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FLUID MECHANICS

A LINEAR ANALYTICAL STUDY OF CORIOLIS FORCE ON SORET-DRIVEN FERROTHERMOHALINE CONVECTION IN A DARCY ANISOTROPIC POROUS MEDIUM WITH MFD VISCOSITY

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Abstract: The double diffusive effect of magnetic field dependent (MFD) viscosity on Soret driven thermoconvective instability in a ferrofluid, saturating a densely packed rotating anisotropic porous medium has been investigated theoretically. The horizontal fluid layer is assumed to be heated from below and salted from above and the Darcy model is used. The effect of salinity has been combined in the magnetization and density of the ferrofluid. An exact solution is obtained for the case of two free boundaries. The critical thermomagnetic Rayleigh number for onset of instability is also determined numerically for sufficient large values of buoyancy magnetization parameter M_1 and results are depicted graphically. The onset of Soret driven ferrothermohaline convection is analyzed both by stationary as well as oscillatory modes using a linear stability analysis and a normal mode technique. In all the cases, it is found that the rotation and MFD viscosity tend to stabilize the system whereas destabilization of the system occurs due to the non-buoyancy magnetization parameter M_3. Further, the ratio of mass transport to heat transport decides the stabilizing factor.

Key words: Coriolis force, Darcy model, Soret effect, MFD viscosity, ferromagnetic fluid, anisotropic porous medium.

1 Introduction

To apply Darcy law, the inertia term in Navier-Stoke's equation is neglected resulting in the fluid motion to be very slow. When the non-dimensional permeability coefficient is greater than 10^{-3}, Brinkman model is used. When it is less than 10^{-3}, Darcy model is used. Walker and Homsy [1] gave a note on the convective instabilities in Boussinesq fluids and porous media. They also discussed the domain of permeability distinguishing the application of Brinkman and Darcy models. When a fluid permeates through a porous material, the flow is analyzed by macroscopic law, called the

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Darcy law as referred by Lapwood [2]. Tyvand *et al*. [3] analyzed the onset of convection in an anisotropic porous medium with oblique principle axes. Venkatasubramanian *et al.* [4] examined the effect of rotation on the thermo convective instability of a horizontal layer of ferrofluids. Alex *et al.* [5] studied the thermal instability in an anisotropic rotating porous medium. McKibbin *et al*. [6] studied the effects of anisotropy and surface boundary conditions of thermal convection in a porous layer. Parthiban *et al*. [7] analyzed the effect of inclined temperature gradient on thermal instability in an anisotropic porous medium. Vaidyanathan *et al.* [8–10] discussed the effect of magnetic field dependent viscosity on ferroconvection in sparsely distributed porous medium with and without rotation.

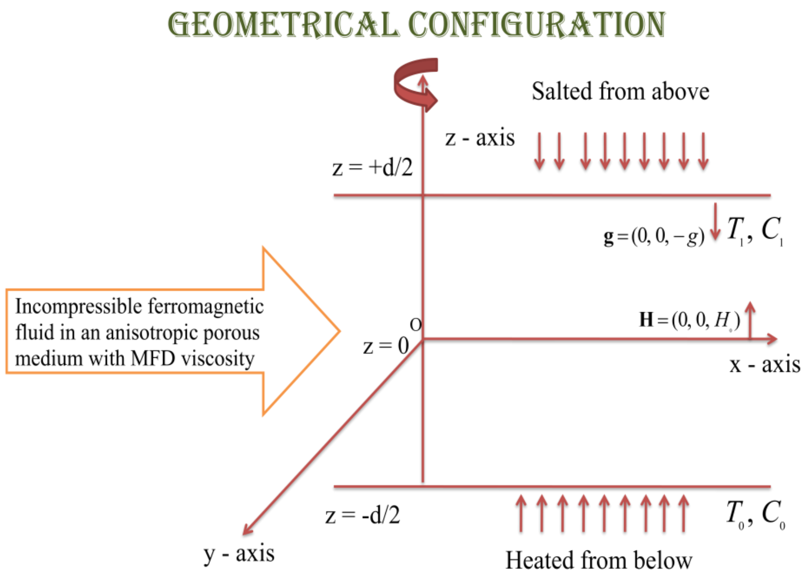
Sekar *et al.* [11] investigated the stability analysis of Soret effect on thermohaline convection in dusty ferrofluid saturating a Darcy porous medium. Nanjundappa *et al.* [12] investigated the influence of coriolis force on onset of thermomagnetic convection in ferrofluid saturating a porous layer in the presence of a uniform vertical magnetic field using both linear and weakly nonlinear analysis. Andrey Ryskin [13] investigated theoretically the influence of a magnetic field on the Soret effect dominated thermal convection in ferrofluids. Ramanathan *et al.* [14] studied the effect of magnetic field dependent viscosity and anisotropy porous medium on ferroconvection. Alam *et al.* [15] carried out Dufour and Soret effects on mixed convection flow past a vertical porous flat plat with variable suction. Nazmul Islam *et al.* [16] investigated the Dufour and Soret effects on steady MHD free convection and mass transfer fluid flow through a porous medium in a rotating system. Suslov [17] discussed the thermomagnetic convection in vertical layer of ferromagnetic fluid. Gaikwad *et al.* [18] studied an analytical study of linear and non-linear double diffusive convection in a fluid saturated anisotropic porous layer with Soret effect. Malashetty *et al.* [19] studied the thermal convection in a rotating viscoelastic fluid saturated porous layer. Sunil *et al.* [20] analyzed the nonlinear stability analysis of onnset of Darcy-Brinkman ferroconvection in a rotating porous layer using a thermal non-equilibrium model. Hemalatha *et al.* [21] carried out the effect of a magnetic field dependent viscosity on ferroconvection in an anisotropic porous medium in the presence of a horizontal thermal gradient.

Sekar *et al.* [22–24] investigated the ferrothermoconvective instability in Soret driven convection saturating a densely packed anisotropic porous medium with and without coriolis force. Sangita *et al.* [25] discussed the natural convection in a spherical porous annulus using Darcy model. Nanjundappa *et al.* [26, 27] analyzed the effect of temperature dependent viscosity on the onset of Benard-Marangoni convection with and without porous layer. Nanjundappa *et al.* [28] investigated the effects of cubic temperature profile and MFD viscosity on onset of Benard-Marangoniferroconvection with a convective surface boundary condition. Gaikwad *et* *al.* [29] studied linear stability analysis of double diffusive convection in a horizontal sparsely packed rotating anisotropic porous layer in presence of Soret effect. Sekar *et al.* [30] investigated the effect of Sparse Distribution Pores in Thermohaline Convection in a Micropolar Ferromagnetic Fluid. Kumar *et al.* [31] carried out the linear stability analysis for ferromagnetic fluids in the presence of magnetic field, compressibility, internal heat source and rotation through a porous medium. Rahman *et al.* [32] examined the thermomagnetic convection in a layer of ferrofluid placed in a uniform oblique external magnetic field. Malashetty *et al.* [33] investigated the effect of rotation on the onset of double diffusive convection in a sparsely packed anisotropic porous layer. Saravanan *et al.* [34] discussed the floquet instability of a gravity modulated Rayleigh-Benard problem in an anisotropic porous medium. Shivakumara *et al.* [35] examined the boundary and thermal non-equilibrium effects on convective instability in an anisotropic porous layer. More recently, Sekar *et al.* [36] carried out the linear stability effect of densely distributed porous medium and coriolis force on soret driven ferrothermohaline convection. Most recently, Sekar *et al.* [37] investigated the stability analysis of ferrothermohaline convection in a Darcy porous medium with Soret and MFD viscosity effects.

In this paper, the effect of magnetic field dependent viscosity on Soret-driven ferrothermohaline convection in the presence of densely packed rotating anisotropic porous medium is studied. The overriding Soret effect on the salinity equation is also studied. Linear stability analysis is used. The conditions for the onset of stationary and oscillatory instabilities are obtained.

2 Mathematical Formulation of the Problem

An infinitely spread layer of Boussinesq ferromagnetic fluid of thickness, d, rotating with uniform constant angular velocity $\Omega = (0,\, 0,\, \Omega)$ and anisotropy along the vertical direction, is taken as z-axis. The entire system is heated from below and salted from above. The temperature and salinity at the bottom and top surfaces $z = \pm d/2$ are $T_0 \pm \Delta T/2$ and $S_0 \pm \Delta S/2$, respectively. Both the boundaries are taken to be free and perfect conductors of heat and solute. Considering the Soret effect on the temperature gradient, the mathematical equations governing the above investigation are as follows: The fluid is assumed to be incompressible fluid having a variable viscosity, given by $\mu = \mu_1(1 + \boldsymbol{\delta} \cdot \boldsymbol{B})$, where μ_1 is taken as viscosity of the fluids when the applied magnetic field is absent. The variation in the coefficient of the magnetic field dependent viscosity δ has been taken to be isotropic, that is, $\delta = \delta_1 = \delta_2 = \delta_3$. Hence the component wise μ can be written as $\mu_x = \mu_1(1+\delta B_1)$, $\mu_y = \mu_1(1+\delta B_2)$ and $\mu_z = \mu_1(1 + \delta B_3)$.



