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FLUID MECHANICS

STUDY OF RADIAL VIBRATIONS IN AN INFINITELY LONG FLUID-FILLED TRANSVERSELY ISOTROPIC THICK-WALLED HOLLOW COMPOSITE POROELASTIC CYLINDERS

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ABSTRACT: In this paper, radial vibrations of an infinitely long fluid-filled transversely isotropic thick-walled hollow composite poroelastic cylinder are investigated in the framework of poroelasticity. The cylinder consists of two concentric cylindrical layers namely, core (inner one) and coating (outer one), each of which retains its own distinctive properties. A comparative study has been made between the thick-walled hollow composite poroelastic cylinder and that of fluid-filled one. Frequency is computed against the ratio between the thickness to inner radius of the composite cylinder at various anisotropic ratios. Another comparative study is made between the results of current case and that of isotropic case by making Young's modulus and Poisson ratio values of isotropic and that of transversely isotropic in the transverse direction equal. Numerical results are depicted graphically and then discussed.

KEY WORDS: poroelasticity, radial vibrations, composite poroelastic cylinder, fluid, anisotropic ratios, frequency, ratio of radii.

LIST OF SYMBOLS

(r, θ, z)	cylindrical coordinate system	σ_{rr}	normal stress components
$\vec{u}(u_r, 0, 0)$	solid displacement component	M	Biot's modulus
$\vec{U}(U_r, 0, 0)$	fluid displacement component	p	pore pressure
α	Biot's effective stress coefficient in the plane of isotropy	b	dissipative coefficient
		e	solid dilatation
M_{ij}	components of the drained elastic modulus	ε	fluid dilatation
		ω	frequency
E	drained Young's modulus in the isotropic plane	t	time
		∇^2	Laplacian operator
E'	drained modulus in the transverse direction	ρ_{ij}	mass coefficients
		ρ_f	density of the fluid
ν	drained Poisson's ratio in the isotropic plane	r_1	inner radius
		a	interface radius

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ν'	drained Poisson's ratio in the transverse direction	r_2	outer radius
ν_u	undrained Poisson's ratio	g	ratio of outer and inner radii
$J_n(x)$	Bessel function of first kind of order n	c_f	velocity of the sound in fluid
$Y_n(x)$	Bessel function of second kind of order n	K_s	bulk modulus of the solid constituents
N_E, N_ν	anisotropic ratios	B	Skempton pore pressure coefficient
		h	thickness of the cylinder

1 Introduction

In the last few decades, deformation problems and fluid flow in porous media has been of great importance due its applications in the diversified areas. The mathematical model of poroelasticity is very useful in describing the interaction between fluid motion and deformation in the porous solid. Linear isotropic poroelasticity theory was developed by Biot [1]. Employing Biot's theory of wave propagation in fluid saturated poroelastic solids, radial vibrations of a thick-walled hollow isotropic poroelastic cylinders are investigated [2]. The frequency equations of these vibrations for a permeable and an impermeable surface are derived. Reddy and Tajuddin [3] studied plane strain vibrations of thick-walled hollow poroelastic cylinders and derived extreme limiting cases of plate and solid cylinder. A study of torsional vibrations of an finite poroelastic composite circular solid cylinder made of two different materials is made [4]. In absence of dissipation, analysis of flexural wave propagation in composite poroelastic solid cylinders that are bonded end to end is presented in the paper [5]. In this study [5], it is seen that the nature of surface does not have influence over shear vibrations unlike in the case of extensional vibrations. Radial vibrations of infinitely long poroelastic isotropic composite hollow circular cylinders are studied by Shanker et al. [6]. Authors of this paper derived frequency equations for radial vibrations of poroelastic composite hollow cylinder with rigid core, poroelastic composite solid cylinder, poroelastic composite solid cylinder with rigid casing and of rigid core. Axially symmetric waves in cylindrical bone filled with marrow is investigated under the assumption that bone exhibits isotropic poroelastic behavior [7]. Vibrations in a fluid-loaded poroelastic cylinder surrounded by a fluid are investigated employing Biot's theory of wave propagation in poroelastic media under plane-strain condition [8]. From the paper [8], it is observed that the shear and dilatational vibrations of the fluid-loaded poroelastic hollow cylinder surrounded by a fluid are uncoupled in the case of axially symmetric vibrations. Analysis of radial vibrations of poroelastic isotropic circular cylindrical shells of infinite extent immersed in an acoustic medium is investigated [9]. In [10], authors investigated axially symmetric vibrations of fluid-filled and empty isotropic poroelastic circular cylindrical shells of infinite extent for different wall-thickness. The analytical solu-

tions for pore pressure and stress fields for inclined bore hole and the cylinder are presented in the papers [11, 12]. Poromechanical analysis of a fully saturated transversely isotropic hollow cylinder is studied by Kanj et al. [13]. In the frame work of transversely isotropic poroelasticity, Reddy and Sandhya [14] studied radial vibrations of cylindrical bone filled with marrow. In this paper, bone is assumed to be transversely isotropic. The poroelastic behavior of rocks is often more complicated than the common assumptions of linearity and isotropy. Berea sandstone and shale rock exhibit transversely isotropic behavior at low effective stresses. Sandhya et al. [15] studied torsional vibrations of transversely isotropic composite poroelastic cylinder consists of two concentric cylindrical layers made of different poroelastic materials. In the paper [15], the limiting cases of torsional vibrations in a poroelastic thick-walled hollow cylinder, thin poroelastic shell and plate are discussed. To the best of authors knowledge, vibrations in composite transversely isotropic poroelastic cylinder filled with another fluid are not yet studied. Because of mathematical complexity, here radial vibrations are considered so that problem becomes a plane strain and therefore can be solved analytically. This kind of model is useful in the problem of bone mechanics and petroleum engineering. Frequency values are computed against ratio between the thickness to inner radius of the composite cylinder at different anisotropic ratios. The numerical results are also obtained for the particular cases.

