

## FRACTIONAL ORDER THEORY OF THERMAL STRESSES IN A TWO DIMENSIONAL TRANSVERSELY ISOTROPIC MAGNETO THERMOELASTIC MATERIAL

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**ABSTRACT:** A mathematical model for homogeneous transversely isotropic magneto-thermoelastic (HTIMT) material and fractional order theory (FOT) of thermal stresses due to ramp type heat and with hall current effect is formulated. To solve the field equations, Fourier and Laplace transforms are used. The mathematical expressions of components of displacement, temperature, current density and stress are solved in the transformed domain and then obtained in physical domain using transform inversion techniques. The fractional order parameter by considering various cases of weak, normal and strong conductivity are represented graphically.

**KEY WORDS:** Fractional-order Theory, Hall current, Magneto Thermoelastic, Transversely Isotropic, Ramp Type Heat.

### NOMENCLATURE

$\delta_{ij}$	: Kronecker delta,	$\vec{u}$	: displacement vector,
$\vec{H}_0$	: magnetic field intensity vector,	$\tau_0$	: relaxation time,
$\tau_v$	: temperature gradient phase lag,	$u_i$	: components of displacement,
$T$	: temperature,	$\varphi$	: conductive temperature,
$c_{ijkl}$	: elastic parameters,	$\omega$	: frequency,
$\rho$	: medium density,	$\beta_{ij}$	: thermal elastic coupling tensor,
$\varepsilon_0$	: electric permeability,	$K_{ij}$	: thermal conductivity,
$\tau_t$	: heat flux phase lag,	$\tau_q$	: phase lag of thermal displacement,
$K_{ij}^*$	: materialistic constant,	$\alpha_{ij}$	: linear thermal expansion coefficient,
$C_E$	: specific heat,	$a_{ij}$	: two temperature parameters,
$e_{ij}$	: strain tensors,	$\delta(t)$	: Dirac's delta function,
$F_i$	: components of Lorentz force,	$\alpha$	: fractional parameter
$T_0$	: reference temperature,	$t_0$	: rise time of heat
$\vec{j}$	: current density vector,	$T_1$	: constant temperature
$t_{ij}$	: stress tensors,	$T_{ij}$	: Maxwell's stress component
$\mu_0$	: magnetic permeability,		

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## 1 INTRODUCTION

In last decade, the attraction of ample number of researchers for magneto-thermoelastic (MT) material (composite material) structures is continuously increasing since magneto-thermoelastic materials exhibits the prerequisite coupling effect among electric and magnetic fields. Due to these electric and magnetic field, Hall voltage is produced in the medium. Similarly, studying the coupling effect in a thermoelastic medium among magnetic fields and stress is known as Magneto-thermoelasticity and has a numerous applications in the domain of science and technology mainly in nuclear reactors, geophysics, and related domains.

Sherief *et al.* [1] proposed a model of thermoelasticity by using fractional calculus and Youseff [2, 3] offered an alternative theory of thermoelasticity by using the fractional calculus with extensive range ( $0 < \alpha \leq 2$ ). Ezzat *et al.* [4–6] established a additional theory on FOT using Taylor series. Bachher *et al.* [7] examined the Caputo time-FOT for MT response of a 2D isotropic solid including rotating. Kumar *et al.* [8] examined the FOT proposed by Sherief [1] and Ezzat & Youssef [2] in a micropolar solid and ramp heat source. Also, Kumar *et al.* [9] studied the thermomechanical exchanges in a rotating transversely isotropic thermoelastic (TIT) solid with two temperatures (2T). Sheoran *et al.* [10] analysed the future predictions of FOT. Abbas [11] discussed 2-D GN-III model with FOT. Additionally, other researcher discussed another theories of thermoelasticity as Marin [12, 13], Marin *et al.* [14, 15], Lata and Kaur [16, 17], Marin [18, 19], Kumar *et al.* [20] Lata and Kaur [21–23], Lata & Kaur [24–26], Marin and Craciun [27], Kaur and Lata [28]. Numerous research is done by researchers to solve different problem using fraction order generalized thermoelasticity. Regardless of these, no considerable research to study fractional order three phase lag (TPL) effect due to Hall current has been taken. In this research, a mathematical model is purposed to study the effect of FOT with hall current and ramp type heat in HTIMT solid.