The continuity equation for an incompressible fluid is

(1) $$\nabla \cdot \boldsymbol{q} = 0.$$

The momentum equation is

(2) $$\rho_0 \frac{D\boldsymbol{q}}{Dt} = -\nabla p + \rho \boldsymbol{g} + \nabla \cdot (\boldsymbol{HB}) + 2\rho_0 (\boldsymbol{q} \times \boldsymbol{\Omega}) + \frac{\rho_0}{2} \nabla (|\boldsymbol{\Omega} \times \boldsymbol{r}|^2) - \frac{\mu_1 (1 + \boldsymbol{\delta} \cdot \boldsymbol{B})}{k} \boldsymbol{q}.$$

The temperature equation for an incompressible ferrofluid is

(3) $$\left[\rho_0 C_{V,H} - \mu_0 \boldsymbol{H} \cdot \left(\frac{\partial \boldsymbol{M}}{\partial T}\right)_{V,H}\right] \frac{dT}{dt} + \mu_0 T \left(\frac{\partial \boldsymbol{M}}{\partial T}\right)_{V,H} \cdot \frac{d\boldsymbol{H}}{dt} = K_1 \nabla^2 T + \phi \, .$$

The mass flux equation is

(4) $$\frac{DS}{Dt} = K_s \nabla^2 S + S_T \nabla^2 T \, ,$$

where ρ_0, $\boldsymbol{q} = (u, v, w)$, $\boldsymbol{g} = (0, 0, -g)$, t, p, μ_0, μ, k, $\boldsymbol{H}$, $\boldsymbol{B}$, $C_{V,H}$, T, $\boldsymbol{M}$, K_1, S, K_S, S_T and ϕ are the density, velocity, acceleration due to gravity, time, pressure, magnetic permeability, viscosity (variable), permeability of the porous medium, magnetic field, magnetic induction, heat capacity at constant volume and magnetic field, temperature, magnetization, thermal conductivity, salinity, mass diffusivity, Soret coefficient and viscous dissipation factor containing second-order terms in velocity, respectively.

Maxwell's equations are

$$\nabla \cdot \boldsymbol{B} = 0\,, \qquad \nabla \times \boldsymbol{H} = 0\,. \tag{5}$$

Further $\boldsymbol{B}$, $\boldsymbol{M}$ and $\boldsymbol{H}$ are related by

$$\boldsymbol{B} = \mu_0(\boldsymbol{M} + \boldsymbol{H}). \tag{6}$$

Using Maxwell's equation for non-conducting fluids one can assume that the magnetization is aligned with the magnetic field and depends on the magnitude of the magnetic field, temperature and salinity, so that

$$\boldsymbol{M} = \frac{\boldsymbol{H}}{H} M(H, T, S). \tag{7}$$

The magnetic equation of state is linearized about the magnetic field H_0, the average temperature T_0 and the average salinity S_0 to become

$$M = M_0 + \chi(H - H_0) - K(T - T_0) + K_2(S - S_0), \tag{8}$$

where $\chi = (\partial M/\partial H)_{H_0,T_0}$ is the susceptibility, $K = -(\partial M/\partial T)_{H_0,T_0}$ is the pyromagnetic coefficient and $K_2 = (\partial M/\partial S)_{H_0,S_0}$ is the salinity magnetic coefficient.

The density equation of state for a Boussinesq two-component fluid is

$$\rho = \rho_0 \left[1 - \alpha_t(T - T_0) + \alpha_s(S - S_0)\right], \tag{9}$$

where $\alpha_t = -(1/\rho)(\partial\rho/\partial T)$ is the thermal expansion coefficient and $\alpha_s = (1/\rho)(\partial\rho/\partial S)$ is the solute analog of α_t.

The basic state is assumed to be the quiescent state and taking the components of the magnetization and magnetic field in the basic state to be $[0, 0, M_0(z)]$ and $[0, 0, H_0(z)]$ the basic state quantities obtained are

$$\begin{aligned}
&\boldsymbol{q} = \boldsymbol{q}_b = 0, \quad p = p_b(z),\\
&\frac{\partial T}{\partial z} = -\beta_t \Rightarrow T_b = T_0 - \beta_t z, \quad \frac{\partial S}{\partial z} = \beta_S \Rightarrow S_b = S_0 + \beta_s z,\\
&\boldsymbol{H}_b(z) = \Big[H_0 + \frac{K(T_b - T_0)}{1+\chi} - \frac{K_2(S_b - S_0)}{1+\chi}\Big]\hat{\boldsymbol{k}}\,,\\
&\boldsymbol{M}_b(z) = \Big[M_0 - \frac{K(T_b - T_0)}{1+\chi} + \frac{K_2(S_b - S_0)}{1+\chi}\Big]\hat{\boldsymbol{k}}\,.
\end{aligned} \tag{10}$$

where β_t and β_S are non-negative constants and $\hat{\boldsymbol{k}} = (0, 0, 1)$ is the unit vector along vertical direction.

3 Linearization of the Problem

The basic state is disturbed by a small thermal perturbation, consider a perturbed state such that $\boldsymbol{q} = \boldsymbol{q}'$, $p = p_b(z) + p'$, $\mu = \mu_b(z) + \mu'$, $T = T_b(z) + T'$, $\boldsymbol{H} = \boldsymbol{H}_b(z) + \boldsymbol{H}'$, $\boldsymbol{M} = \boldsymbol{M}_b(z) + \boldsymbol{M}'$, where $\boldsymbol{q}'$, p', μ', T', $\boldsymbol{H}'$, and $\boldsymbol{M}'$ are perturbed variables and are assumed too small. The perturbed state temperature and solute are $T = T_0 - \beta_t z + T'$ and $S = S_0 + \beta_s z + S'$. Let the components of the perturbed magnetization and the magnetic field be $(M_1', M_2', M_0(z) + M_3')$ and $(H_1', H_2', H_0(z) + H_3')$, respectively.

$$H_i' + M_i' = \Big(1 + \frac{M_0}{H_0}\Big) H_i' \quad (i = 1, 2), \tag{11}$$

$$H_3' + M_3' = (1 + \chi) H_3' - KT' + K_2 S' + S_T K T' \,. \tag{12}$$

Let (B_1, B_2, B_3) denote the components of $\boldsymbol{B}$, using Eq. (5), one gets the result $B_i = \mu_0 (M_i' + H_i')$ and Eqs. (11) and (12) become

$$B_i = \mu_0 \Big(1 + \frac{M_0}{H_0}\Big) H_i' \qquad (i = 1, 2), \tag{13}$$

$$B_3 = \mu_0 \left[(1 + \chi) H_3' - KT' + K_2 S' + S_{\mathrm{T}} K T' + M_0 + H_0\right] . \tag{14}$$

The procedure for finding the perturbation equation is given in Appendix A.

4 Normal Mode Analysis

We now proceed to a normal mode analysis of the above stability problem. Let us take

$$\begin{aligned} & f(x, y, z, t) = f(z, t) e^{i(k_x x + k_y y)} \,, \\ & \phi = \phi(z, t) e^{i(k_x x + k_y y)} \,, \qquad w = w(z, t) e^{i(k_x x + k_y y)} \\ & T' = \theta(z, t) e^{i(k_x x + k_y y)} \,, \qquad S' = S(z, t) e^{i(k_x x + k_y y)} \end{aligned} \tag{15}$$

with the wave number $k_0^2 = k_x^2 + k_y^2$.

The non-dimensional numbers can be written using

$$t^* = \frac{\nu t}{d^2} \,, \quad w^* = \frac{wd}{\nu} \,, \quad T^* = \Big(\frac{K_1 a R^{1/2}}{\rho_0 C_{V,H} \beta_t \nu d}\Big) \theta \,, \quad \phi^* = \Big(\frac{(1+\chi) K_1 a R^{1/2}}{K \rho_0 C_{V,H} \beta_t \nu d^2}\Big) \phi \,,$$

$$z^* = \frac{z}{d} \,, \quad a = k_0 d \,, \quad D = \frac{\partial}{\partial z^*} \,, \quad S^* = \Big(\frac{K_s a R_s^{1/2}}{\rho_0 C_{V,H} \beta_s \nu d}\Big) S \,,$$

$$\gamma = \frac{\mu}{\rho_0} \,, \quad k_1^* = \frac{k_1}{d^2} \,, \quad k_2^* = \frac{k_2}{d^2} \,, \quad \delta^* = \mu_0 \delta H_0 (1 + \chi) \,.$$

The non-dimensional form of constitutive equations can be written as

$$\frac{\partial}{\partial t^*}(D^2 - a^2)w^* = aR^{1/2}[M_1 D\phi^* - (1 + M_1(1 - S_T))T^*] \\ + aR^{1/2}M_1M_5D\phi^* - aR^{1/2}M_1M_5(1 - S_T)T^* \\ + aR_S^{1/2}\Big[1+M_4+\frac{M_4}{M_5}\Big]S^* - \Big(\frac{D^2}{k_1^*} - \frac{a^2}{k_2^*}\Big)w^* \\ + \frac{a^2}{k_2*}M_3\delta^* w^* - (T_a)^{1/2}D\xi^*\,, \tag{16}$$

$$\Big(\frac{\partial}{\partial t^*} + \frac{1}{k^*}\Big)\xi^* = (T_a)^{1/2}Dw^*\,, \tag{17}$$

$$P_r\Big[\frac{\partial T^*}{\partial t^*} - M_2\frac{\partial}{\partial t^*}(D\phi^*)\Big] = (D^2 - a^2)T^* + aR^{1/2}(1 - M_2 - M_2M_5)w^*, \tag{18}$$

$$P_r\frac{\partial S^*}{\partial t^*} = \tau(D^2 - a^2)S^* - aR_S^{1/2}M_6w^* + S_T\Big(\frac{M_5}{M_6}\Big)\Big(\frac{R_S}{R}\Big)^{1/2}(D^2 - a^2)T^*, \tag{19}$$

$$D^2\phi^* - M_3a^2\phi^* - (1 - S_T)DT^* + \frac{M_5}{M_6}\Big(\frac{R}{R_S}\Big)^{1/2}DS^* = 0, \tag{20}$$

where the magnetic numbers are given in Appendix B.