This paper is arranged as follows. Transversely isotropic thick-walled hollow composite cylinder is considered in Section 2. In Section 3, composite cylinder filled with fluid is investigated. Some particular cases are discussed in Section 4. In Section 5, numerical results are presented. In Section 6, concluding remarks are given.

2 Thick-Walled Hollow Composite Poroelastic Cylinder

Consider an infinitely long transversely isotropic cylinder having axis in the direction of the z-axis of the cylindrical coordinate system (r, θ, z). Assume that z-axis is along the axis of rotational symmetry. Poroelastic composite thick-walled hollow cylinder, under consideration consists of two concentric cylindrical layers with two different materials. The radius of core and coating are r_1 and r_2, respectively whereas interface radius is a. Let $\vec{u}(u_r, 0, 0)$ and $\vec{U}(U_r, 0, 0)$ be the solid and fluid displacements of radial vibrations, respectively. These displacements are functions of $r$and time t only. The pertinent equations of motion for transversely isotropic poroelastic solid saturated with fluid are [14]:

$$(1)\quad \frac{M_{11}-M_{12}}{2}\Big(\nabla^2-\frac{1}{r^2}\Big)u_r+\Big(M_{12}+M\alpha^2+\frac{M_{11}-M_{12}}{2}\Big)\frac{\partial e}{\partial r}-\alpha M\frac{\partial \varepsilon}{\partial r} = \frac{\partial^2}{\partial t^2}(\rho_{11}u_r+\rho_{12}U_r)+b\frac{\partial}{\partial t}(u_r-U_r)\ ,$$

$$M\frac{\partial \varepsilon}{\partial r} - \alpha M\frac{\partial e}{\partial r} = \frac{\partial^2}{\partial t^2}(\rho_{12}u_r + \rho_{22}U_r) - b\frac{\partial}{\partial t}(u_r - U_r)\,.$$

In Eq. (1), ∇^2 is the Laplacian, e and ε are the dilatations of solid and fluid, respectively; ρ_{jk} are mass coefficients, α is Biot's effective stress coefficients in the isotropic plane (r–θ plane), which can be expressed in terms of elastic moduli M_{ij} and the bulk modulus of solid constituents (K_s). The components of elastic modulus M_{ij}, Biot's effective stress coefficient (α), Biot's modulus (M) and the bulk modulus of solid constituents (K_s) can be determined from the following relations [13]:

$$M_{11} = \frac{E(E' - E\nu'^2)}{(1+\nu)(E' - E'\nu - 2E\nu'^2)}\,, \quad M_{12} = \frac{E(E'\nu + E\nu'^2)}{(1+\nu)(E' - E'\nu - 2E\nu'^2)}\,,$$
$$M_{13} = \frac{EE'\nu'}{E' - E'\nu - 2E\nu'^2}\,, \quad K_s = \frac{B(1+\nu_u)E}{3B(1-2\nu)(1+\nu_u) - 9(\nu_u - \nu)}\,,$$
$$\alpha = 1 - \frac{M_{11} + M_{12} + M_{13}}{3K_s}\,, \quad M = \frac{B^2(1-2\nu)(1+\nu_u)^2 E}{9(1-2\nu_u)(1+\nu)(\nu_u - \nu)}\,.$$

Here E and ν are drained Young's modulus and Poisson ratio in the isotropic plane, E' and ν' are similar quantities as that of E and ν pertaining to the transverse direction of the axis of symmetry. For given anisotropic ratios $N_E = E'/E$, $N_\nu = \nu'/\nu$ [13], E' and ν' can be determined. Different N_E and N_ν ratios define different degrees of anisotropy. If the substitutions $E = E'$, $\nu = \nu'$ $(N_E = N_\nu = 1)$ are made, then Eq. (1) will be reduced to Biot's equations of motion for isotropic solids which are used by many researchers in this domain [2, 6, 9]. The solid displacement u_r and the fluid displacement U_r in the radial direction are assumed as follows:

$$(2) \qquad {}_ju_r(r,t) = {}_jf(r)\mathrm{e}^{i\omega t}\,, \quad {}_jU_r(r,t) = {}_jF(r)\mathrm{e}^{i\omega t}\,, \qquad j = 1,\, 2.$$

In Eq. (2), the prefixes $j = 1, 2$ are used to denote two layers of composite cylinder. The quantities with prefix 1 refer to the core while 2 refers to the coating, ω is the frequency of wave, i is the complex unity and t is time. Substituting Eq. (2) in Eqs. (1), then Eqs. (1) become

$$(3) \qquad \begin{aligned} ({}_jM_{11} + {}_jM_j\alpha^2)\Delta_j f - {}_j\alpha_j M\Delta_j F &= -\omega^2({}_jK_{11j}f + {}_jK_{12j}F)\,,\\ -{}_j\alpha_j M\Delta_j f + {}_jM\Delta_j F &= -\omega^2({}_jK_{12j}f + {}_jK_{22j}F)\,, \end{aligned}$$

where,

$$\Delta = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2}\,,$$
$${}_jK_{11} = {}_j\rho_{11} - \frac{ib}{\omega}\,, \quad {}_jK_{12} = {}_j\rho_{12} + \frac{ib}{\omega}\,, \quad {}_jK_{22} = {}_j\rho_{22} - \frac{ib}{\omega}\,.$$

