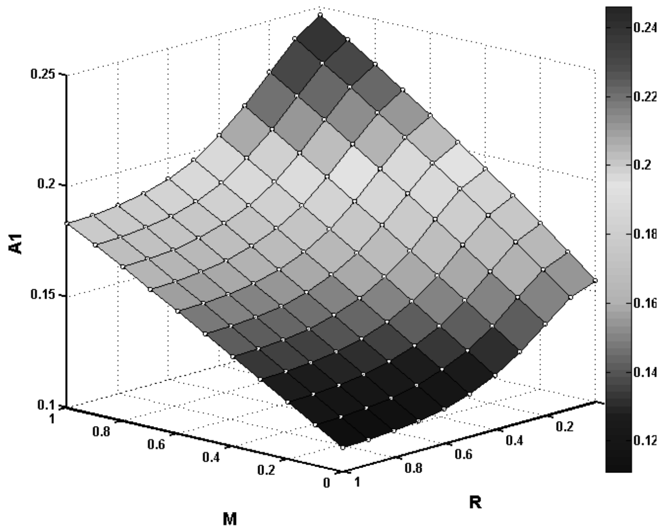

Fig. 6. Surface plot of attenuation exponent A_1 against Alfven wave velocity parameter M and rotational parameter R

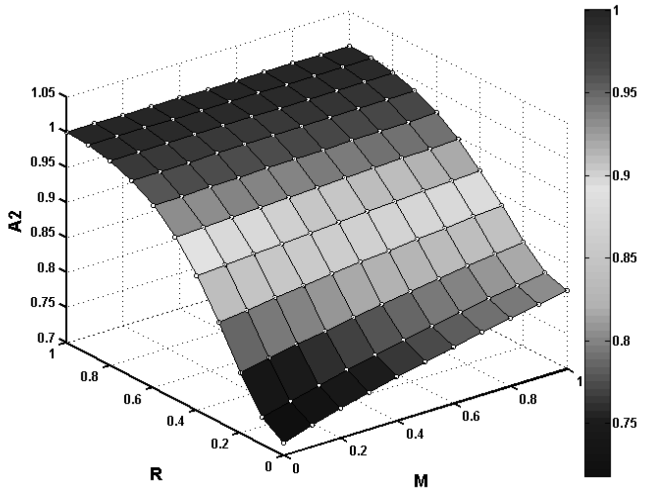

Fig. 7. Surface plot of attenuation exponent A_2, against Alfven wave velocity parameter M and rotational parameter R

8. Conclusions

In the present work, the combined effect of rotation and magnetic field on the propagation of Rayleigh surface waves has been investigated in the context of linear nonlocal elasticity. The results obtained in this paper may be considered as more general in the sense that it includes rotation of the medium and the presence of magnetic field. Special cases have also been investigated with due importance. Variations of wave velocity, attenuation exponents have been analyzed and discussed elaborately. Based on these studies, it can be concluded that rotation and magnetic field play significant role on nonlocal Rayleigh wave propagation. The technique, applied in this research may be helpful to investigate similar problems for piezoelectric/piezomagnetic composite materials. This approach may also be extended for Love and Stoneley type of waves.

REFERENCES

[1] Ivanov, T. P. On the Propagation of Thermoelastic Rayleigh Waves. *Wave Motion*, **10** (1988), No. **1**, 73–82.

[2] Abd-Alla, A. M., S. M. Abo-Dahab, H. A. H. Hammad. Propagation of Rayleigh Waves in Generalized Magneto-thermoelastic Orthotropic Material under Initial Stress and Gravity Field. *Appl. Math. Model*, **35** (2011), No. **6**, 2981–3000.

[3] Shams, M., R. Ogden. On Rayleigh-type Surface Waves in an Initially Stressed Incompressible Elastic Solid. *IMA J. Appl. Math.*, **79** (2014), No. **2**, 360–376.

[4] Acharya, D. P., P. R. Sengupta. Two-dimensional Wave Propagation in a Micropolar Thermo-elastic Layer. *Int. J. Eng. Sci.*, **15** (1977), No. **3**, 211–217.

[5] Acharya, D. P., P. R. Sengupta. Two-dimensional Wave Propagation in a Micropolar Thermo-elastic Layer with Stretch. *Int. J. Eng. Sci.*, **17** (1979), No. **10**, 1109–1116.

[6] Love, A. E. H. A Treatise on Mathematical Theory of Elasticity, New York, Dover, 1944.

[7] Gazis, D. C., R. Herman, R. F. Wallis. Surface Elastic Waves in Cubic Crystals. *Phys. Rev.*, **119** (1960), No. **2**, 533–544.