5 Analysis of Solution at Free Boundaries

The free-free boundary conditions on velocity, temperature and salinity are

$$w^* = D^2w^* = T^* = D\phi^* = S^* = \xi^* = D\xi^* = 0 \;\text{ at }\; z^* = \pm 1/2. \tag{21}$$

The exact solutions satisfying above Eq.(21) are

$$w^* = Ae^{\sigma t^*}\cos\pi z^*, \quad T^* = Be^{\sigma t^*}\cos\pi z^*, \quad S^* = Ce^{\sigma t^*}\cos\pi z^*, \\ D\phi^* = Ee^{\sigma t^*}\cos\pi z^*, \qquad \phi^* = \frac{E}{\pi}e^{\sigma t^*}\sin\pi z^*, \tag{22}$$

where A, B, C and E are constants. These functions substituted in the set of Eqs. (16) – (20) give the following four linear homogeneous algebraic equations in the constant A, B, C and E are obtained upon $k_2^* = \varepsilon k_1^*$ and removing the asterisks for our convenience, where ε is non-dimensional parameter governing anisotropy leads to:

$$\Big[\sigma(\pi^2 + a^2) + \Big(\frac{\pi^2\varepsilon + a^2}{k_1\varepsilon}\Big) + \frac{T_a\pi^2}{(\sigma + 1/k_1)} + \frac{1}{k_1\varepsilon}a^2M_3\delta\Big]A- \tag{23}$$

$$- aR^{1/2}\left[1 + M_1(1 - S_T) + M_1 M_5(1 - S_T)\right]B + aR_S^{1/2}(1 + M_4 + M_4 M_5^{-1})C + aR^{1/2}M_1(1 + M_5)E = 0,$$

(24) $aR^{1/2}(1 - M_2 - M_2M_5)A - (\pi^2 + a^2 + P_r\sigma)B + P_r\sigma M_2 E = 0,$

(25) $aR_S^{1/2}M_6 A + S_T\left(\frac{M_5}{M_6}\right)\left(\frac{R_S}{R}\right)^{1/2}(\pi^2 + a^2)B + [\tau(\pi^2 + a^2) + \sigma P_r]C = 0,$

(26) $-R_S^{1/2}\pi^2(1 - S_T)B + R^{1/2}\pi^2 M_5 M_6^{-1}C + R_S^{1/2}(\pi^2 + a^2 M_3)E = 0\,.$

For the existence of non-trivial Eigen functions, the determinant of the coefficients of A, B, C and E in Eqs. (23) – (26) must vanish. Following the techniques and analysis of Sekar *et al.* [22–24], Eqs. (23) – (26) lead to

(27) $$U\sigma^4 + V\sigma^3 + W\sigma^2 + X\sigma + Y = 0\,.$$

The coefficients U, V, W, X, and Y are given in Appendix C.

6 Stability Behaviour of Stationary Convection and Oscillatory Convection and Mathematical Approach Used for Investigation

To obtain stationary instability, the time-independent term $Y = 0$. Equation (27) helps one to obtain Eigenvalue R_{SC} for which a solution exists:

$$R_{SC} = \frac{N_r}{D_r}\,,$$

where

$$N_r = (\pi^2 + a^2)\left(T_a\pi^2 k_1 + \frac{\varepsilon\pi^2 + a^2}{\varepsilon k_1} + \frac{1}{\varepsilon k_1}a^2 M_3\delta\right) - a^2 R_s\tau^{-1}\left(1 + \mathrm{M}_4 + \frac{M_4}{M_5}\right)\left[S_{\mathrm{T}}\left(\frac{\mathrm{M}_5}{\mathrm{M}_6}\right) + \mathrm{M}_6\right]$$

and

$$D_r = a^2\left[1 + (1 - S_{\mathrm{T}})\,\mathrm{M}_1\,(1 + \mathrm{M}_5)\right] - \pi^2\left[\frac{a^2\mathrm{M}_1\,(1 + \mathrm{M}_5)}{\pi^2 + a^2\mathrm{M}_3}\right]\left[S_{\mathrm{T}}\left(\frac{\mathrm{M}_5}{\mathrm{M}_6}\right)^2\tau^{-1} + (1 - S_{\mathrm{T}}) + \mathrm{M}_5\tau^{-1}\right].$$

For M_1 very large, one gets the results for the magnetic mechanism, and the critical thermo magnetic Rayleigh number for stationary mode is obtained using

$$N_{SC} = M_1 R_{SC} = \frac{N_r}{D_r}\,,$$

where

$$N_r = \left(\pi^2 + a^2\right)\left(T_a\pi^2 k_1 + \frac{\varepsilon\pi^2 + a^2}{\varepsilon k_1} + \frac{1}{\varepsilon k_1}a^2 M_3\delta\right) - a^2 R_s \tau^{-1}\left(1 + \mathrm{M}_4 + \frac{M_4}{M_5}\right)\left[S_{\mathrm{T}}\left(\frac{\mathrm{M}_5}{\mathrm{M}_6}\right) + \mathrm{M}_6\right]$$

and

$$D_r = a^2\left[\left(1 - S_{\mathrm{T}}\right)\left(1 + \mathrm{M}_5\right)\right] - \pi^2\left[\frac{a^2\left(1 + \mathrm{M}_5\right)}{\pi^2 + a^2\mathrm{M}_3}\right]\left[S_{\mathrm{T}}\left(\frac{\mathrm{M}_5}{\mathrm{M}_6}\right)^2\tau^{-1} + \left(1 - S_{\mathrm{T}}\right) + \mathrm{M}_5\tau^{-1}\right].$$

The conditions for the onset of oscillatory stabilities are obtained as follows. Taking $\sigma = i\sigma$ and $\sigma^2 > 0$, in Eq. (27) and following the analysis and techniques of Sekar *et al.* [22–24], the critical Rayleigh number for oscillatory mode has been calculated using

$$R_{OC} = \frac{C_2 A_2 + B_2 D_2}{A_2^2 + B_2^2}.$$

The Soret-driven thermoconvective instability of ferromagnetic fluid layer heated from below and salted from above rotating a densely packed anisotropic porous medium with magnetic field dependent (MFD) viscosity has been analyzed using Darcy model. Perturbation method is applied and Normal mode analysis is adopted. In the perturbation method, due to the application of magnetic field, the system is perturbed from the basic state (quiescent state). Accordingly, the governing and other equations are modified. Linear stability analysis is considered. Normal mode analysis is taken, Non-dimensional analysis is carried out and the exact solutions satisfying the appropriate boundary conditions are taken yielding algebraic equations. For getting non-trivial solution for the system of linear homogeneous equations, the coefficients of the dynamic variables are equated to zero and on simplification, the expression for R_{SC} is obtained. Varying the values of the parameters in the allowable range and getting the corresponding R_{SC} values, we get the stability pattern.

7 Results and Discussions

Before discussing the significant results of the convective system, we turn our attention to the possible range of values of various parameters arising in the study. The Prandtl number P_r is assumed to be 0.01. The Soret parameter S_T is assumed to take values from -0.002 to 0.002, the salinity Rayleigh number R_s is varied from -500 to 500. The values of ratio of the mass transport to heat transport τ is assumed to be

0.03, 0.05, 0.07, 0.09 and 0.11. The Taylor number Ta is assumed from 10 to 10^5. The coefficient of MFD viscosity δ is assumed from 0.01 to 0.09. The magnetization parameter M_1 is assumed to be 1000; for a very large value of M_1, the effect of magnetic mechanism is very large, when compared to buoyancy effect. For such fluids, M_2 is assumed to have negligible value and hence taken to be zero. M_3 is varied from 1 to 25 because M_3 cannot take a value less than one. M_6 is taken to be 0.1. M_4 is the effect on magnetization due to salinity. This is allowed to vary from 0.1 to 0.5 taking values less than the magnetization parameter M_3. M_5 represents the ratio of the salinity effect on magnetic field and pyromagnetic coefficient. This is varied between 0.1 and 0.5. The permeability of porous medium k is assumed to take the values 0.001, 0.003, 0.005, 0.007, 0.009 and 0.0001 (Darcy numbers).

The critical thermal Rayleigh number is calculated for both stationary and oscillatory modes. When the salinity Rayleigh number $R_S = 0$, the critical Rayleigh number obtained by Finlayson [38] for single component ferrofluid. When $M_1 = 1000$, the classical Rayleigh problem for buoyancy-induced convection is and obtained Chandrasekhar [39]. When $k \to \infty$, the critical Rayleigh number exactly same as Sekar*et al.* [40]. When $\delta = 0$, $\varepsilon = 0$, $T_a = 0$ and $k_1 \to \infty$ this tends to critical Rayleigh number obtained by Vaidyanathan*et al.* [41]. When $\delta = 0$, $\varepsilon = 1$ and

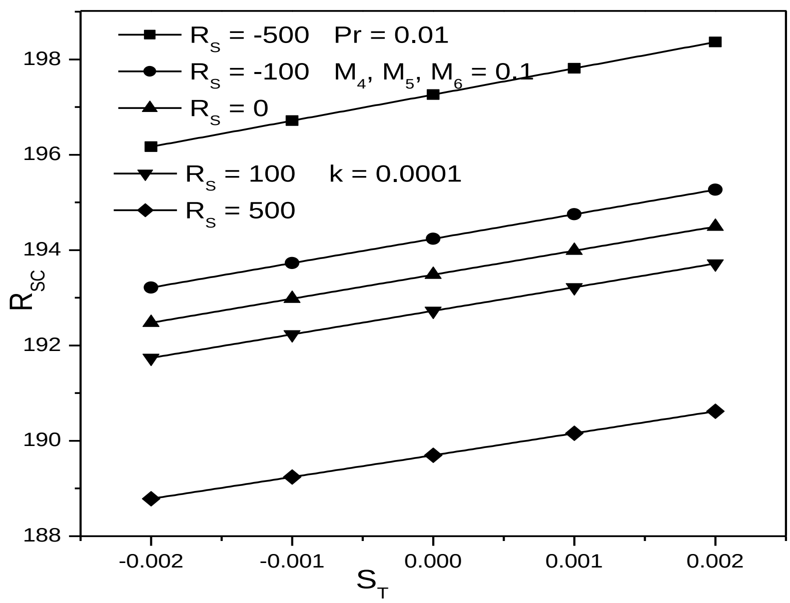


Fig. 1. Critical thermal Rayleigh number R_{SC} versus Soret coefficient S_T for different values of Salinity Rayleigh number R_S with $\tau = 0.03$, $\varepsilon = 10$, $S_T = -0.002$ and $M_3 = 5$.