Solving the above equations, we obtain

$$ {}_jF = \frac{-{}_jM_j M_{11}}{\omega^2({}_jM_jK_{12} - {}_j\alpha_j M_j K_{22})}\Delta_j f - \Big(\frac{{}_jM_jK_{11} + {}_j\alpha_j M_j K_{12}}{{}_jM_jK_{12} + {}_j\alpha_j M_j K_{22}}\Big){}_jf. $$

Substitution of ${}_jF$ into Eq. (3), gives

$$ \Delta^2{}_jf + \omega^2\Big(\frac{({}_jM_{11} + {}_jM_j\alpha^2){}_jK_{22} + {}_jM_jM_{11} - {}_j\alpha_jM_jK_{12}}{{}_jM_jM_{11}}\Big)\Delta_j f + \omega^2\Big(\frac{{}_jK_{11}{}_jK_{22} - {}_jK_{12}^2}{{}_jM_jM_{11}}\Big){}_jf = 0, \tag{4} $$

Equation (4) can also be written as

$$ \Delta^2{}_jf + ({}_j\zeta_1^2 + {}_j\zeta_2^2)\Delta_j f + ({}_j\zeta_1^2{}_j\zeta_2^2){}_jf = 0. \tag{5} $$

Equation (5) can also written as

$$ (\Delta + {}_j\zeta_1^2){}_jf = 0, \qquad (\Delta + {}_j\zeta_2^2){}_jf = 0. \tag{6} $$

Solving Eq. (6), we obtain ${}_jf$ for ${}_j\zeta_1$ and ${}_j\zeta_2$. Similarly we can obtain ${}_jF$. Substituting these in Eq. (2), we obtain the displacement components as follows:

$$ \begin{aligned} {}_ju_r(r,t) &= ({}_jc_{1j}D_{11}(r) + {}_jc_{2j}D_{12}(r) + {}_jc_{3j}D_{13}(r) + {}_jc_{4j}D_{14}(r))e^{i\omega t}, \\ {}_jU_r(r,\,t) &= -({}_jc_{1j}\delta_{1j}^2D_{11}(r) + {}_jc_{2j}\delta_{1j}^2D_{12}(r) + {}_jc_{3j}\delta_{2j}^2D_{13}(r) \\ &\quad + {}_jc_{4j}\delta_{2j}^2D_{14}(r))e^{i\omega t}\,, \end{aligned} \tag{7} $$

where ${}_jc_1$, ${}_jc_2$, ${}_jc_3$ and ${}_jc_4$ are arbitrary constants,

$$ \begin{aligned} {}_jD_{11}(r) &= J_1({}_j\xi_1 r), \qquad {}_jD_{12}(r) = Y_1({}_j\xi_1 r), \\ {}_jD_{13}(r) &= J_1({}_j\xi_2 r), \qquad {}_jD_{14}(r) = Y_1({}_j\xi_2 r), \end{aligned} $$

J_n and Y_n are Bessel functions of first and second kind of order n, respectively, and δ_l, ξ_l $(l = 1, 2)$ are

$$ \begin{aligned} {}_j\delta_l^2 &= \Big(({}_jM_jK_{11} + {}_j\alpha_jM_jK_{12}) - \frac{{}_j\xi_l^2}{\omega^2}{}_jM_jM_{11}\Big)({}_jM_jK_{12} + {}_j\alpha_jM_jK_{22})^{-1}, \\ {}_j\xi_1 &= \frac{({}_jL_1 + {}_jL_2)^{1/2}}{{}_jL_3}, \qquad {}_j\xi_2 = \frac{({}_jL_1 - {}_jL_2)^{1/2}}{{}_jL_3}, \\ {}_jL_1 &= \omega^2(({}_jM_{11} + x_j{}_jM_j\alpha^2){}_jK_{22} + {}_jM_jK_{11} + 2_j\alpha\,{}_jM_jK_{12}), \\ {}_jL_2 &= (\omega^4(({}_jM_{11} + {}_jM_j\alpha^2)\,{}_jK_{22} + {}_jM_jK_{11} + 2\,{}_j\alpha\,{}_jM_jK_{12})^2 \\ &\quad - 4\omega^2{}_jM_jM_{11}\,({}_jK_{11}\,{}_jK_{22} - {}_jK_{12}^2))^{1/2}, \\ {}_jL_3 &= (2_jM_jM_{11})^{1/2}, \qquad \text{for } j = 1, 2. \end{aligned} \tag{8} $$

From the stress-displacement relations [13] and the displacement functions in Eq. (7), the following stresses are obtained:

$$
\begin{aligned}
{}_j(\sigma_{rr}+s) &= \big({}_jc_{1j}A_{11}(r)+{}_jc_{2j}A_{12}(r)+{}_jc_{3j}A_{13}(r)+{}_jc_{4j}A_{14}(r)\big)e^{i\omega t},\\
{}_js &= \big({}_jc_{1j}A_{21}(r)+{}_jc_{2j}A_{22}(r)+{}_jc_{3j}A_{23}(r)+{}_jc_{4j}A_{24}(r)\big)e^{i\omega t}, \quad \text{for } j=1,2.
\end{aligned} \tag{9}
$$

