

ROTATIONAL EFFECTS ON PROPAGATION OF RAYLEIGH
WAVE IN A MICROPOLAR PIEZOELECTRIC MEDIUM

BALJEET SINGH^{1*}, RITU SINDHU²

¹*Department of Mathematics, Post Graduate Government College, Sector 11,
Chandigarh, India*

²*Department of Mathematics, Maharishi Dayanand University, Rohtak, India*

[Received 16 October 2017. Accepted 10 January 2018]

ABSTRACT: In this paper, the governing equations of a linear, homogeneous and transversely isotropic rotating micropolar piezoelectric medium are solved for surface wave solutions. The appropriate solutions satisfying the radiation conditions are obtained in a half-space. These solutions are applied to suitable boundary conditions at the free surface of the half- space. A frequency equation for Rayleigh wave is obtained for both charge free and electrically shorted cases. Using iteration method, the non-dimensional wave speed of Rayleigh wave is computed for relevant material constants modelling the medium. The effects of rotation, piezoelectricity, frequency and material parameters are observed graphically on the propagation speed.

KEY WORDS: Micropolar, piezoelectric, rayleigh wave, rotation, iteration method.

1 INTRODUCTION

The study of linear coupling between mechanical and electrical fields is called piezoelectricity. Surface waves are propagation modes of elastic energy along the surface of a half-space, whose displacement amplitudes decay exponentially with the distance from the surface. The propagation of surface waves in piezoelectric media finds its applications in various engineering fields. In a rotating and vibrating elastic or piezoelectric body, the Coriolis and centrifugal forces changes wave speed or vibration frequency. This phenomenon is useful in designing of rotation sensors, which measure the angular rate of a rotating body. These rotation sensors are used extensively in automobiles, motion cameras, smart weapon systems, machine control, robotics, and navigation.

Rotation effects on wave speed in isotropic elastic surface wave resonators was analyzed in Tiersten et al. [1], Lao [2], Wren and Burdess [3], Clarke and Burdess [4]. Elastic analysis of surface waves in rotating bodies was extended to anisotropic materials by Destrade [5] and Ting [6]. Fang et al. [7,8] analyzed the surface acoustic waves propagating over a piezoelectric half-space rotating at a constant angular

*Corresponding author e-mail: bsinghgc11@gmail.com

rate about a fixed axis by using the linear theory of piezoelectricity with inclusion of Coriolis and centrifugal forces. Yang [9] presented a review of analyses on vibrations of rotating piezoelectric structures for applications in piezoelectric angular rate sensors. He mentioned that two types of sensors, accelerometers and gyroscopes are needed for detecting the complete motion of a moving object, where accelerometers sense linear motions and gyroscopes sense rotations. Various plane wave problems including Kyame [10], Pailloux [11], Auld [12], Galassi, et al. [13], Singh [14] and Yuan [15] are studied by using theory of piezoelectricity.

Eringen [16-18] developed the micro-continuum field theories of solids involving interactions between electromagnetic and thermal fields. He also derived the field equations of micropolar piezoelectricity in a special case. Zhilin and Kolpakov [19] also proposed a theory of micropolar piezoelectric materials. Iesan [20] established a uniqueness result and a reciprocal theorem by using the linear theory of microstretch piezoelectricity. Aouadi [21] propounded a linear dynamic theory of micropolar piezoelectricity and established a reciprocity relation. Gales [22] formulated an initial boundary value problem by using the linear theory of micromorphic piezoelectricity. Chen [23] derived the linear constitutive equations for micropolar electromagnetic elastic solids.

The propagation of a Rayleigh wave in a rotating transversely isotropic micropolar piezoelectric solid half-space is not attempted yet in literature. The equations of a rotating micropolar piezoelectric body govern the basic behaviour of a piezoelectric gyroscope. These equations consist of the equations of linear piezoelectricity with rotation-related Coriolis and centrifugal accelerations. Following Aouadi [21], the governing equations for a rotating and transversely isotropic micropolar piezoelectric medium are formulated. The surface wave solutions are obtained in a half-space. Finally, a frequency equation in non-dimensional speed of Rayleigh wave is derived for both charge free and electrically shorted cases. The dependence of non-dimensional speed on rotation, frequency, material constants and piezoelectric field is shown graphically.

2 FORMULATION OF THE PROBLEM

We consider a linear, homogeneous and transversely isotropic micropolar piezoelectric half space as shown in Fig. 1. We take the origin O of the coordinate system on the free surface and z axis is poling axis pointing normally into the half-space ($z \geq 0$). We assume the components of the displacement and the microrotation vector of the form $\vec{u} = (u_1, 0, u_3)$ and $\vec{\varphi} = (0, \varphi_2, 0)$. We assume that the medium is transversely isotropic in such a way that the plane of isotropy is perpendicular to z -axis. It is assumed that the entire half-space is rotating with constant angular rate Ω about y -axis.