$T_a = 0$, the thermal Rayleigh number is identical to Sekar*et al.* [42]. When $\delta = 0$, one gets the critical Rayleigh number calculated for Sekar *et al.* [43]. When all the magnetic parameters M_1 to M_6 vanish, this reduces to double diffusive convection (Baines and Gill, [44]).

Figure 1 shows variation of R_{SC} with S_T for different values of R_S, keeping the values of M_3 fixed as 5, k as 0.0001 and τ as 0.03. It has been observed that as S_T increases, the system gets stabilized, whereas when salinity Rayleigh number R_S increases, the system gets destabilized. When increasing effect of salt on the convective system, the system gets high energy and it is dominated by Soret effect on the onset of instability.

Figure 2 depicts variation of R_{SC} with R_S for differentM_3, keeping the values of S_T to be fixed as -0.002, k as 0.0001 and τ as 0.03. It has been observed that the system gets destabilized as M_3 increases. Due to an increasing of non-buoyancy magnetization parameter M_3, convection is not much pronounced.

From Fig. 3 it is seen that the vertical anisotropy of the permeability of the porous medium destabilizes the system. This is because of the decrease in R_{SC} when ε is increased. As far as the magnetization M_3 is concerned, the increase in M_3 (5, 10, 15, 20, 25), decreases R_{SC} for a fixed ε. This is because the high magnetization tends to release large energy to the system causing instability to set earlier. The same effect is found when ε is increased from 10 to 70. This indicates the destabilizing nature of the system.

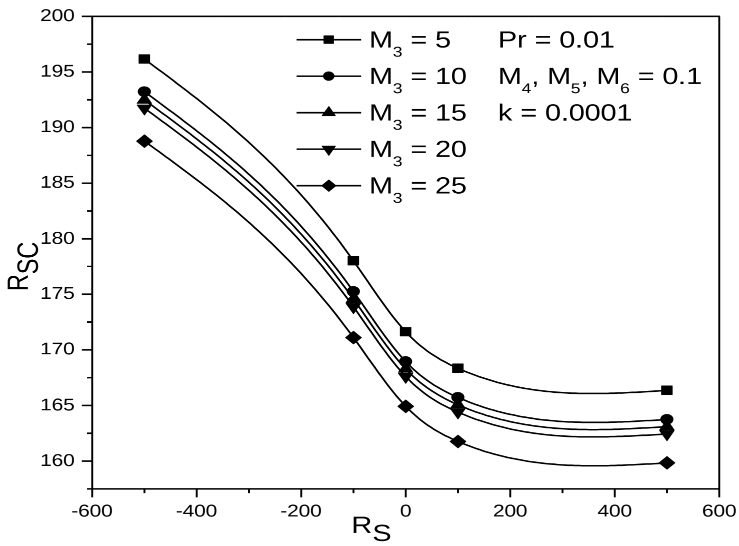


Fig. 2. Critical thermal Rayleigh number R_{SC} versus Salinity Rayleigh number R_s for different values of M_3 with $\tau = 0.03$, $\varepsilon = 10$, $R_S = -500$ and $S_T = -0.002$.

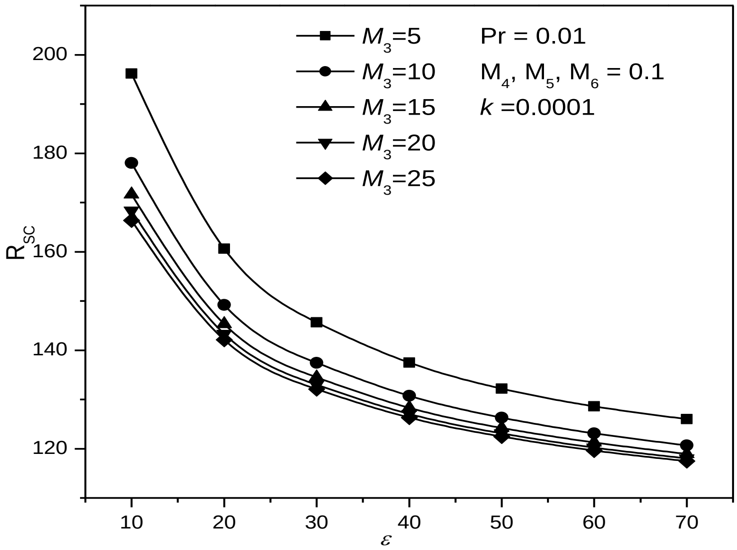


Fig. 3. Critical thermal Rayleigh number R_{SC} versus anisotropic parameter ε for different magnetization parameter M_3 with $R_S = -500$, $S_T = -0.002$, $\delta = 0.01$ and $\tau = 0.03$.

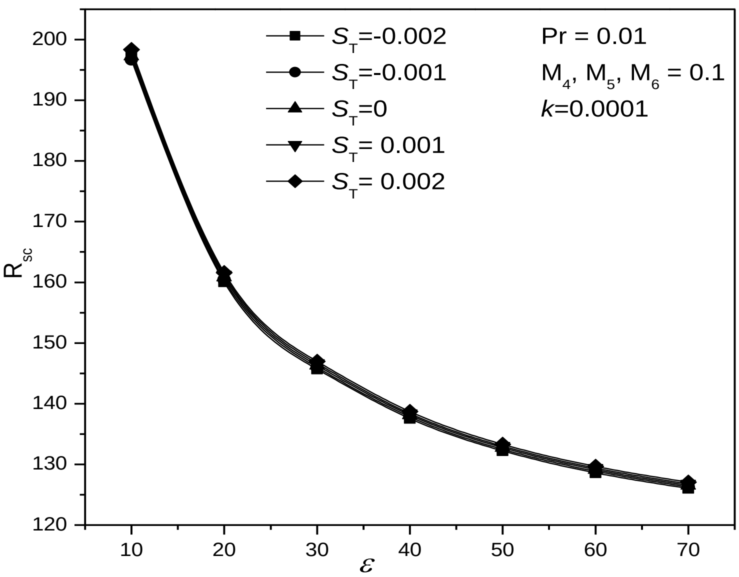


Fig. 4. Critical thermal Rayleigh number R_{SC} versus anisotropic parameter ε for different Soret coefficient S_T with $R_S = -500$, $\delta = 0.01$, $M_3 = 5$ and $\tau = 0.03$.

Figure 4 shows the variation of R_{SC} with ε for different Soret coefficient S_T (-0.002, -0.001, 0, 0.001, 0.002), while keeping the values of M_3 fixed as 5, R_S fixed as -500, k fixed as 0.0001 and τ as 0.03. It has been noticed that as ε increases, the system gets destabilized. The increase in anisotropic favours the onset of instability. But negligible change is observed as S_T increases from -0.002 to 0.002. Anyhow the trend is slightly upward thereby stabilizing the system with respect to S_T. This is because, increase in S_T provides additional temperature gradient due to cross diffusion of salinity on temperature.

Figure 5 represent the variation of R_{SC} versus anisotropic ratio ε for various values (-500, -100, 0 100, 500), of salinity Rayleigh number R_S. It has been observed that as ε increases from 10 to 70, there is decrease in R_{SC}. It is clear that the system gets destabilized. As R_S increases from -500 to 500, the critical thermal Rayleigh number R_{SC} values tend to decrease leading to destabilization. This is because, adding salt from above makes the system heavier at the bottom, thereby delays the onset of convection.

Figure 6 give the variation ofcritical thermal Rayleigh number R_{SC} with anisotropic ratio ε for different values (0.03, 0.05, 0.07, 0.09, 0.11), of the ratio of the mass transport to heat transport τ, where the value of salinity Rayleigh number R_s is fixed as -500, the magnetization parameter M_3 as 5 and Soret parameter S_T is fixed as -0.002. It has been observed that increase in ε decreases R_{SC} and thereby destabi-

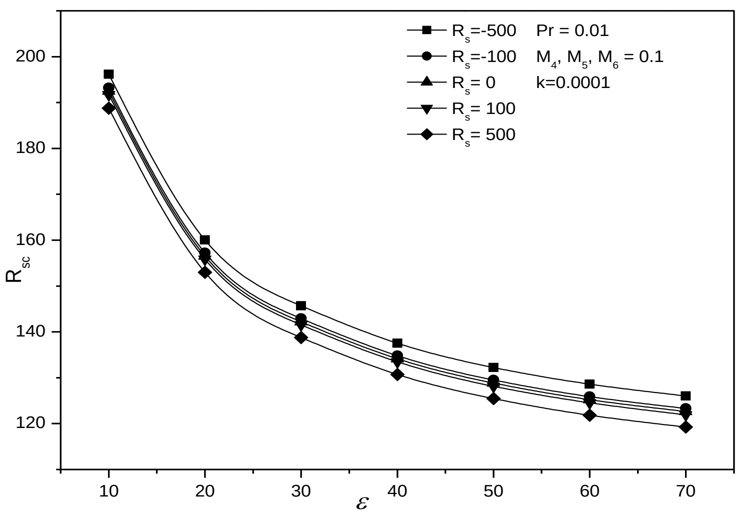


Fig. 5. Critical thermal Rayleigh number R_{SC} versus anisotropic parameter ε for different values of Salinity Rayleigh number R_s with $S_T = -0.002$, $\delta = 0.01$, $M_3 = 5$ and $\tau = 0.03$.

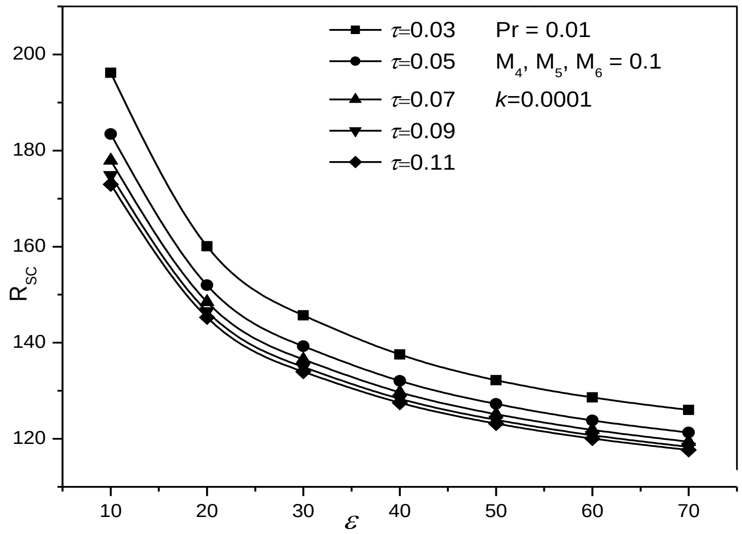


Fig. 6. Critical thermal Rayleigh number R_{SC} versus anisotropic parameter ε for various τ, $R_S = -500$, $\delta = 0.01$, $M_3 = 5$ and $S_T = -0.002$.