Where,

$$
\begin{aligned}
{}_jA_{11}(r) &= \frac{-2}{r}({}_jM_j\delta_1^2(1-{}_j\alpha) - \frac{1}{2}({}_jM_{11}+{}_jM_{12}+2{}_jM_j\alpha({}_j\alpha-1))J_1({}_j\xi_1 r)\\
&\quad + ({}_jM_j\delta_1^2({}_j\alpha-1) - ({}_jM_{11}+{}_jM_j\alpha({}_j\alpha-1)))_j\xi_1 J_2({}_j\xi_1 r)\\
{}_jA_{21}(r) &= \frac{-2}{r}({}_jM_j\delta_1^2+{}_j\alpha_jM)J_1({}_j\xi_1 r)+({}_jM_j\delta_1^2+{}_j\alpha_jM)_j\xi_1 J_2({}_j\xi_1 r),
\end{aligned} \tag{10}
$$

${}_jA_{12}(r)$, ${}_jA_{22}(r)$ are similar expressions as ${}_jA_{11}(r)$, ${}_jA_{21}(r)$ with J_1, J_2 replaced by Y_1, Y_2, respectively, ${}_jA_{13}(r)$, ${}_jA_{23}(r)$ are similar expressions as ${}_jA_{11}(r)$, ${}_jA_{21}(r)$ with ${}_j\xi_1$, ${}_j\delta_1$ replaced by ${}_j\xi_2$, ${}_j\delta_2$, respectively, ${}_jA_{14}(r)$, ${}_jA_{24}(r)$ are similar expressions as ${}_jA_{11}(r)$, ${}_jA_{21}(r)$ with J_1, J_2, ${}_j\xi_1$, ${}_j\delta_1$ replaced by Y_1, Y_2, ${}_j\xi_2$, ${}_j\delta_2$, respectively.

With regard to the boundary conditions, the inner and outer surface of the composite hollow cylinder are stress free and there is a perfect bonding at the interface. Moreover, pores at the interface assumed to be sealed. These conditions can be expressed mathematically as follows:

$$
\begin{aligned}
{}_1(\sigma_{rr}+s) = 0, \quad {}_1s = 0 &\qquad\qquad &&\text{at } r, = r_1,\\
{}_1(\sigma_{rr}+s) - {}_2(\sigma_{rr}+s) = 0, \quad {}_1s = {}_2s = 0, \quad {}_1u_2 - {}_2u_2 = 0 &&&\text{at } r = a,\\
{}_2(\sigma_{rr}+s) = 0, \quad {}_2s = 0 &\qquad\qquad &&\text{at } r = r_2.
\end{aligned} \tag{11}
$$

From the first and third set of conditions, it is clear that the inner surface $r = r_1$ and outer surface $r = r_2$ are traction free. From the second set of conditions, it is clear that the effective stress and displacement of inner solid are intact with that of outer solid at the interface. Also, pores are assumed to be sealed. That is, there is no flow across the interface $r = a$. Equations (7), (9) and (11) gives a homogeneous system of eight equations in eight constants ${}_jc_1$, ${}_jc_2$, ${}_jc_3$ and ${}_jc_4$ $(j = 1, 2)$. A non-zero solution exists if the determinant of the coefficient matrix vanishes, which leads to the following frequency equation:

$$
|p_{mn}(r)| = 0, \qquad (m, n = 1, 2, 3, \ldots, 8), \tag{12}
$$

where

$$|p_{mn}(r)| = \begin{vmatrix} {}_1A_{11}(r_1) & {}_1A_{12}(r_1) & {}_1A_{13}(r_1) & {}_1A_{14}(r_1) & 0 & 0 & 0 & 0 \\ {}_1A_{21}(r_1) & {}_1A_{22}(r_1) & {}_1A_{23}(r_1) & {}_1A_{24}(r_1) & 0 & 0 & 0 & 0 \\ {}_1A_{11}(a) & {}_1A_{12}(a) & {}_1A_{13}(a) & {}_1A_{14}(a) & -{}_2A_{11}(a) & -{}_2A_{12}(a) & -{}_2A_{13}(a) & -{}_2A_{14}(a) \\ {}_1A_{21}(a) & {}_1A_{22}(a) & {}_1A_{23}(a) & {}_1A_{24}(a) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & {}_2A_{21}(a) & {}_2A_{21}(a) & {}_2A_{21}(a) & {}_2A_{21}(a) \\ {}_1D_{11}(a) & {}_1D_{12}(a) & {}_1D_{13}(a) & {}_1D_{14}(a) & -{}_2D_{11}(a) & -{}_2D_{12}(a) & -{}_2D_{13}(a) & -{}_2D_{14}(a) \\ 0 & 0 & 0 & 0 & {}_2A_{11}(r_2) & {}_2A_{12}(r_2) & {}_2A_{13}(r_2) & {}_2A_{14}(r_2) \\ 0 & 0 & 0 & 0 & {}_2A_{21}(r_2) & {}_2A_{22}(r_2) & {}_2A_{23}(r_2) & {}_2A_{24}(r_2) \end{vmatrix},$$

where the elements ${}_jA_{1l}$, ${}_jA_{2l}$, ${}_jD_{1l}$ $(j = 1, 2,\ l = 1, 2, 3, 4)$ are defined in Eqs. (10) and (8).