## 2 BASIC EQUATIONS

The Maxwell's stress components and simplified linear equations for slow moving conducting elastic solid following [29] are:

$$(1) \quad \text{curl } \vec{h} = \vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t},$$

$$(2) \quad \text{curl } \vec{E} = -\mu_0 \frac{\partial \vec{h}}{\partial t},$$

$$(3) \quad \vec{E} = -\mu_0 \left( \frac{\partial \vec{u}}{\partial t} + \vec{H}_0 \right),$$

(4) 
$$\operatorname{div} \vec{h} = 0.$$

(5) 
$$T_{ij} = \mu_0(H_i h_j + H_j h_i - H_k h_k \delta_{ij}).$$

The equation of motion [30] for a TITM with the Lorentz force  $F_i = \mu_0 (\vec{j} \times \vec{H}_0)$  is given as under:

(6) 
$$t_{ij} = c_{ijkl} e_{kl} - \beta_{ij} T,$$

(7) 
$$t_{ij,j} + F_i = \rho \ddot{u}_i.$$

The equations (6) and (7) using the universal Ohm’s law with Hall current and with constant conductivity is given by

(8) 
$$J = \frac{\sigma_0}{1 + m^2} \left( E + \mu_0 \left( \dot{u} \times H - \frac{1}{en_e} J \times H_0 \right) \right).$$

The TPL heat conduction equation with FOT is

(9) 
$$\begin{aligned} K_{ij} \left( 1 + \frac{(\tau_t)^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) \dot{T}_{,ji} + K_{ij}^* \left( 1 + \frac{(\tau_v)^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) T_{,ji} \\ = \left( 1 + \frac{(\tau_q)^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} + \frac{(\tau_q)^{2\alpha}}{2\alpha!} \frac{\partial^{2\alpha}}{\partial t^{2\alpha}} \right) \left[ \rho C_E \ddot{T} + \beta_{ij} T_0 \dot{e}_{ij} \right], \end{aligned}$$

where

(10) 
$$\beta_{ij} = c_{ijkl} \alpha_{ij}, \quad e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), \quad \beta_{ij} = \beta_i \delta_{ij}, \quad K_{ij} = K_i \delta_{ij},$$

where  $i, j = 1, 2, 3$  and  $i, j$  is not summed. Here  $c_{ijkl} = c_{klij} = c_{jikl} = c_{ijlk}$ .

### 3 FORMULATION AND SOLUTION OF THE PROBLEM

We consider a HTIMT material with TPL fractional order heat transfer at a constant temperature  $T_0$ , and with a magnetic field  $\vec{H}_0 = (0, H_0, 0)$  along  $y$ -axis. Furthermore, the Cartesian coordinates  $x, y, z$  are considered with origin at  $z = 0$  towards  $z$ -axis with ramp heat directing vertically downwards.

For a 2-D model in the  $xz$ -plane, we take

$$\mathbf{u} = (u, 0, w)$$

Also, we consider that

$$\mathbf{E} = 0.$$

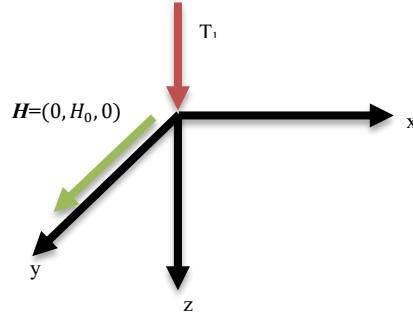


Fig. 1: Geometry of the problem.