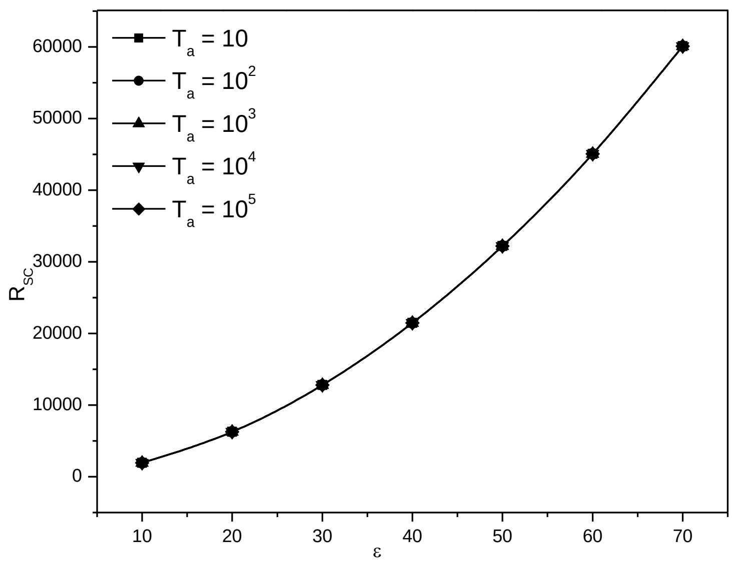


Fig. 7. Critical thermal Rayleigh number R_{SC} versus anisotropic parameter ε for various T_a $\tau = 0.03$, $S_T = -0.002$, $k = 0.001$, $R_S = -500$ and $M_3 = 5$.

lizes the system. This is because the increase in anisotropic ratio results in instability irrespective of the increasing values of mass transport to heat transport. Therefore the system has a destabilizing behavior with respect to the mass transport to heat transport τ.

Figure 7 shows variation of R_{SC} versus ε for different T_a (10, 10^2, 10^3, 10^4, 10^5). When T_a, increases there is no notable variation. But increase in Taylor number T_a increases the critical thermal Rayleigh number R_{SC}. Therefore the system gets stabilized. For different T_a the curves do not show major difference.

Figure 8 shows that the vertical anisotropy of the permeability of the porous medium destabilizes the system. This is because of the decrease in R_{SC} when ε is increased. As far as the MFD viscosity δ is concerned, the increase in δ (0.001, 0.003, 0.005, 0.007, 0.009), increases R_{SC} for a fixed ε. The same effect is found when ε is increased from 10 to 70. This indicates the stabilizing nature of the system.

Figure 9 shows the variation of R_{SC} versus δ for different values of ε. From Fig. 9, one can find that as the coefficient of magnetic field dependent viscosity is increased from 0.01 to 0.09, the critical thermal Rayleigh number increases. This means that the system is stabilized through viscosity variation with respect to magnetic field. This leads to the conclusion that the magnetic field dependent viscosity delays the onset of convection for ferrofluid in a densely distributed porous medium.

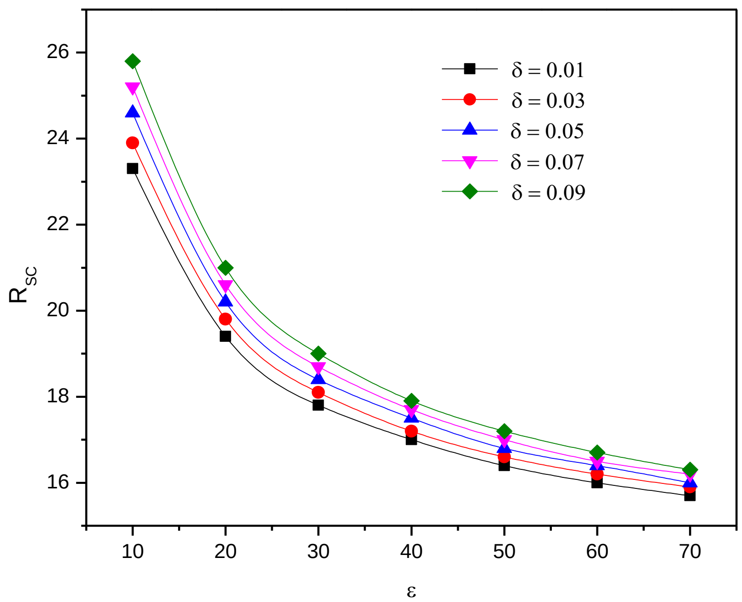


Fig. 8. Critical thermal Rayleigh number R_{SC} versus anisotropic parameter ε for various δ, $\tau = 0.03$, $S_T = -0.002$, $k = 0.001$, $R_S = -500$ and $M_3 = 5$.

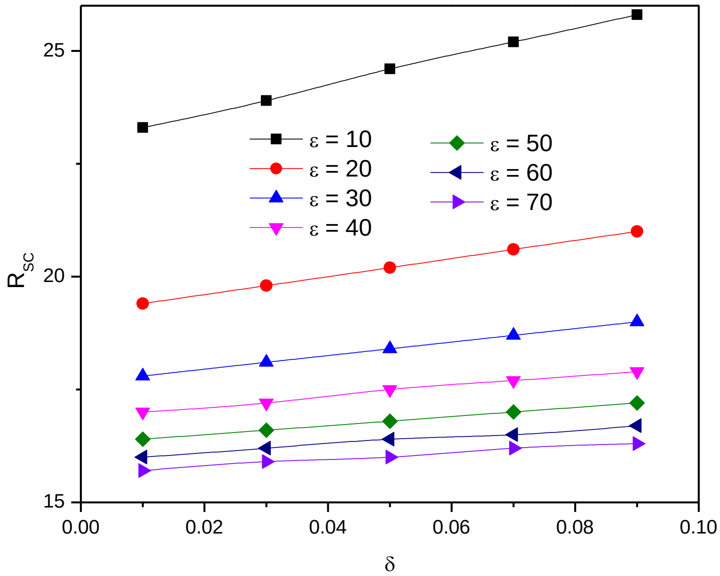


Fig. 9. Critical thermal Rayleigh number R_{SC} versus coefficient of MFD viscosity δ for various ε, $R_S = -500$, $S_T = -0.002$, $k = 0.001$, $\tau = 0.03$ and $M_3 = 5$.

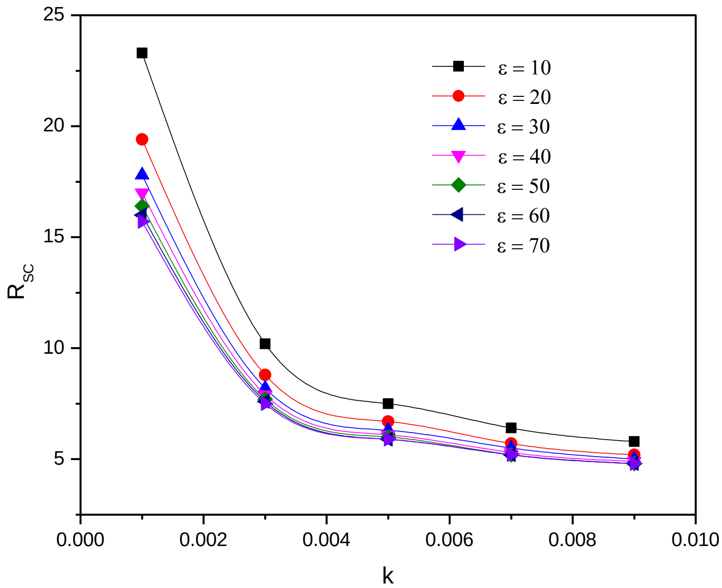


Fig. 10. Critical thermal Rayleigh number R_{SC} versus permeability of porous medium k for various ε, $R_S = -500$, $S_T = -0.002$, $\delta = 0.01$, $\tau = 0.03$ and $M_3 = 5$.

Figure 10 indicates the variation of critical Rayleigh number R_{SC} with respect to the permeability of the porous medium k for different ε. It is clear that the system destabilizes as the permeability of the porous medium k increases. This is indicated by a decrease in R_{SC} values. The reason is that as the pore size increases, it becomes easier for the flow to destabilize the system. It is observed from the figure that the effect of anisotropic parameter ε is to destabilize the system.

Figure 11 represents the variation of R_{SC} versus R_S for different values of ε. When the salinity Rayleigh number R_S increases from -500 to 500, the critical thermal Rayleigh number R_{SC} decreases. Therefore the system gets a destabilizing behaviour. It is observed from figure that anisotropic parameter ε is found to destabilize the system.

Figure 12 indicates the variation of the critical Rayleigh number R_{SC} with respect to the Soret parameter S_T for various ε. It is found that the increase in Soret effect stabilizes the system, thereby delaying the onset of convection. The figure exhibits a stabilizing trend. This is due to the fact that the modulation of the salinity gradient by temperature gradient promotes stabilization. Positive values ofS_T stabilize the system more. The destabilizing trend of ε is seen from figure, as would mean adding salt from top.