3 Thick-Walled Hollow Composite Poroelastic Cylinder Filled with Fluid

In this section, thick-walled hollow composite cylinder is filled with fluid is considered. The example for this model is bone filled with marrow. First, the behaviour of waves in fluid is studied independently from the solid cylinder. The equation of motion of the fluid in the inner cylinder for the case of radial vibrations are [16]

$$\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} = \frac{1}{c_f^2}\frac{\partial^2\varphi}{\partial t^2}\,. \tag{13}$$

In Eq. (13), c_f is the velocity of the sound in fluid, φ is the displacement potential function and satisfies $\varphi = J_0(r\omega/c_f)e^{i\omega t}$. The fluid pressure (p_f) and the radial displacement (D_r) of the fluid are [16]

$$\begin{aligned} p_f &= -\rho_f\frac{\partial^2\varphi}{\partial t^2} = -\rho_f\omega^2 J_0\Big(\frac{r\omega}{c_f}\Big)\,e^{i\omega t}, \\ D_r &= \frac{\partial\varphi}{\partial r} = \frac{\omega}{c_f}J_1\Big(\frac{r\omega}{c_f}\Big)e^{i\omega t}. \end{aligned} \tag{14}$$

In Eq. (14), ρ_f is mass density of the fluid, $J_0(x)$ and $J_1(x)$ are the Bessel function of first kind of order 0 and 1, respectively. The boundary conditions for fluid-filled composite poroelastic cylinder are considered as follows:

$$\begin{aligned} \frac{{}_1(\sigma_{rr}+s)}{{}_1u_r} = \frac{-p_f}{D_r} = 0, \quad {}_1s = 0 \qquad & \text{at } r = r_1, \\ {}_1(\sigma_{rr}+s) - {}_2(\sigma_{rr}+s) = 0, \quad {}_1s = {}_2s = 0, \quad {}_1u_r - {}_2u_r = 0 \qquad & \text{at } r = a, \\ {}_2(\sigma_{rr}+s) = 0, \quad {}_2s = 0 \qquad & \text{at } r = r_2. \end{aligned} \tag{15}$$

From the first set of conditions, it is clear that the effective stress and displacement of solid are intact with that of fluid at inner surface $r = r_1$. From the second set of conditions, it is clear that the effective stress and displacement of inner solid are intact with that of outer solid at the interface. Also, pores are assumed to be sealed. Then, there is no flow across the interface $r = a$. From the third set of conditions, it is clear that the outer surface $r = r_2$ is traction free. Equations (7), (9), (14) and (15) result in a homogeneous system of eight equations in eight constants ${}_jc_1$, ${}_jc_2$, ${}_jc_3$ and ${}_jc_4$ $(j = 1, 2)$. For a non-trivial solution, the determinant of the coefficient matrix must be vanished. Accordingly, the frequency equation is obtained and which is given below

$$|q_{mn}(r)| = 0\,, \qquad (m, n = 1, 2, 3, \ldots, 8), \tag{16}$$

where

$$|q_{mn}(r)| = \begin{vmatrix}
{}_1A'_{11}(r_1) & {}_1A'_{12}(r_1) & {}_1A'_{13}(r_1) & {}_1A'_{14}(r_1) & 0 & 0 & 0 & 0 \\
{}_1A_{21}(r_1) & {}_1A_{22}(r_1) & {}_1A_{23}(r_1) & {}_1A_{24}(r_1) & 0 & 0 & 0 & 0 \\
{}_1A_{11}(a) & {}_1A_{12}(a) & {}_1A_{13}(a) & {}_1A_{14}(a) & -{}_2A_{11}(a) & -{}_2A_{12}(a) & -{}_2A_{13}(a) & -{}_2A_{14}(a) \\
{}_1A_{21}(a) & {}_1A_{22}(a) & {}_1A_{23}(a) & {}_1A_{24}(a) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & {}_2A_{21}(a) & {}_2A_{21}(a) & {}_2A_{21}(a) & {}_2A_{21}(a) \\
{}_1D_{11}(a) & {}_1D_{12}(a) & {}_1D_{13}(a) & {}_1D_{14}(a) & -{}_2D_{11}(a) & -{}_2D_{12}(a) & -{}_2D_{13}(a) & -{}_2D_{14}(a) \\
0 & 0 & 0 & 0 & {}_2A_{11}(r_2) & {}_2A_{12}(r_2) & {}_2A_{13}(r_2) & {}_2A_{14}(r_2) \\
0 & 0 & 0 & 0 & {}_2A_{21}(r_2) & {}_2A_{22}(r_2) & {}_2A_{23}(r_2) & {}_2A_{24}(r_2)
\end{vmatrix},$$

$$\begin{aligned}
{}_1A'_{11}(r) &= -{}_1A_{11}(r)\omega/c_f J_1(r\omega/c_f) + \rho_f\omega^2 J_0(r\omega/c_f)J_1({}_1\xi_1 r), \\
{}_1A'_{12}(r) &= -{}_1A_{12}(r)\omega/c_f J_1(r\omega/c_f) + \rho_f\omega^2 J_0(r\omega/c_f)Y_1({}_1\xi_1 r), \\
{}_1A'_{13}(r) &= -{}_1A_{13}(r)\omega/c_f J_1(r\omega/c_f) + \rho_f\omega^2 J_0(r\omega/c_f)J_1({}_1\xi_2 r), \\
{}_1A'_{14}(r) &= -{}_1A_{14}(r)\omega/c_f J_1(r\omega/c_f) + \rho_f\omega^2 J_0(r\omega/c_f)Y_1({}_1\xi_2 r),
\end{aligned} \tag{17}$$

and the elements ${}_jA_{1l}$, ${}_jA_{2l}$, ${}_jD_{1l}$ $(j = 1, 2,\ l = 1, 2, 3, 4)$ are defined in Eqs. (10) and (8).