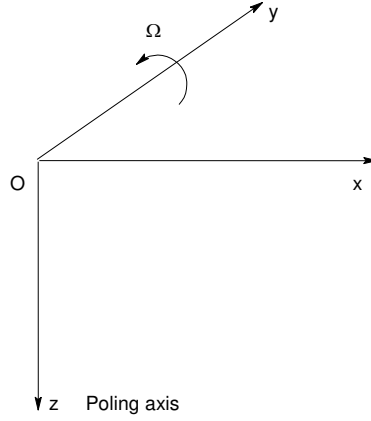


Fig. 1. A rotating micropolar piezoelectric half-space.

The fixed coordinate system in the rotating micropolar piezoelectric medium introduces additional terms in the equations of motion: a centripetal and a Coriolis acceleration. Following Aouadi [21] and Schoenberg and Censor [24], the governing equations for a linear, homogeneous and transversely isotropic rotating micropolar piezoelectric medium in x - z plane take the following form:

$$(1) \quad A_{11} \frac{\partial^2 u_1}{\partial x^2} + (A_{13} + A_{56}) \frac{\partial^2 u_3}{\partial x \partial z} + A_{55} \frac{\partial^2 u_1}{\partial z^2} + K_1 \frac{\partial \varphi_2}{\partial z} - (\lambda_{15} + \lambda_{31}) \frac{\partial^2 \psi}{\partial x \partial z} = \rho \left[\frac{\partial^2 u_1}{\partial t^2} - \Omega^2 u_1 + 2\Omega \frac{\partial u_3}{\partial t} \right],$$

$$(2) \quad A_{66} \frac{\partial^2 u_3}{\partial x^2} + (A_{13} + A_{56}) \frac{\partial^2 u_1}{\partial x \partial z} + A_{33} \frac{\partial^2 u_3}{\partial z^2} + K_2 \frac{\partial \varphi_2}{\partial x} - \lambda_{15} \frac{\partial^2 \psi}{\partial x^2} - \lambda_{33} \frac{\partial^2 \psi}{\partial z^2} = \rho \left[\frac{\partial^2 u_3}{\partial t^2} - \Omega^2 u_3 - 2\Omega \frac{\partial u_1}{\partial t} \right],$$

$$(3) \quad B_{77} \frac{\partial^2 \varphi_2}{\partial x^2} + B_{66} \frac{\partial^2 \varphi_2}{\partial z^2} - \chi \varphi_2 - K_1 \frac{\partial u_1}{\partial z} - K_2 \frac{\partial u_3}{\partial x} = \rho j \frac{\partial^2 \varphi_2}{\partial t^2},$$

$$(4) \quad \lambda_{15} \frac{\partial^2 u_3}{\partial x^2} + \lambda_{33} \frac{\partial^2 u_3}{\partial z^2} + (\lambda_{31} + \lambda_{15}) \frac{\partial^2 u_1}{\partial x \partial z} + \gamma_{11} \frac{\partial^2 \psi}{\partial x^2} + \gamma_{33} \frac{\partial^2 \psi}{\partial z^2} = 0$$

The equations of motion (1) and (2) in such a rotating frame have two additional terms that do not appear in the non-rotating frame. In above equations, ρ is the mass density, j is micro-inertia, ψ is electrostatic potential, A_{11} , A_{13} , A_{33} , A_{55} , A_{56} , A_{66} , B_{66} , B_{77} are micropolar elastic constants, λ_{15} , λ_{31} , λ_{33} , λ_{35} are piezoelectric constants, γ_{11} , γ_{33} are dielectric constants and $K_1 = A_{56} - A_{55}$, $K_2 = A_{66} - A_{56}$, $\chi = K_2 - K_1$.

3 SOLUTION PROCEDURE

We seek the following surface wave type solutions of Eqs. (1) to (4):

$$(5) \quad \{u_1, u_3, \varphi_2, \psi\} = \{\bar{u}_1(z), \bar{u}_3(z), \bar{\varphi}_2(z), \bar{\psi}(z)\} e^{ik(x-ct)},$$

where k is the wave number in x -direction, c is phase velocity of the wave, and $\omega = kc$ is the angular frequency.

Making use of Eq. (5) in Eqs. (1) to (4), we obtain four homogeneous equations in $\bar{u}_1(z)$, $\bar{u}_3(z)$, $\bar{\varphi}_2(z)$ and $\bar{\psi}(z)$, which have non-trivial solution if

$$(6) \quad D^8 - S_1 D^6 + S_2 D^4 - S_3 D^2 + S_4 = 0,$$

where $D = d/dz$ and S_j , ($j = 1, 2, \dots, 4$) are given in Appendix.