And thus from the universal Ohm's law, we have

$$(11) \quad J_2 = 0,$$

$J_1$  and  $J_3$  using (8) are given as

$$(12) \quad J_1 = \frac{\sigma_0 \mu_0 H_0}{1 + m^2} \left( m \frac{\partial u}{\partial t} - \frac{\partial w}{\partial t} \right),$$

$$(13) \quad J_3 = \frac{\sigma_0 \mu_0 H_0}{1 + m^2} \left( \frac{\partial u}{\partial t} + m \frac{\partial w}{\partial t} \right).$$

Following Slaughter [31] apply transformation on Eqs. (7 – 9) yeilds

$$(14) \quad c_{11} \frac{\partial^2 u}{\partial x^2} + c_{13} \frac{\partial^2 w}{\partial x \partial z} + c_{44} \left( \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x \partial z} \right) - \beta_1 \frac{\partial}{\partial x} T - \mu_0 J_3 H_0 = \rho \left( \frac{\partial^2 u}{\partial t^2} \right),$$

$$(15) \quad (c_{13} + c_{44}) \frac{\partial^2 u}{\partial x \partial z} + c_{44} \frac{\partial^2 w}{\partial x^2} + c_{33} \frac{\partial^2 w}{\partial z^2} - \beta_3 \frac{\partial}{\partial z} T - \mu_0 J_1 H_0 = \rho \left( \frac{\partial^2 w}{\partial t^2} \right),$$

$$(16) \quad K_1 \left( 1 + \frac{(\tau_t)^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) \frac{\partial^2 \dot{T}}{\partial x^2} + K_3 \left( 1 + \frac{(\tau_t)^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) \frac{\partial^2 \dot{T}}{\partial z^2} \\ + K_1^* \left( 1 + \frac{(\tau_v)^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) \frac{\partial^2 T}{\partial x^2} + K_3^* \left( 1 + \frac{(\tau_v)^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) \frac{\partial^2 T}{\partial z^2} \\ = \left( 1 + \frac{(\tau_q)^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} + \frac{(\tau_q)^{2\alpha}}{2\alpha!} \frac{\partial^{2\alpha}}{\partial t^{2\alpha}} \right) \left[ \rho C_E \ddot{T} + T_0 \left\{ \beta_1 \frac{\partial \ddot{u}}{\partial x} + \beta_1 \frac{\partial \ddot{w}}{\partial z} \right\} \right]$$

and

$$(17) \quad t_{xx} = c_{11} e_{xx} + c_{13} e_{xz} - \beta_1 T,$$

$$(18) \quad t_{zz} = c_{13}e_{xx} + c_{33}e_{zz} - \beta_3 T,$$

$$(19) \quad t_{xz} = 2c_{44}e_{xz},$$

where

$$\beta_1 = (c_{11} + c_{12})\alpha_1 + c_{13}\alpha_3, \quad \beta_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3.$$

Following dimensionless (DL) quantities are used to solve the problem:

$$(20) \quad \begin{aligned} x' &= \frac{x}{L}, & u' &= \frac{\rho c_1^2}{L\beta_1 T_0} u, & t' &= \frac{C_1}{L} t, & w' &= \frac{\rho c_1^2}{L\beta_1 T_0} w, \\ T' &= \frac{T}{T_0}, & t'_{xx} &= \frac{t_{xx}}{\beta_1 T_0}, & t'_{zz} &= \frac{t_{zz}}{\beta_1 T_0}, & t'_{xz} &= \frac{t_{xz}}{\beta_1 T_0}, \\ z' &= \frac{z}{L}, & \tau'_T &= \frac{C_1}{L} \tau_T, & \tau'_v &= \frac{C_1}{L} \tau_v, & \tau'_q &= \frac{C_1}{L} \tau_q. \end{aligned}$$