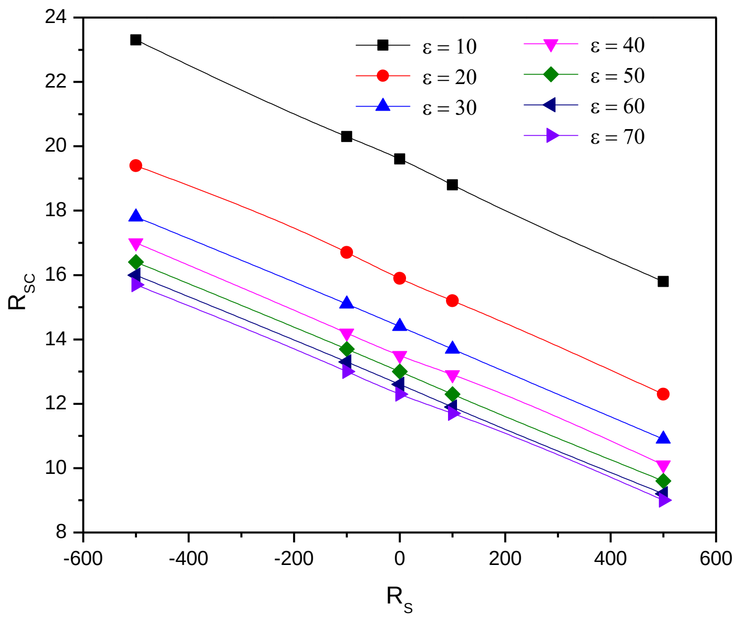


Fig. 11. Critical thermal Rayleigh number R_{SC} versus salinity Rayleigh number R_s for various ε, $\delta = 0.01$, $S_T = -0.002$, $k = 0.001$, $\tau = 0.03$ and $M_3 = 5$.

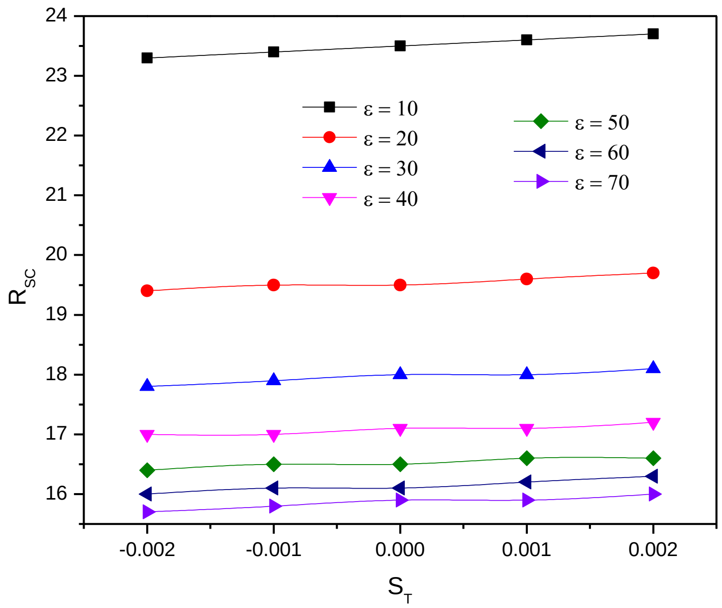


Fig. 12. Critical thermal Rayleigh number R_{SC} versus Soret parameter S_T for various ε, $\delta = 0.01$, $R_S = -500$, $k = 0.001$, $\tau = 0.03$ and $M_3 = 5$.

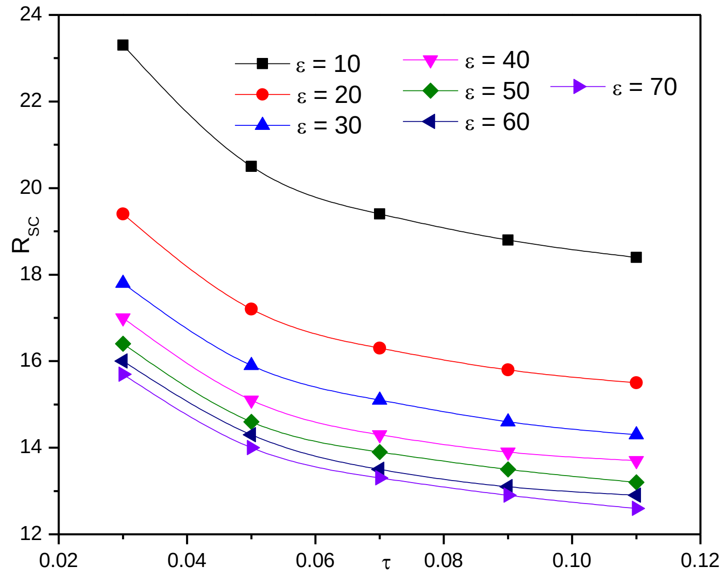


Fig. 13. Critical thermal Rayleigh number R_{SC} versus ratio of the mass transport to heat transport τ for various ε, $\delta = 0.01$, $R_S = -500$, $k = 0.001$, $S_T = -0.002$, and $M_3 = 5$.

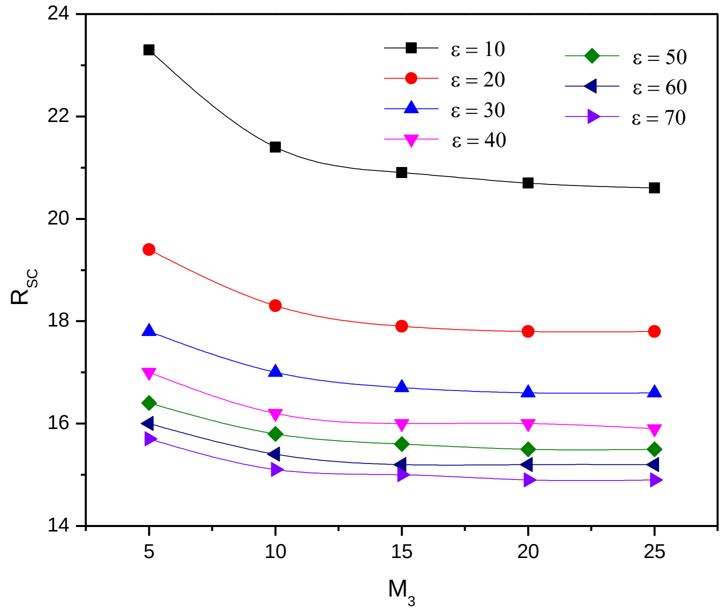


Fig. 14. Critical thermal Rayleigh number R_{SC} versus magnetization M_3 for various ε, $\delta = 0.01$, $R_S = -500$, $k = 0.001$, $S_T = -0.002$, and $\tau = 0.03$.

It is very clear from Fig. 12 that the system gets destabilizing effect with respect to Soret coefficient S_T for different anisotropy parameter ε. Thus, the system is dominated by Soret coefficient.

Figure 13 shows the variation of critical Rayleigh number R_{SC} versus the mass transport to heat transport $\tau$for different ε. It is seen from the figure that the system destabilizes as the mass transport to heat transport τ increases. This is shown by a fall in R_{SC} values. It is observed from the figure that the anisotropic parameter ε is found to destabilize the system.

Figure 14 gives the variation of the critical Rayleigh number R_{SC} versus the non-buoyancy magnetization parameter M_3 for different anisotropic parameter ε. The figure indicates destabilizing trend of anisotropic parameter ε. Magnetization releases extra energy which adds up to thermal energy to promote convection.

Figure 15 shows the plot of critical thermal Rayleigh number R_{SC} versus Taylor number T_a for various values of anisotropic ratio ε. As Taylor number T_a increases, the critical thermal Rayleigh number R_{SC} is almost constant. But increase in anisotropic ratio ε increases the critical thermal Rayleigh number R_{SC}. Therefore anisotropy leads to stability of the system.

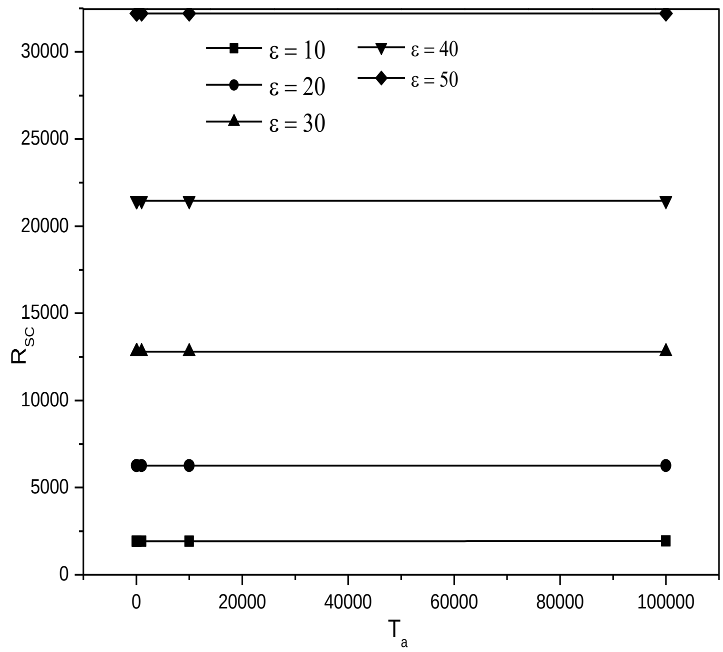


Fig. 15. Critical thermal Rayleigh number R_{SC} versus Taylor number T_a for various anisotropic parameter ε, $\delta = 0.01$, $R_S = -500$, $k = 0.001$, $S_T = -0.002$, and $\tau = 0.03$.