Let m_1, m_2, m_3, m_4 be the roots of auxiliary equation corresponding to Eq. (6). With radiation conditions $u_1 \rightarrow 0, u_3 \rightarrow 0, \phi_2 \rightarrow 0, \psi \rightarrow 0$ as $z \rightarrow \infty$, we obtain the particular solutions in half-space ($z \geq 0$) as

$$(7) \quad u_1 = (A_1 e^{-m_1 z} + A_2 e^{-m_2 z} + A_3 e^{-m_3 z} + A_4 e^{-m_4 z}) e^{ik(x-ct)},$$

$$(8) \quad u_3 = (\zeta_1 A_1 e^{-m_1 z} + \zeta_2 A_2 e^{-m_2 z} + \zeta_3 A_3 e^{-m_3 z} + \zeta_4 A_4 e^{-m_4 z}) e^{ik(x-ct)},$$

$$(9) \quad \varphi_2 = (\eta_1 A_1 e^{-m_1 z} + \eta_2 A_2 e^{-m_2 z} + \eta_3 A_3 e^{-m_3 z} + \eta_4 A_4 e^{-m_4 z}) e^{ik(x-ct)},$$

$$(10) \quad \psi = (\xi_1 A_1 e^{-m_1 z} + \xi_2 A_2 e^{-m_2 z} + \xi_3 A_3 e^{-m_3 z} + \xi_4 A_4 e^{-m_4 z}) e^{ik(x-ct)}$$

where the relations between m_j^2 ($j = 1, 2, \dots, 4$) and the expressions for ζ_i, ξ_i and η_i/k are given in Appendix.

The mechanical boundary conditions at $z = 0$ are vanishing of the normal force stress component, tangential force stress component, tangential couple stress component, normal dielectric displacement component and electrostatic potential are

$$(11) \quad \sigma_{33} = 0, \quad \sigma_{31} = 0, \quad m_{32} = 0,$$

$$(12) \quad D_3 = 0, \quad (\text{charge free case})$$

$$(13) \quad \psi = 0, \quad (\text{electrically shorted case})$$

where

$$\sigma_{33} = A_{13} u_{1,1} + A_{33} u_{3,3} - \lambda_{35} \psi_{,1} - \lambda_{33} \psi_{,3},$$

$$\sigma_{31} = A_{56} u_{3,1} + A_{55} u_{1,3} + (A_{56} - A_{55}) \varphi_2 - \lambda_{31} \psi_{,1} - \lambda_{35} \psi_{,3},$$

$$m_{32} = B_{66} \varphi_{2,3},$$

$$D_3 = \lambda_{15} u_{1,1} + \lambda_{33} u_{3,3} + \gamma_{33} \psi_{,3}.$$

The particular solutions (7) to (10) satisfy the boundary conditions (11) to (13) at the free surface $z = 0$ and we obtain the following frequency equation:

$$(14) \quad A_1^* B_2^* C_3^* D_4^* - A_1^* B_2^* C_4^* D_3^* - A_1^* B_3^* C_2^* D_4^* + A_1^* B_3^* C_4^* D_2^* \\ + A_1^* B_4^* C_2^* D_3^* - A_1^* B_4^* C_3^* D_2^* - A_2^* B_1^* C_3^* D_4^* + A_2^* B_1^* C_4^* D_3^* \\ + A_2^* B_3^* C_1^* D_4^* - A_2^* B_3^* C_4^* D_1^* - A_2^* B_4^* C_1^* D_3^* + A_2^* B_4^* C_3^* D_1^* \\ + A_3^* B_1^* C_2^* D_4^* - A_3^* B_1^* C_4^* D_2^* - A_3^* B_2^* C_1^* D_4^* + A_3^* B_2^* C_4^* D_1^* \\ + A_3^* B_4^* C_1^* D_2^* - A_3^* B_4^* C_2^* D_1^* - A_4^* B_1^* C_2^* D_3^* + A_4^* B_1^* C_3^* D_2^* \\ + A_4^* B_2^* C_1^* D_3^* - A_4^* B_2^* C_3^* D_1^* - A_4^* B_3^* C_1^* D_2^* + A_4^* B_3^* C_2^* D_1^* = 0,$$

where

$$A_i^* = ikA_{13} - m_i \zeta_i A_{33} + m_i \xi_i \left(\lambda_{33} - \frac{ik\lambda_{35}}{m_i} \right) \quad (i = 1, 2, \dots, 4), \\ B_i^* = ik\zeta_i A_{56} - m_i A_{55} + (A_{56} - A_{55}) \eta_i - \lambda_{31} ik \xi_i, \\ C_i^* = -m_i \eta_i B_{66}, \\ D_i^* = ik\lambda_{15} - m_i \zeta_i \lambda_{33} - m_i \xi_i \gamma_{33}, \quad (\text{charge free case}), \\ D_i^* = \xi_i, \quad (\text{electrically shorted case}).$$