By using these DL quantities defined in (20) in the Eqs. (14 – (16), and thereafter suppressing primes, yields

$$(21) \quad \frac{\partial^2 u}{\partial x^2} + \delta_1 \frac{\partial^2 w}{\partial x \partial z} + \delta_2 \frac{\partial^2 u}{\partial z^2} - \frac{\partial T}{\partial x} = \frac{M}{1+m^2} \left[ \frac{\partial u}{\partial t} + m \frac{\partial w}{\partial t} \right] + \frac{\partial^2 u}{\partial t^2},$$

$$(22) \quad \delta_1 \frac{\partial^2 u}{\partial x \partial z} + \delta_2 \frac{\partial^2 w}{\partial x^2} + \delta_3 \frac{\partial^2 w}{\partial z^2} - \frac{\beta_3}{\beta_1} \frac{\partial T}{\partial z} = -\frac{M}{1+m^2} \left[ m \frac{\partial u}{\partial t} - \frac{\partial w}{\partial t} \right] + \frac{\partial^2 w}{\partial t^2},$$

$$(23) \quad \begin{aligned} &\left( 1 + \frac{C_1(\tau_t)^\alpha}{\alpha! L} \frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}} \right) \left( K_1 \frac{\partial^2 T}{\partial x^2} \right. \\ &\quad \left. + K_3 \frac{\partial^2 T}{\partial z^2} \right) + \left( 1 + \frac{(\tau_v)^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) \left( K_1^* \frac{\partial^2 T}{\partial x^2} + K_3^* \frac{\partial^2 T}{\partial z^2} \right) \\ &= \left( 1 + \frac{(\tau_q)^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} + \frac{(\tau_q)^{2\alpha}}{2\alpha!} \frac{\partial^{2\alpha}}{\partial t^{2\alpha}} \right) \left[ \rho C_E \ddot{T} + \frac{\beta_1}{\rho} T_0 \left\{ \beta_1 \frac{\partial \ddot{u}}{\partial x} + \beta_1 \frac{\partial \ddot{w}}{\partial z} \right\} \right], \end{aligned}$$

where

$$\delta_1 = \frac{c_{13} + c_{44}}{c_{11}}, \quad \delta_2 = \frac{c_{44}}{c_{11}}, \quad \delta_3 = \frac{c_{33}}{c_{11}}, \quad M = \left( \frac{L\sigma_0\mu_0^2 H_0^2}{\rho C_1} \right).$$

According to initial and symmetry conditions

$$\mathbf{u}(x, z, 0) = \dot{u}(x, z, 0) = 0,$$

$$\mathbf{w}(x, z, 0) = \dot{w}(x, z, 0) = 0,$$

$$\mathbf{T}(x, z, 0) = \dot{T}(x, z, 0) = 0.$$

For  $z \geq 0$  and  $-\infty \leq x \leq \infty$   $\mathbf{w}(x, z, t) = u(x, z, t) = \mathbf{T}(x, z, t) = 0$ , when  $t > 0$  while  $z \rightarrow \infty$ .

The Laplace and Fourier transforms are defined as

$$(24) \quad \tilde{f}(x, z, s) = \int_0^{\infty} f(x, z, t) e^{-st} dt,$$

$$(25) \quad \hat{f}(\xi, z, s) = \int_{-\infty}^{\infty} \tilde{f}(x, z, s) e^{i\xi x} dx.$$

On Eqs. (17) – (19) we get a set of equations

$$(26) \quad \left[ -\xi^2 - s^2 + \delta_2 D^2 - \frac{Ms}{1+m^2} \right] \hat{u}(\xi, z, s) + \left[ \delta_1 Di\xi - \frac{mMs}{1+m^2} \right] \hat{w}(\xi, z, s) + (-i\xi) \hat{T}(\xi, z, s) = 0,$$

$$(27) \quad \left[ \delta_1 Di\xi + \frac{mMs}{1+m^2} \right] \hat{u}(\xi, z, s) + \left[ -\delta_2 \xi^2 + \delta_3 D^2 - s^2 - \frac{Ms}{1+m^2} \right] \hat{w}(\xi, z, s) - \frac{\beta_3}{\beta_1} D \hat{T}(\xi, z, s) = 0,$$