4 Particular Cases

4.1 Transversely isotropic thick-walled hollow poroelastic cylinder filled with fluid

When the material constants of both coating and core are equal, the composite poroelastic cylinder results in a thick walled hollow poroelastic cylinder with thickness $h\ (= r_2 - r_1)$. Then the frequency equation Eq. (16) reduces to

$$|E_{mn}(r)| = 0, \qquad m, n = 1, 2, 3, 4, \tag{18}$$

where

$$|E_{mn}| = \begin{vmatrix} {}_1A'_{11}(r_1) & {}_1A'_{12}(r_1) & {}_1A'_{13}(r_1) & {}_1A'_{14}(r_1) \\ {}_1A_{21}(r_1) & {}_1A_{22}(r_1) & {}_1A_{23}(r_1) & {}_1A_{24}(r_1) \\ {}_1A_{11}(r_2) & {}_1A_{12}(r_2) & {}_1A_{13}(r_2) & {}_1A_{14}(r_2) \\ {}_1A_{21}(r_2) & {}_1A_{22}(r_2) & {}_1A_{23}(r_2) & {}_1A_{24}(r_2) \end{vmatrix}$$

and the elements ${}_1A'_{1l}$, ${}_1A_{1l}$, ${}_1A_{2l}$ $(l = 1, 2, 3, 4)$ are defined in Eq. (10) and (17). This case is discussed in the paper [14].

4.2 Thick-walled hollow composite isotropic poroelastic cylinder filled with fluid

If the substitutions $E = E'$, $\nu = \nu'$ are made in Eq. (16), then transversely isotropic thick-walled hollow composite cylinder filled with fluid reduces to isotropic thick-walled hollow composite cylinder filled with fluid. In this case, the frequency equation is

$$|F_{mn}(r)| = 0, \qquad m, n = 1, 2, 3, 4, \ldots, 8, \tag{19}$$

where the elements F_{mn} are similar to the elements q_{mn} in Eqs. (16) and (17) with $E = E'$, $\nu = \nu'$.

4.3 Thick-walled hollow isotropic cylinder filled with fluid

Apart from the above substitutions in Section 4.2, if the material constants of both coating and core are equal then the case of isotropic thick-walled hollow poroelastic cylinder filled with fluid is obtained [9].

4.4 Isotropic thick-walled hollow composite empty cylinder

Apart from the above substitutions in Section 4.2, if $p_f = 0$ then the case of isotropic thick-walled hollow composite cylinder is obtained [6].

5 Numerical Results

All the frequency equations are investigated in absence of dissipation. If the dissipation coefficient (b) is non-zero, then frequency equations will be complex valued and implicit. Therefore, b is made to be zero so that frequency equation will be real valued and the roots will be obtained easily that explicitly give frequency. Even if we make b zero, the problem would be poroelastic in nature as the coefficients M_{11}, M_{12}, M_{13}, α and M would not vanish. In this case, mass coefficients K_{11}, K_{12} and K_{13} are real. For numerical process, two materials are considered. The composite poroelastic cylinder consists of two cylindrical layers namely core made up of Berea sandstone and the coating made up of shale rock. The material constants for Berea

sandstone and shale rock are taken from the papers [13,17]. The parameter values are computed from the available data in the literature and the values are given in Tables 1 and 2.

Table 1. Parameter values

Parameters	Berea sandstone	Shale rock
E	14.4 Gpa	1.854 Gpa
ν	0.20	0.22
B	0.62	0.903
ν_u	0.33	0.46
M	12.307 Gpa	8.5602 Gpa
K_s	37.7823 Gpa	44.60346 Gpa
ρ_{11}	2407.64 kg/m^3	1398.72 kg/m^3
ρ_{12}	-266 kg/m^3	-257.28 kg/m^3
ρ_{22}	456 kg/m^3	771.84 kg/m^3

Table 2. Parameter values

Parameters	Water	Kerosene
c_f	1.432×10^3 m/s	1.432×10^3 m/s
ρ_f	1000 kg/m^3	820.1 kg/m^3

Employing these values in Eqs. (12), (16), (18) and (19), implicit relation between frequency (ω), ratio of thickness to inner radius (h/r_1) and anisotropic ratios $(N_E = E'/E,\ N_\nu = \nu'/\nu)$ is obtained. The frequency is computed against h/r_1 for both thick-walled hollow composite cylinder and fluid filled composite cylinder. The numerical values are presented graphically in Figs. 1–4. Variation of frequency against h/r_1 when the anisotropic ratio $N_E = 1$ at various N_ν values in the case of $g = 2$ is depicted in Fig. 1. Variation of frequency against h/r_1 when the anisotropic ratio when $N_\nu = 1$ at various N_E values in the case of $g = 2$ is depicted in Fig. 2. From Figs. 1–2, it is seen that the frequency values of the fluid-filled composite cylinder are, in general, greater than that of the thick-walled hollow composite cylinder and the values depend on the anisotropic ratios.