4 RESULTS AND DISCUSSION

To the best of authors' knowledge, the micromechanics based data for transversely isotropic micropolar piezoelectric material is not available in literature. In present study, the relevant values of physical constants (satisfying the inequalities among these constants) of a transversely isotropic composite material modelled as micropolar piezoelectric medium are taken to compute the non-dimensional speed of Rayleigh wave

$$A_{11} = 17.8 \times 10^{10} \text{ Nm}^{-2}, \quad A_{33} = 18.43 \times 10^{10} \text{ Nm}^{-2}, \\ A_{13} = 7.59 \times 10^{10} \text{ Nm}^{-2}, \quad A_{56} = 1.89 \times 10^{10} \text{ Nm}^{-2}, \\ A_{55} = 4.357 \times 10^{10} \text{ Nm}^{-2}, \quad A_{66} = 4.42 \times 10^{10} \text{ Nm}^{-2}, \\ A_{65} = 1.99 \times 10^{10} \text{ Nm}^{-2}, \quad B_{77} = 0.278 \times 10^9 \text{ N}, \\ B_{66} = 0.268 \times 10^9 \text{ N}, \quad \lambda_{15} = 37 \text{ C m}^{-2}, \\ \lambda_{31} = 12 \text{ C m}^{-2}, \quad \lambda_{33} = 1.33 \text{ C m}^{-2}, \\ \lambda_{35} = 0.23 \text{ C m}^{-2}, \quad \gamma_{11} = 0.000852 \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}, \\ \gamma_{33} = 0.000287 \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}, \quad \rho = 1.74 \times 10^3 \text{ Kg/m}^3, \\ j = 0.196 \text{ m}^2.$$

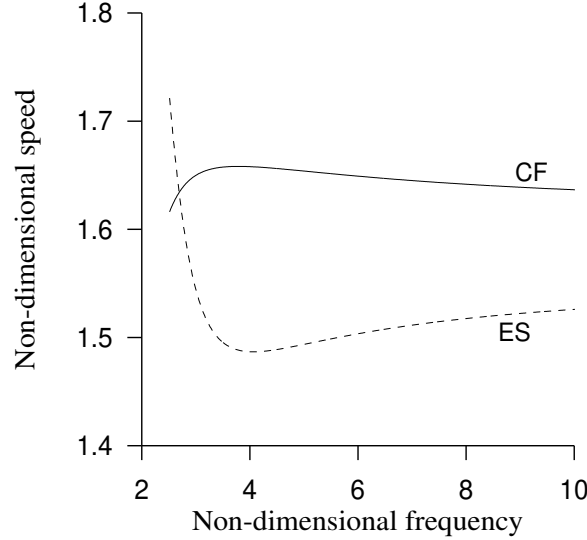


Fig. 2. Variation of non-dimensional speed $\sqrt{\frac{\rho c^2}{A_{33}}}$ versus non-dimensional frequency $\omega^* = \omega^2 / \frac{\chi}{\rho j}$ for charge free (CF) and electrically shorted (ES) cases when $\Omega/\omega = 2$.

An iteration method is used to compute the propagation speed from frequency equation (14). The variation of non-dimensional speed $\sqrt{\frac{\rho c^2}{A_{33}}}$ versus non-dimensional frequency $\{\omega^* = \omega^2 / \frac{\chi}{\rho j}\}$ are shown graphically in Fig. 2 for charge free (CF) and electrically shorted (ES) cases, when $\Omega/\omega = 2$. For CF case, the value of speed at $\omega^* = 2.5$ is 1.6167. It increases to a value 1.6588 at $\omega^* = 3.77$ and then decreases to a value 1.6372 at $\omega^* = 10$. This variation is shown by the solid line in Fig. 2. For ES case, the value of speed at $\omega^* = 2.5$ is 1.7220. It decreases to value 1.4874 at $\omega^* = 4.04$ and then increases to 1.5266 at $\omega^* = 10$. This variation is shown by dotted line in Fig. 2. Comparing the solid and dotted lines in Fig. 2, we can observe the effect of different surfaces on non-dimensional speed of the Rayleigh wave in a Transversely Isotropic Rotating Micropolar Piezoelectric (TIRMP) solid half-space.