8 Conclusions

The linear stability of Soret-driven thermohaline convection in ferromagnetic fluid layer heated from below and salted from above rotating a densely packed anisotropic porous medium subject to a transverse uniform magnetic field has been considered with magnetic field dependent (MFD) viscosity using Darcy model. In this analysis, we have investigated the effect of various parameters like permeability of the porous medium, anisotropic parameter, MFD viscosity parameter, non – buoyancy magnetization, Prandtl number, thermal Rayleigh number and salinity Rayleigh number on the onset of convection. The thermal critical magnetic Rayleigh numbers for the onset of instability are also determined numerically for sufficient large values of buoyancy magnetization parameter M_1 and results are depicted graphically. Also the principle of exchange of instability is applied to find out the mode of attaining instability. In order to investigate our results, we must review the results and physical explanations. It is well known that in case of Newtonian fluid the rotation introduces vorticity into the fluid. Then, the fluid moves in the horizontal planes with higher velocities. On account of this motion, the velocity of the fluid perpendicular to the planes reduces, and hence delays the onset of convection. When the fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium, free from rotation or small rate of rotation, then the permeability of porous medium has a destabilizing effect. As permeability of porous medium increases, the void space increases and as a result of this, the flow quantities perpendicular to the planes will clearly be increased. Thus, increasing Darcy's number leads to decrease in critical thermal Rayleigh number. In case of high rotation, the motion of the fluid prevails essentially in the horizontal planes. This motion is increased as permeability of porous medium increases. Thus the component of the velocity perpendicular to the horizontal planes reduces, leading to delay in the onset of convection. Hence permeability of porous medium has a stabilizing effect in the case of high rotation.

In this investigation, it is clear that the system gets destabilized with respect to
(1) variation in magnetization parameter M_3.
(2) variation in salinity Rayleigh number R_s.
(3) variation in an anisotropic parameter ε.
(4) variation in the ratio of the mass transport to heat transport τ.

For the case of stationary convection, the porosity and anisotropy effects have destabilizing behaviour in the absence of MFD viscosity and rotation. The destabilizing behaviour of medium permeability is virtually unaffected by magnetization parameter. The presence of MFD viscosity and rotation have stabilizing behaviour.From the above investigation, one can conclude that the variable viscosity and rotation tend to stabilize the system, when compared with the constant viscosity of the system. The increase in magnetization tends to destabilize the system. The MFD viscosity, rotation and anisotropicity have a profound influence on the onset of convection in a porous medium.

APPENDIX A

When of Eq. (5) is used in Eq. (1) and resulting equation is linearized with B_i ($i = 1, 2, 3$) given by Eqs. (13) and (14), we obtain the following components:

$$\text{(A.1)}\quad \rho_0\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu_0(M_0 + H_0)\frac{\partial H_1'}{\partial z} + 2\rho_0 v\Omega - \frac{\mu_1}{k_1}u\,,$$

$$\text{(A.2)}\quad \rho_0\frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \mu_0(M_0 + H_0)\frac{\partial H_2'}{\partial z} - 2\rho_0 u\Omega - \frac{\mu_1}{k_1}v\,,$$

$$\begin{aligned}\text{(A.3)}\quad \rho_0\frac{\partial w}{\partial t} = &-\frac{\partial p}{\partial z} + \mu_0(M_0 + H_0)\frac{\partial H_3'}{\partial z} - \mu_0 H_3' K\beta_t + \frac{\mu_0 K^2\beta_t T'}{1+\chi}(1 - S_T)\\ &+ \mu_0 H_3' K_2\beta_s + \rho_0 g\alpha_t T' - \frac{\mu_0 K K_2\beta_t S'}{1+\chi} + \frac{\mu_0 K_2^2\beta_s S'}{1+\chi} -\\ &- \frac{\mu_0 K K_2\beta_s T'}{1+\chi}(1 - S_{\mathrm{T}}) - \rho_0 g\alpha_s S' - \frac{\mu_1}{k_2}w - \frac{\mu_1}{k_2}\delta\mu_0(M_0 + H_0)w\,.\end{aligned}$$

Differentiating Eqs. (A.1) – (A.3) with respect to x, y and z, respectively, and adding, the following equation is obtained upon using Eq. (2):

$$\begin{aligned}(\text{A.4})\quad \nabla^2 p &= \mu_0(M_0+H_0)\frac{\partial}{\partial z}(\nabla\cdot\boldsymbol{H}') + \mu_0 K_2\beta_s\frac{\partial H_3'}{\partial z} - \mu_0 K\beta_t\frac{\partial H_3'}{\partial z}\\ &+ \frac{\mu_0 K^2\beta_t}{1+\chi}(1-S_T)\frac{\partial T'}{\partial z} - \frac{\mu_0 K K_2\beta_s}{1+\chi}(1-S_T)\frac{\partial T'}{\partial z} + \frac{\mu_0 K_2^2\beta_s}{1+\chi}\frac{\partial S'}{\partial z}\\ &- \frac{\mu_0 K K_2\beta_t}{1+\chi}\frac{\partial S'}{\partial z} + \rho_0 g\alpha_t\frac{\partial T'}{\partial z} - \rho_0 g\alpha_s\frac{\partial S'}{\partial z} + \frac{\mu_1}{k_1}\Big(\frac{\partial w}{\partial z}\Big)\\ &- \frac{\mu_1}{k_2}\Big(\frac{\partial w}{\partial z}\Big) - \frac{\mu_1}{k_2}\delta\mu_0(M_0+H_0)\frac{\partial w}{\partial z} + 2\rho_0\Omega\xi\,,\end{aligned}$$

where $\boldsymbol{H}'$ has the components (H_1', H_2', H_3').

From Eq. (5), $\boldsymbol{H}' = \nabla\phi$ where ϕ is a scalar potential. Elimination of p from Eq. (A.1) – (A.3) and using Eq. (A.4), we get

$$\begin{aligned}(\text{A.5})\quad \rho_0\frac{\partial}{\partial t}(\nabla^2 w) &= \mu_0 K_2\beta_s\frac{\partial}{\partial z}(\nabla_1^2\phi) - \mu_0 K\beta_t\frac{\partial}{\partial z}\left(\nabla_1^2\phi\right)\\ &+ \frac{\mu_0 K^2\beta_t}{1+\chi}(1-S_T)\nabla_1^2 T' + \rho_0 g\alpha_t\nabla_1^2 T' - \frac{\mu_0 K K_2\beta_s}{1+\chi}(1-S_T)\nabla_1^2 T'\\ &+ \frac{\mu_0 K_2^2\beta_s}{1+\chi}\nabla_1^2 S' - \frac{\mu_0 K K_2\beta_t}{1+\chi}\nabla_1^2 S' - \rho_0 g\alpha_s\nabla_1^2 S'\\ &- \frac{\mu_1}{k_1}\Big(\frac{\partial^2 w}{\partial z^2}\Big) - \frac{\mu_1}{k_2}\nabla_1^2 w - \frac{\mu_1}{k_2}\delta\mu_0(M_0+H_0)\nabla_1^2 w - 2\rho_0\Omega\frac{\partial\xi}{\partial z}\,,\end{aligned}$$

where $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ and $\nabla^2 = \nabla_1^2 + \frac{\partial^2}{\partial z^2}$.

APPENDIX B

Using Eq. (20) in Eq. (A.5), one gets the vertical component of the momentum equation, which can be written as

$$\begin{aligned}(\text{B.1})\quad \rho_0\frac{\partial}{\partial t}\Big(\frac{\partial^2}{\partial z^2} - k_0^2\Big)w &= \frac{\mu_0 K\beta_t}{1+\chi}\Big[(1+\chi)\frac{\partial\phi}{\partial z} - K\theta(1-S_T)\Big]k_0^2 - \rho_0 g\alpha_t k_0^2\theta\\ + \rho_0 g\alpha_s k_0^2 S &+ \frac{\mu_0 K_2\beta_s}{1+\chi}\Big[(1+\chi)\frac{\partial\phi}{\partial z} + K_2 S\Big]k_0^2 - \frac{\mu_0 K K_2}{1+\chi}[\beta_s(1-S_T)\theta - \beta_t S]k_0^2\\ &- \frac{\mu_1}{k_1}\Big(\frac{\partial^2 w}{\partial z^2}\Big)w + \frac{\mu_1}{k_2}k_0^2 w + \frac{\mu_1}{k_2}k_0^2\delta\mu_0(M_0+H_0)w - 2\rho_0\Omega\frac{\partial\xi}{\partial z}\,,\end{aligned}$$

(B.2)
$$\Big(\rho_0\frac{\partial}{\partial t}+\frac{\mu}{k}\Big)\xi = 2\rho_0\Omega\frac{\partial w}{\partial z}\,.$$

The modified Fourier heat conduction equation is

(B.3)
$$\rho_0 C_{V,H}\frac{\partial\theta}{\partial t}-\mu_0 K T_0\frac{\partial}{\partial t}\Big(\frac{\partial\phi}{\partial z}\Big)=K_1\left(\frac{\partial^2}{\partial z^2}-k_0^2\right)\theta + \Big[\rho_0 C_{V,H}\beta_t-\frac{\mu_0 K^2 T_0^2\beta_t}{1+\chi}+\frac{\mu_0 K K_2 T_0\beta_s}{1+\chi}\Big]w\,,$$

where $\rho_0 C=\rho_0 C_{V,H}+\mu_0 K H_0$.

The salinity equation is

(B.4)
$$\frac{\partial S}{\partial t}+\beta_s w = K_s\Big(\frac{\partial^2}{\partial z^2}-k_0^2\Big)S+S_T\Big(\frac{\partial^2}{\partial z^2}-k_0^2\Big)\theta\,.$$

The magnetic potential equation is

(B.5)
$$(1+\chi)\frac{\partial^2\phi}{\partial z^2}-\Big(1+\frac{M_0}{H_0}\Big)k_0^2\phi-K\frac{\partial\theta}{\partial z}+K_2\frac{\partial S}{\partial z}+S_T K\frac{\partial\theta}{\partial z}=0\,,$$

where the non-dimensional parameters used are

(B.6)
$$\begin{aligned}
&M_1=\frac{\mu_0K^2\beta_t}{(1+\chi)\rho_0 g\alpha_t}\,,\quad M_2=\frac{\mu_0K^2T_0}{(1+\chi)\rho_0 C_{v,H}}\,,\quad M_3=\frac{1+M_0/H_0}{(1+\chi)}\,,\\
&M_4=\frac{\mu_0K^2\beta_s}{(1+\chi)\rho_0 g\alpha_s}\quad M_5=\frac{K_2\beta_s}{K\beta_t}\,,\quad M_6=\frac{K_S}{K_1}\,,\quad \tau=\rho_0 C_{v,H}\Big(\frac{K_S}{K_1}\Big),\\
&P_r=\frac{\mu C_{v,H}}{K_1}\,,\quad R_S=\frac{\rho_0 C_{v,H}\beta_s\alpha_s g d^4}{\nu K_S}\,,\quad R=\frac{\rho_0 C_{v,H}\beta_t\alpha_t g d^4}{\nu K_1}\,,
\end{aligned}$$

where R_S is the salinity Rayleigh number, R is the thermal Rayleigh number, P_r is the Prandtl number and other parameters describe non-dimensional parameters.