For the comparative study, the numerical results of current case and that of isotropic case are presented in Figs. 3 and 4. Fig. 3 depicts the variation of frequency against h/r_1 in the cases of transversely isotropic and isotropic composite cylinder when $g = 2$. For each case, two different fluids water and kerosene filled in the cylinders are considered. From this figure, it is seen that the frequency values of composite cylinder filled with water are, in general, greater than that of composite cylinder filled

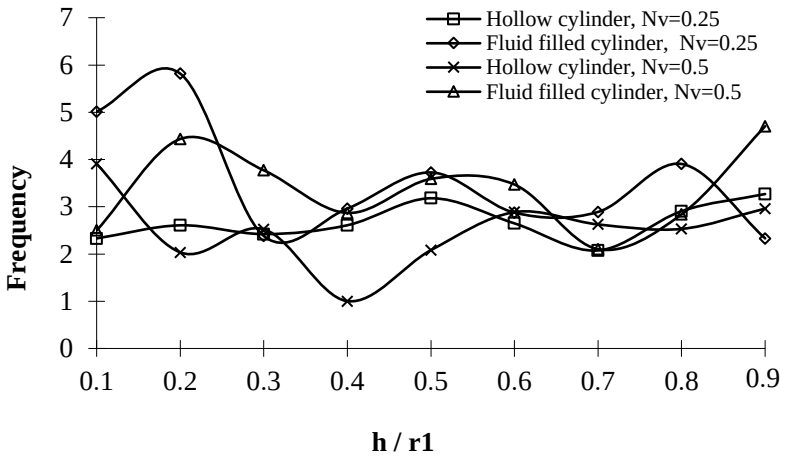

Fig. 1. Variation of frequency with ratio of thickness to inner radius of core (h/r_1) in the cases of thick-walled hollow and fluid filled composite cylinders when $N_E = 1$.

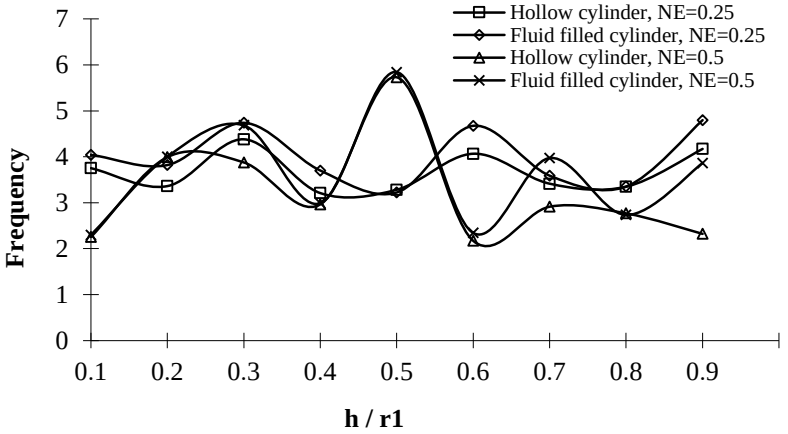

Fig. 2. Variation of frequency with ratio of thickness to inner radius of core (h/r_1) in the cases of thick-walled hollow and fluid filled composite cylinders when $N_\nu = 1$.

with kerosene in the case of both isotropic and transversely isotropic. Also, it is seen that frequency values of isotropic composite cylinder are, in general, greater than that of transversely isotropic composite cylinder in both the cases of water and kerosene. Fig. 4 depicts the variation of frequency against h/r_1 in the cases of transversely isotropic and isotropic cylinder for both hollow composite and fluid filled cylinders. In Fig. 4, the notations IHCC, TIHCC, IFFC, TIFFC stand for isotropic hollow composite cylinder, transversely isotropic hollow composite cylinder, isotropic fluid filled

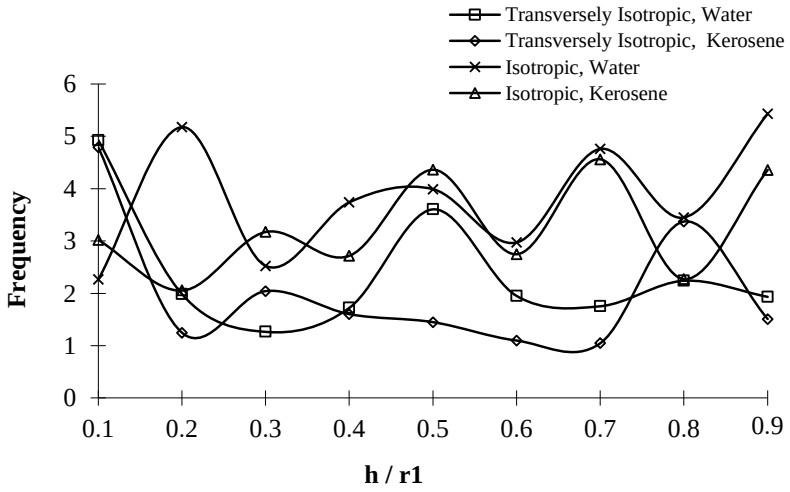

Fig. 3. Variation of frequency with ratio of thickness to inner radius of core (h/r_1) in the case of fluid-filled composite cylinder.

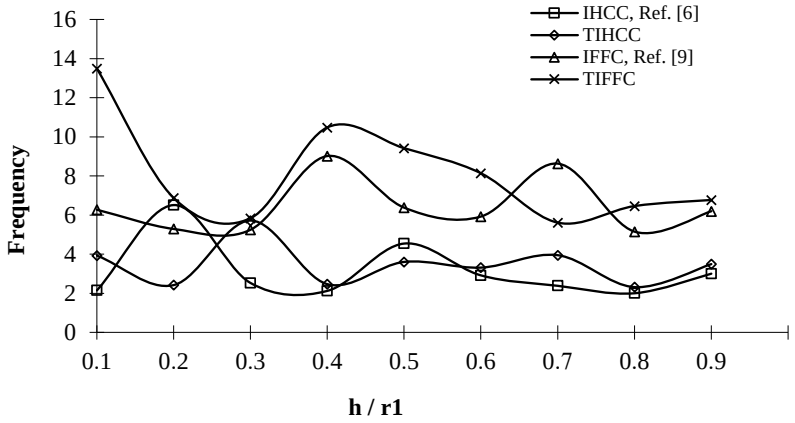

Fig. 4. Variation of frequency with h/r_1 in the cases of isotropic and transversely isotropic cylinders.