The variation of non-dimensional speed $\sqrt{\frac{\rho c^2}{A_{33}}}$ is shown graphically in Fig. 3 versus non-dimensional frequency $\{\omega^* = \omega^2 / \frac{\chi}{\rho j}\}$ for charge free (CF) case to observe the piezoelectric and rotational effects when $\Omega/\omega = 2$. The variation of non-

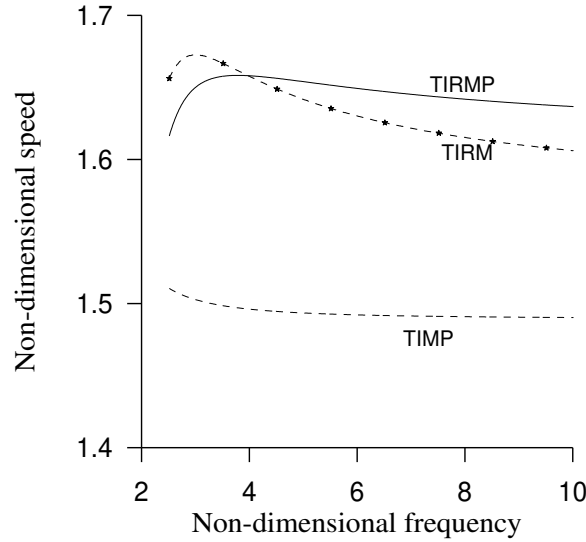


Fig. 3. Piezoelectric and rotational effects on non-dimensional speed $\sqrt{\frac{\rho c^2}{A_{33}}}$ versus non-dimensional frequency $\{\omega^* = \omega^2 / \frac{\chi}{\rho j}\}$ for charge free (CF) case, when $\Omega/\omega = 2$ (TIMP-transversely isotropic micropolar piezoelectric, TIRMP-transversely isotropic rotating micropolar piezoelectric, TIRM-transversely isotropic rotating micropolar).

dimensional speed as shown by the solid line (TIRMP case) in Fig. 3 is same as solid line shown in Fig. 2. For Transversely Isotropic Micropolar Piezoelectric (TIMP) case, the variation of non-dimensional speed is shown by the dotted line in Fig. 3. It has value 1.5110 at $\omega^* = 2.5$ and it decreases to value 1.4907 at $\omega^* = 10$. For Transversely Isotropic Rotating Micropolar (TIRM) case, the variation of non-dimensional speed is shown by the dotted line with center symbols in Fig. 3. It has value 1.6566 at $\omega^* = 2.5$ and it increases to value 1.6728 at $\omega^* = 3.07$ and then decreases to value 1.6066 at $\omega^* = 10$. The comparison of the solid and dotted lines in Fig. 3 shows the piezoelectric and rotational effects on non-dimensional speed of Rayleigh wave in a transversely isotropic micropolar piezoelectric solid half-space with charge free surface.

The variation of non-dimensional speed $\sqrt{\frac{\rho c^2}{A_{33}}}$ is shown graphically in Fig. 4 versus non-dimensional frequency $\{\omega^* = \omega^2 / \frac{\chi}{\rho j}\}$ for electrically shorted (ES) case to observe the piezoelectric and rotational effects when $\Omega/\omega = 2$. The variation

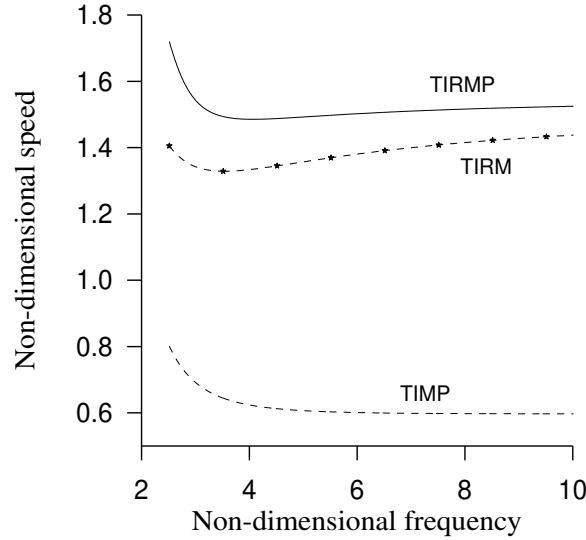


Fig. 4. Piezoelectric and rotational effects on non-dimensional speed $\sqrt{\frac{\rho c^2}{A_{33}}}$ versus non-dimensional frequency $\{\omega^* = \omega^2 / \frac{\chi}{\rho j}\}$ for electrically shorted (ES) case, when $\Omega/\omega = 2$ (TIMP-transversely isotropic micropolar piezoelectric, TIRMP-transversely isotropic rotating micropolar piezoelectric, TIRM-transversely isotropic rotating micropolar).