APPENDIX C

$$\begin{aligned}
U&=(\pi^2+a^2)(\pi^2+a^2M_3)P_r^2\,,\\
V&=(\pi^2+a^2M_3)\Big[(\pi^2+a^2)^2(1+\tau)\\
&\qquad+P_r\Big(\frac{1}{k_1}(\pi^2+a^2)+\frac{\varepsilon\pi^2+a^2}{\varepsilon k_1}+\frac{1}{\varepsilon k_1}a^2M_3\delta\Big)\Big]P_r\,,
\end{aligned}$$

$$
\begin{aligned}
W = {} & (\pi^2 + a^2 M_3)(\pi^2 + a^2)\Big[\tau(\pi^2 + a^2)^2 + P_r(1+\tau)\Big(\frac{1}{k_1}(\pi^2 + a^2) \\
& + \frac{\varepsilon\pi^2 + a^2}{\varepsilon k_1} + \frac{1}{\varepsilon k_1} a^2 M_3 \delta\Big)\Big] + a^2 R P_r(\pi^2 + a^2 M_3)[1 + M_1(1+M_5)(1-S_T)] \\
& - a^2 R P_r \pi^2 M_1(1+M_5)[(1-S_T) + M_5] + a^2 R_s P_r(\pi^2 + a^2 M_3) \\
& \times \Big(1 + M_4 + \frac{M_4}{M_5}\Big) M_6 + (\pi^2 + a^2 M_3)\Big[T_a \pi^2 + \frac{1}{k_1}\Big(\frac{\varepsilon\pi^2 + a^2}{\varepsilon k_1} + \frac{a^2 M_3 \delta}{\varepsilon k_1}\Big)\Big] P_r^2\,,
\end{aligned}
$$

$$
\begin{aligned}
X = {} & (\pi^2 + a^2 M_3)(\pi^2 + a^2)(1+\tau)\Big[\frac{1}{k_1}\Big(\frac{\varepsilon\pi^2 + a^2}{\varepsilon k_1} + \frac{a^2 M_3 \delta}{\varepsilon k_1}\Big) + \pi^2 T_a\Big] P_r \\
& + \tau(\pi^2 + a^2 \mathrm{M}_3)(\pi^2 + a^2)^2\Big(\frac{1}{k_1}(\pi^2 + a^2) + \frac{\varepsilon\pi^2 + a^2}{\varepsilon k_1} + \frac{1}{\varepsilon k_1} a^2 M_3 \delta\Big) \\
& - a^2 R\tau(\pi^2 + a^2 M_3)(\pi^2 + a^2)[1 + (1-S_T) M_1(1+M_5)] \\
& - \frac{1}{k_1} a^2 R P_r\Big[(\pi^2 + a^2 M_3)\{1 + M_1(1+M_5)(1-S_T)\} \\
& - \pi^2 M_1(1+M_5)\{(1-S_T) + M_5\}\Big] + a^2 R(\pi^2 + a^2) M_1(1+M_5)\pi^2 \\
& \times \Big[S_T\Big(\frac{M_5}{M_6}\Big)^2 + \tau(1-S_T) + M_5\Big] - a^2 R_s(\pi^2 + a^2 M_3)(\pi^2 + a^2) \\
& \times \Big(1 + M_4 + \frac{M_4}{M_5}\Big)\Big[\frac{1}{k_1} M_6 P_r + (\pi^2 + a^2)\Big\{S_T\Big(\frac{M_5}{M_6}\Big) + M_6\Big\}\Big],
\end{aligned}
$$

$$
\begin{aligned}
Y = {} & (\pi^2 + a^2 M_3)(\pi^2 + a^2)^2 \tau\Big[\frac{1}{k_1}\Big(\frac{\varepsilon\pi^2 + a^2}{\varepsilon k_1} + \frac{a^2 M_3 \delta}{\varepsilon k_1}\Big) + \pi^2 T_a\Big] P_r \\
& - \frac{1}{k_1} a^2 R\tau(\pi^2 + a^2 M_3)(\pi^2 + a^2)[1 + (1-S_T) M_1(1+M_5)] \\
& + \frac{1}{k_1} a^2 R(\pi^2 + a^2) M_1(1+M_5)\pi^2\Big[S_T\Big(\frac{M_5}{M_6}\Big)^2 + \tau(1-S_T) + M_5\Big] \\
& - \frac{1}{k_1} a^2 R_s(\pi^2 + a^2 M_3)(\pi^2 + a^2)\Big(1 + M_4 + \frac{M_4}{M_5}\Big)\Big[S_T\Big(\frac{M_5}{M_6}\Big) + M_6\Big].
\end{aligned}
$$

APPENDIX D

$$
\begin{aligned}
& A_1 = U_2 W_1 - U_1 V_2, \quad B_1 = V_1 V_2 + U_1 X_1 - W_1 W_2, \quad C_1 = W_1 Y_1 - V_1 X_1 \\
& A_2 = -U_1 \sigma_1^2 + V_1, \quad B_2 = W_1 \sigma_1, \quad C_2 = -U_2 \sigma_1^4 + W_2 \sigma_1^2 - Y_1 \\
& D_2 = V_2 \sigma_1^3 - X_1 \sigma_1, \quad \sigma_1^2 = \frac{-B_1 \pm \sqrt{B_1^2 - 4 A_1 C_1}}{2 A_1}
\end{aligned}
$$

$$U_1 = a^2 P_r M_1(1+M_5)[(1-S_T)(\pi^2+a^2M_3) - \pi^2(1-S_T+M_5)] + a^2 R P_r(\pi^2+a^2M_3),$$

$$V_1 = \frac{1}{k_1}a^2\tau(\pi^2+a^2M_3)(\pi^2+a^2)[1+(1-S_T)M_1(1+M_5)] - \frac{1}{k_1}a^2(\pi^2+a^2)M_1(1+M_5)\pi^2\Big[S_T\Big(\frac{M_5}{M_6}\Big)^2 + \tau(1-S_T)+M_5\Big]$$

$$U_2 = (\pi^2+a^2)(\pi^2+a^2M_3)P_r^2$$

$$V_2 = (\pi^2+a^2M_3)\Big[(\pi^2+a^2)^2(1+\tau) + P_r\Big\{\frac{1}{k_1}(\pi^2+a^2) + \Big(\frac{\varepsilon\pi^2+a^2}{\varepsilon k_1}\Big) + \frac{a^2M_3\delta}{\varepsilon k_1}\Big\}\Big]P_r$$

$$W_1 = a^2\tau(\pi^2+a^2M_3)(\pi^2+a^2)[1+(1-S_T)M_1(1+M_5)] + \frac{1}{k_1}a^2P_r\Big[(\pi^2+a^2M_3)\{1+M_1(1+M_5)(1-S_T)\} - \pi^2M_1(1+M_5) \times\{(1-S_T)+M_5\}\Big] - a^2(\pi^2+a^2)M_1(1+M_5)\pi^2\Big[S_T\Big(\frac{M_5}{M_6}\Big)^2 + \tau(1-S_T)+M_5\Big],$$

$$W_2 = (\pi^2+a^2M_3)(\pi^2+a^2)\Big[\tau(\pi^2+a^2)^2 + P_r(1+\tau)\Big\{\Big(\frac{\varepsilon\pi^2+a^2}{\varepsilon k_1} + \frac{a^2M_3\delta}{\varepsilon k_1}\Big) + \frac{1}{k_1}(\pi^2+a^2)\Big\}\Big] + a^2R_sP_r(\pi^2+a^2M_3)\Big(1+M_4+\frac{M_4}{M_5}\Big)M_6 + (\pi^2+a^2M_3)\Big[T_a\pi^2 + \frac{1}{k_1}\Big(\frac{\varepsilon\pi^2+a^2}{\varepsilon k_1} + \frac{a^2M_3\delta}{\varepsilon k_1}\Big)\Big]P_r^2,$$

$$X_1 = (\pi^2+a^2M_3)(\pi^2+a^2)(1+\tau)\Big[\frac{1}{k_1}\Big(\frac{\varepsilon\pi^2+a^2}{\varepsilon k_1} + \frac{a^2M_3\delta}{\varepsilon k_1}\Big) + \pi^2T_a\Big]P_r + (\pi^2+a^2M_3)(\pi^2+a^2)^2\tau\Big[\Big(\frac{\varepsilon\pi^2+a^2}{\varepsilon k_1} + \frac{a^2M_3\delta}{\varepsilon k_1}\Big) + \frac{1}{k_1}(\pi^2+a^2)\Big] + a^2R_s(\pi^2+a^2M_3)\Big(1+M_4+\frac{M_4}{M_5}\Big)\Big[\frac{1}{k_1}M_6P_r + (\pi^2+a^2)\Big\{S_T\Big(\frac{M_5}{M_6}\Big)+M_6\Big\}\Big],$$

$$Y_1 = \tau(\pi^2+a^2M_3)(\pi^2+a^2)^2\Big[T_a\pi^2 + \frac{1}{k_1}\Big(\frac{\varepsilon\pi^2+a^2}{\varepsilon k_1} + \frac{a^2M_3\delta}{\varepsilon k_1}\Big)\Big] + \frac{1}{k_1}a^2R_s(\pi^2+a^2M_3)(\pi^2+a^2)\Big(1+M_4+\frac{M_4}{M_5}\Big)\Big[S_T\Big(\frac{M_5}{M_6}\Big)+M_6\Big].$$

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