[8] Eringen, A. C., B. S. Kim. Relation Between Non-local Elasticity and Lattice Dynamics. *Cryst. Latt. Def.*, **7** (1977), 51–57.

[9] Maradudin, A. A., E. W. Montroll, G. H. Weiss, I. P. Ipatova. Theory of Lattice Dynamics in the Harmonic Approximation, **531** (2^{nd} edition, Hampton), USA, NH, Academic Press, 1971.

[10] Eringen, A. C. On Rayleigh Surface Waves with Small Wave Lengths. *Letters in Applied and Engineering Sciences*, **1** (1973), 11–17.

[11] Inan, E., A. C. Eringen. Nonlocal Theory of Wave Propagation in Thermoelastic Plates. *Int. J. Eng. Sci.*, **29** (1991), No. **7**, 831–843.

[12] Inan, E. Nonlocal Theory of Longitudinal Waves in Thermoelastic Bars. *Le Matematiche*, **46** (1991), No. **1**, 203–212.

[13] Acharya, D. P., A. Mondal. Propagation of Rayleigh Surface Waves with Small Wavelengths in Nonlocal Visco-elastic Solids. *Sādhanā*, **27** (2002), No. **6**, 605–612.

[14] Lazar, M., G. A. Maugin, E. C. Aifantis. On a Theory of Nonlocal Elasticity of Bi-Helmholtz Type and Some Applications. *Int. J. Solids Struct.*, **43** (2006), No. **6**, 1404–1421.

[15] Zhou, Z., B. Wang. Nonlocal Theory Solution of Two Collinear Cracks in the Functionally Graded Materials. *Int. J. Solids Struct.*, **43** (2006), No. **5**, 887–898.

[16] Chakraborty, A. Wave Propagation in Anisotropic Media with Non-local Elasticity. *Int. J. Solids Struct*, **44** (2007), No. **17**, 5723–5741.

[17] Schoenberg, M., D. Censor. Elastic Waves in Rotating Media. *Q. J. Appl. Math.*, **31** (1973), No. **1**, 115–125.

[18] Chandrasekharaiah, D. S. Temperature-rate Dependent Electro-magneto-thermoelastic Plane Waves in a Rotating Solid. *Int. J. Eng. Sci.*, **24** (1986), No. **7**, 1173–1181.

[19] Sharma, J. N., M. I. A. Othman. Effect of Rotation on Generalized Thermo-viscoelastic Rayleigh-Lamb Waves. *Int. J. Solids Struct.*, **44** (2007), No. **13**, 4243–4255.

[20] Tomar, S. K., R. W. Ogden. Two-dimensional Wave Propagation in a Rotating Elastic Solid with Voids. *J. Sound Vib.*, **333** (2014), No. **7**, 1945–1952.

[21] Ogden, R. W., B. Singh. The Effect of Rotation and Initial Stress on the Propagation of Waves in a Transversely Isotropic Elastic Solid. *Wave Motion*, **51** (2014), No. **7**, 1108–1126.

[22] Acharya, D. P., A. Mondal. Effect of Rotation on Rayleigh Surface Waves under the Linear Theory of Non-local Elasticity. *Indian Journal of Theoretical Physics*, **52** (2004), No. **1**, 81–89.

[23] Ezzat, M. A., H. M. Youssef. Generalized Magneto-thermoelasticity in a Perfectly Conducting Medium. *Int. J. Solids Struct.*, **42** (2005), No. **24–25**, 6319–6334.

[24] Bakshi, A., R. K. Bera, L. Debnath. A Study of Magneto-thermoelastic Problems with Thermal Relaxation and Heat Sources in a Three-Dimensional Infinite Rotating Elastic Medium. *Int. J. Eng. Sci.*, **43** (2005), No. **19–20**, 1419–1434.

[25] Singh, B., S. Kumari, J. Singh. Rayleigh Wave in a Rotating Magneto-thermo-elastic Half-plane. *Journal of Theoretical and Applied Mechanics*, **42** (2012), No. **4**, 75–92.

[26] Shekhar, S., I. A. Parvez. Effect of Rotation, Magnetic Field and Initial Stresses on Propagation of Plane Waves in Transversely Isotropic Dissipative Half Space. *Applied Mathematics*, **4** (2013), No. **8A**, 107–113.

[27] Hajdo, L. E., A. C. Eringen. Application of Nonlocal Theory to Electromagnetic Dispersion. *Lett. Appl. Eng. Sci.*, **17** (1979), No. **6**, 785–791.

[28] Othman, M. I. A., Y. Song. The Effect of Rotation on the Reflection of Magneto-thermoelastic Waves under Thermoelasticity without Energy Dissipation. *Acta Mech.*, **184** (2006), No. **1-4**, 189–204.