of non-dimensional speed for TIRMP case is shown by solid line in Figure 4 and is similar to dotted line in Fig. 2. For TIMP case, the variation of non-dimensional speed is shown by dotted line in Fig. 4. It has value 0.8036 at $\omega^* = 2.5$ and it decreases to value 0.5991 at $\omega^* = 10$. For TIRM case, the variation of non-dimensional speed is shown by dotted line with center symbols in Fig. 4. It has value 1.4074 at $\omega^* = 2.5$ and it decreases to value 1.3306 at $\omega^* = 3.5$ and then increases to value 1.4396 at $\omega^* = 10$. The comparison of the solid and dotted lines in Fig. 4 shows the piezoelectric and rotational effects on non-dimensional speed of Rayleigh wave in a transversely isotropic micropolar piezoelectric solid half-space with electrically shorted surface.

The variation of non-dimensional speed $\sqrt{\frac{\rho c^2}{A_{33}}}$ is shown graphically in Fig. 5 versus non-dimensional constant $\frac{A_{11}}{A_{33}}$ for charge free (CF) case when $\omega^* = 5$ and $\Omega/\omega = 2, 5$ and 8. For $\Omega/\omega = 2$, the non-dimensional speed is 1.4674 at $\frac{A_{11}}{A_{33}} = 0$.

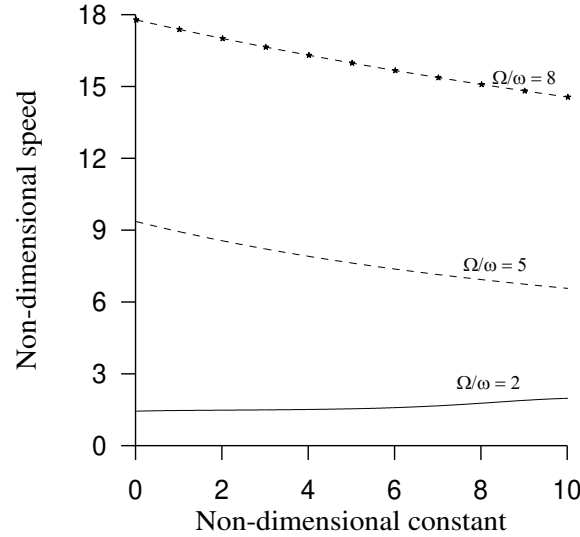


Fig. 5. Variation of non-dimensional speed $\sqrt{\frac{\rho c^2}{A_{33}}}$ versus non-dimensional constant $\frac{A_{11}}{A_{33}}$ for charge free (CF) case when $\omega^* = 5$.

It increases to its maximum value 2.0027 at $\frac{A_{11}}{A_{33}} = 10$. For $\Omega/\omega = 5$, the non-dimensional speed is 9.3844 at $\frac{A_{11}}{A_{33}} = 0$. It decreases to its minimum value 6.5939 at $\frac{A_{11}}{A_{33}} = 10$. For $\Omega/\omega = 8$, the non-dimensional speed is 17.8053 at $\frac{A_{11}}{A_{33}} = 0$. It decreases to its minimum value 14.5828 at $\frac{A_{11}}{A_{33}} = 10$. The comparison of the solid ($\Omega/\omega = 2$), dotted ($\Omega/\omega = 5$) and dotted with star ($\Omega/\omega = 8$) lines in Fig. 5 show the effect of rotation and non-dimensional material constant on non-dimensional speed of Rayleigh wave in a transversely isotropic micropolar piezoelectric solid half-space with charge free surface.

5 CONCLUSION

The governing equations for a linear, homogeneous and transversely isotropic rotating micropolar piezoelectric medium are formulated in two-dimension by assuming the components of the displacement and microrotation vectors in the form $\vec{u} = (u_1, 0, u_3)$ and $\vec{\varphi} = (0, \varphi_2, 0)$. Particular solutions of these governing equations which satisfy the radiation conditions in a half-space are obtained. The frequency equation for non-dimensional propagation speed of Rayleigh wave is obtained in the

medium. The frequency equation is solved for non-dimensional speed with the help of iteration method. The non-dimensional propagation speed is computed for relevant material parameters and is shown graphically against non-dimensional frequency and non-dimensional material constant for different values of rotation rate. The effects of piezoelectricity, charge free surface, electrically shorted surface, non-dimensional frequency and non-dimensional material constant are observed on non-dimensional speed.