cylinder, transversely isotropic fluid filled cylinder, respectively. From Fig. 4, it is observed that frequency values of fluid filled cylinder are greater than that of hollow composite cylinder for both the cases isotropic and transversely isotropic. Also, it is seen that frequency values of transversely isotropic cylinder are, in general, greater than that of isotropic cylinder in both the cases of hollow composite and fluid filled cylinder.

6 Conclusion

Radial vibrations in an infinitely long thick-walled hollow fluid-filled transversely isotropic composite poroelastic cylinder are studied. A comparative study has been made between the composite thick-walled hollow cylinder and the composite cylinder filled with fluid. Frequency as a function of ratio between the thickness to inner radius of the composite cylinder at various anisotropic ratios is obtained. The numerical results are also obtained for the pertinent isotropic case and depicted graphically for the case of comparison. From the figures, one can infer that the dispersive behaviors in thick-walled hollow cylinder and fluid filled cylinder are different in phenomenon. Also, it is seen that the anisotropic ratio effects frequency values in all the cases.

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References

[1] Biot M. A. The Theory of Propagation of Elastic Waves in Fluid-Saturated Porous Solid. *Journal of Acoustical Society of America*, **28** (1956), 168-178.

[2] Tajuddin, M., S. A. Shah. Radial Vibrations of Thick-Walled Hollow Poroelastic Cylinders. *Journal of Porous Media*, **13** (2010) No. 4, 307-318.

[3] Malla Reddy, P., M. Tajuddin. Exact Analysis of the Plane Strain Vibrations of Thick Walled Hollow Poroelastic Cylinders. *International Journal of Solids Structures*, **37**2 (2000), 3439-3456.

[4] Tajuddin, M. Torsional Vibrations of Finite Composite Poroelastic Cylinders. *Journal of Pure and Applied Maths*, **13** (1982) No. 3, 375-381.

[5] Sandhya Rani, B., A. Srisailam, P. Malla Reddy. Flexural Vibrations of Finite Poroelastic Composite Cylinders. *Sadhana-Academy Proceedings in Engineering Sciences Indian Academy of Sciences*, **40** (2010), 591-604.

[6] Shanker, B., J. Manoj Kumar, S. Ahmed Shah, C. Nageswara Nath. Radial Vibrations of an Infinitely Long Poroelastic Composite Hollow Circular Cylinder. *International Journal of Engineering, Science and Technology*, **4** (2012), 17-33.

[7] Malla Reddy, P., B. Sandhya Rani, M. Tajuddin. Dispersion Study of Axially Symmetric Waves in Cylindrical Bone Filled with Marrow. *International Journal of Biomathematics*, **4** (2011), 109-118.

[8] Shanker, B., C. N. Nath, S. A. Shah, P. M. Reddy. Vibrations in a Fluid-Loaded Poroelastic Hollow Cylinder Surrounded by a Fluid in Plane-Strain Form. *International Journal of Applied Mechanics and Engineering*, **18** (2013) No. 1, 189-216.

[9] RAMESH, M., S. A. SHAH, M. V. RAMANA MURTHY. Analysis of Radial Vibrations of Poroelastic Circular Cylindrical Shells Immersed in an Acoustic Medium. *International Journal of Engineering, Science and Technology*, **6** (2014) No. 5, 26-35.

[10] AHMED SHAH, S. Axially Symmetric Vibrations of Fluid - Filled Poroelastic Circular Cylindrical Shells. *Journal of Sound and Vibration*, **318** (2008), 389-405.

[11] CUI, L., A. H-D. CHENG, Y. ABOUSLEIMAN. Poroelastic Solutions of an Inclined Bore Hole. *Transations of ASME, Journal of Applied Mechanics*, **64** (1997), 32-38.

[12] ABOUSLEIMAN, Y., L. CUI. Poroelastic Solutions in Transversely Isotropic Media for Wellbore Cylinder. *International Journal of solids and structures*, **35** (1998), 4905-4929.

[13] KANJ, M., Y. ABOUSLEIMAN, R. GHANEM. Poromechanics of Anisotropic Hollow Cylinders. *Journal of Engineering Mechanics, ASCE*, **129** (2003), 1277-1287.

[14] MALLA REDDY, P., B. SANDHYA RANI. Study of Radial Vibrations in Cylindrical Bone in the Framework of Transversely Isotropic Poroelasticity. *Journal of Vibration and Control*, **22** (2016), 1276-1287.

[15] SANDHYARANI, B., P. MALLA REDDY, G. SHAMANTHA REDDY. Study of Torsional Vibrations in Composite Transversely Isotropic Poroelastic Cylinders. *Procedia Engineering*, **127** (2015), 462-468.

[16] BIOT, M. A. Propagation of Elastic Waves in a Cylindrical Bore Containing a Fluid. *Journal of Applied Physics*, **23** (1952) No. 8, 997-1005.

[17] ANIL, K.V., K. POONAM. Wave Propagation Along a Cylindrical Borehole in an Anisotropic Poroelastic Solid. *Geophysics Journal International*, **161** (2005), 295-302.