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APPENDIX

The expressions for S_j , ($j = 1, 2, \dots, 4$) are

$$\begin{aligned}
 S_1 = & \left[k^2 \left(A_{33} A_{55} \gamma_{33} P + A_{33} A_{55} B_{66} \gamma_{11} + A_{55} B_{66} \gamma_{33} N \right. \right. \\
 & + L A_{33} B_{66} \gamma_{33} + L B_{66} \lambda_{33}^2 - B_{66} \gamma_{33} M^2 + P A_{55} \lambda_{33}^2 \\
 & - \rho \frac{\Omega^2}{k^2} A_{55} B_{66} \gamma_{33} - \rho \frac{\Omega^2}{k^2} A_{33} B_{66} \gamma_{33} - \rho \frac{\Omega^2}{k^2} \lambda_{33}^2 B_{66} \\
 & \left. + A_{33} B_{66} R^2 + 2 A_{55} B_{66} \lambda_{33} \lambda_{15} - 2 M B_{66} \lambda_{33} R \right) \\
 & \left. - K_1^2 A_{33} \gamma_{33} - K_1^2 \lambda_{33}^2 \right] / \left(A_{33} A_{55} B_{66} \gamma_{33} + A_{55} B_{66} \lambda_{33}^2 \right),
 \end{aligned}$$

$$\begin{aligned}
S_2 = & \left[k^4 \left(A_{55} B_{66} \gamma_{11} N + A_{33} A_{55} \gamma_{11} P + A_{55} \gamma_{33} NP + A_{33} \gamma_{33} LP \right. \right. \\
& + B_{66} \gamma_{33} LN + A_{55} B_{66} \lambda_{15}^2 + \lambda_{33}^2 LP + A_{33} B_{66} \gamma_{11} L + A_{33} PR^2 \\
& + B_{66} NR^2 - B_{66} \lambda_{11} M^2 - \gamma_{33} PM^2 + \rho^2 \frac{\Omega^4}{k^4} B_{66} \gamma_{33} - \rho \frac{\Omega^2}{k^2} \lambda_{33}^2 P \\
& - \rho \frac{\Omega^2}{k^2} B_{66} R^2 - \rho \frac{\Omega^2}{k^2} A_{55} B_{66} \gamma_{11} - \rho \frac{\Omega^2}{k^2} A_{55} \gamma_{33} P - L \rho \frac{\Omega^2}{k^2} B_{66} \gamma_{33} \\
& - \rho \frac{\Omega^2}{k^2} A_{33} B_{66} \gamma_{11} - \rho \frac{\Omega^2}{k^2} A_{33} \gamma_{33} P - \rho \frac{\Omega^2}{k^2} B_{66} \gamma_{33} N - 4 \rho \frac{\Omega^2}{k^2} c^2 B_{66} \gamma_{33} \\
& \left. + 2 A_{55} \lambda_{33} \lambda_{15} P + 2 B_{66} \lambda_{33} \lambda_{15} L - 2 B_{66} \lambda_{15} MR - 2 \lambda_{33} MPR \right) \\
& - k^2 \left(A_{55} \gamma_{33} K_2^2 + A_{33} \gamma_{11} K_1^2 + \gamma_{33} N K_1^2 + 2 \lambda_{33} \lambda_{15} K_1^2 - 2 \gamma_{33} M K_1 K_2 \right. \\
& \left. - 2 \lambda_{33} R K_1 K_2 - \rho \frac{\Omega^2}{k^2} \gamma_{33} K_1^2 \right) \Big/ \left(A_{33} A_{55} B_{66} \gamma_{33} + A_{55} B_{66} \lambda_{33}^2 \right),
\end{aligned}$$

$$\begin{aligned}
S_3 = & \left[k^6 \left(A_{55} \gamma_{11} NP + B_{66} \gamma_{11} LN + A_{33} \gamma_{11} LP - \rho \frac{\Omega^2}{k^2} A_{55} \gamma_{11} P \right. \right. \\
& - L \rho \frac{\Omega^2}{k^2} \gamma_{11} B_{66} - L \rho \frac{\Omega^2}{k^2} \gamma_{33} P - \rho \frac{\Omega^2}{k^2} A_{33} \gamma_{11} P - \rho \frac{\Omega^2}{k^2} B_{66} \gamma_{11} N \\
& - \rho \frac{\Omega^2}{k^2} P \gamma_{33} N + \rho^2 \frac{\Omega^4}{k^4} B_{66} \gamma_{11} + \rho^2 \frac{\Omega^4}{k^4} P \gamma_{33} - 2 \rho \frac{\Omega^2}{k^2} \lambda_{33} \lambda_{15} P \\
& - \rho \frac{\Omega^2}{k^2} B_{66} \lambda_{15}^2 - 4 B_{66} \gamma_{11} \rho^2 c^2 \frac{\Omega^2}{k^2} - 4 P \gamma_{33} \rho^2 c^2 \frac{\Omega^2}{k^2} + \gamma_{33} LNP \\
& + A_{55} \lambda_{15}^2 P + B_{66} \lambda_{15}^2 L - \gamma_{11} M^2 P + PR^2 N + 2 \lambda_{33} \lambda_{15} LP \\
& \left. - 2 \lambda_{15} MPR \right) - k^4 \left(A_{55} \gamma_{11} K_2^2 + \gamma_{33} L K_2^2 + \gamma_{11} N K_1^2 \right. \\
& + \lambda_{15}^2 K_1^2 + K_2^2 R^2 - 2 K_1 K_2 \gamma_{11} M - 2 K_1 K_2 \lambda_{15} R \\
& \left. - \rho \frac{\Omega^2}{k^2} K_2^2 \gamma_{33} - \rho \frac{\Omega^2}{k^2} K_1^2 \gamma_{11} \right) \Big/ \left(A_{33} A_{55} B_{66} \gamma_{33} + A_{55} B_{66} \lambda_{33}^2 \right)
\end{aligned}$$

$$\begin{aligned}
S_4 = & \left[k^8 \left(\gamma_{11} LNP + \lambda_{15}^2 LP - L \rho \frac{\Omega^2}{k^2} P \gamma_{11} - \rho \frac{\Omega^2}{k^2} NP \gamma_{11} - \rho \frac{\Omega^2}{k^2} \lambda_{15}^2 P \right. \right. \\
& \left. + \rho^2 \frac{\Omega^4}{k^4} P \gamma_{11} - 4 P \gamma_{11} \rho^2 c^2 \frac{\Omega^2}{k^2} \right) + k^6 \left(-\gamma_{11} L K_2^2 + \rho \frac{\Omega^2}{k^2} K_2^2 \gamma_{11} \right) \Big] \\
& \Big/ \left(A_{33} A_{55} B_{66} \gamma_{33} + A_{55} B_{66} \lambda_{33}^2 \right)
\end{aligned}$$

where

$$L = (A_{11} - \rho c^2), \quad M = (A_{13} + A_{56}), \quad N = (A_{66} - \rho c^2)$$

$$P = \left(B_{77} - \rho j c^2 + \frac{\chi}{k^2} \right) R = (\lambda_{15} + \lambda_{31}).$$

The expressions for m_j^2 ($j = 1, 2, \dots, 4$) are obtained from the following relations:

$$m_1^2 + m_2^2 + m_3^2 + m_4^2 = S_1,$$

$$m_1^2 m_2^2 + m_2^2 m_3^2 + m_3^2 m_4^2 + m_4^2 m_1^2 = S_2,$$

$$m_1^2 m_2^2 m_3^2 + m_2^2 m_3^2 m_4^2 + m_3^2 m_4^2 m_1^2 = S_3,$$

$$m_1^2 m_2^2 m_3^2 m_4^2 = S_4$$

and the expressions for ζ_i , ξ_i and η_i/k are

$$\zeta_i = \frac{p_i q_i - v_i r_i}{q_i s_i + t_i r_i}, \quad \xi_i = \frac{p_i - s_i \zeta_i}{r_i},$$

$$\frac{\eta_i}{k} = \frac{\left[w_i + i R \xi_i \frac{m_i}{k} + \left(2i \rho c \frac{\Omega}{k} - i M \frac{m_i}{k} \right) \zeta_i \right]}{K_1 \frac{m_i}{k}}$$

where

$$p_i = i K_2 A_{55} \frac{m_i^2}{k^2} - i K_2 L - i M K_1 \frac{m_i^2}{k^2} + \rho \frac{\Omega^2}{k^2} K_2 - 2i \rho c \frac{\Omega}{k} \frac{m_i}{k} K_1,$$

$$q_i = \left(\gamma_{33} \frac{m_i^2}{k^2} - \gamma_{11} \right) \left(B_{66} \frac{m_i^2}{k^2} - P \right),$$

$$r_i = R K_2 \frac{m_i}{k} - \lambda_{15} K_1 \frac{m_i}{k} + \lambda_{33} K_1 \frac{m_i^3}{k^3},$$

$$s_i = N K_1 \frac{m_i}{k} - M K_2 \frac{m_i}{k} - A_{33} K_1 \frac{m_i^3}{k^3} - \rho \frac{\Omega^2}{k^2} \frac{m_i}{k} K_1 + 2\rho c \frac{\Omega}{k} K_2,$$

$$t_i = - \left(\lambda_{33} \frac{m_i^2}{k^2} - \lambda_{15} \right) \left(B_{66} \frac{m_i^2}{k^2} - P \right),$$

$$v_i = i R \frac{m_i}{k} \left(B_{66} \frac{m_i^2}{k^2} - P \right),$$

$$w_i = A_{55} \frac{m_i^2}{k^2} - L + \rho \frac{\Omega^2}{k^2}.$$