$$(28) \quad \frac{\beta_1^2 T_0 s^2 i \xi}{\rho} \left[ 1 + \frac{\tau_q^\alpha s^\alpha}{\alpha!} + \frac{\tau_q^{2\alpha} s^{2\alpha}}{2\alpha!} \right] \hat{u}(\xi, z, s) + \frac{\beta_1 \beta_3 T_0 s^2}{\rho} \left[ 1 + \frac{\tau_q^\alpha s^\alpha}{\alpha!} + \frac{\tau_q^{2\alpha} s^{2\alpha}}{2\alpha!} \right] D \hat{w}(\xi, z, s) + \left\{ \rho C_E C_1^2 s^2 \left( 1 + \frac{\tau_q^\alpha s^\alpha}{\alpha!} + \frac{\tau_q^{2\alpha} s^{2\alpha}}{2\alpha!} \right) + K_1 \xi^2 \left( 1 + \frac{C_1 \tau_T^\alpha s^{\alpha+1}}{\alpha! L} \right) + K_1^* \xi^2 \left( 1 + \frac{\tau_v^\alpha s^\alpha}{\alpha! L} \right) - \left[ K_3 \left( 1 + \frac{C_1 \tau_T^\alpha s^{\alpha+1}}{\alpha! L} \right) + K_3^* \left( 1 + \frac{\tau_v^\alpha s^\alpha}{\alpha! L} \right) D^2 \right] \right\} \hat{T}(\xi, z, s) = 0.$$

The non-trivial solution of (26) – (28) yields

$$(29) \quad AD^6 + BD^4 + CD^2 + E = 0,$$

where

$$\begin{aligned}
 D &= \frac{d}{dz}, & A &= -\delta_2\delta_3A_4, \\
 B &= -A_4A_9\delta_2 + \delta_2\delta_3A_3 - A_5A_4\delta_3 - A_2A_{10}\delta_2 + \delta_1^2A_7^2A_4, \\
 C &= A_3A_9\delta_2 - A_9A_4A_5 + A_5A_3\delta_3 - A_2A_{10}A_5 + A_8A_4A_6 - \delta_1^2A_7^2A_3 \\
 &\quad + A_7A_1\delta_1A_{10} - \delta_1A_7^2A_2 + A_1A_7\delta_3, \\
 E &= A_5A_9A_3 - A_6A_8A_3 + A_1A_9A_7, \\
 A_1 &= \frac{\beta_1^2T_0s^2i\xi}{\rho} \left[ 1 + \frac{\tau_q^\alpha s^\alpha}{\alpha!} + \frac{\tau_q^{2\alpha} s^{2\alpha}}{2\alpha!} \right], \\
 A_2 &= \frac{\beta_1\beta_3T_0s^2}{\rho} \left[ 1 + \frac{\tau_q^\alpha s^\alpha}{\alpha!} + \frac{\tau_q^{2\alpha} s^{2\alpha}}{2\alpha!} \right], \\
 A_3 &= \rho C_E C_1^2 s^2 \left[ 1 + \frac{\tau_q^\alpha s^\alpha}{\alpha!} + \frac{\tau_q^{2\alpha} s^{2\alpha}}{2\alpha!} \right] + K_1 \xi^2 \left[ 1 + \frac{C_1 \tau_T^\alpha s^{\alpha+1}}{\alpha! L} \right] \\
 &\quad + K_1^* \xi^2 \left[ 1 + \frac{\tau_v^\alpha s^\alpha}{\alpha! L} \right], \\
 A_4 &= K_3 \left[ 1 + \frac{C_1 \tau_T^\alpha s^{\alpha+1}}{\alpha! L} \right] + K_3^* \left[ 1 + \frac{\tau_v^\alpha s^\alpha}{\alpha! L} \right], \\
 A_5 &= -\xi^2 - s^2 - \frac{Ms}{1+m^2}, & A_6 &= -\frac{mMs}{1+m^2}, \\
 A_7 &= i\xi, & A_8 &= \frac{mMs}{1+m^2}, & A_9 &= -\xi^2\delta_2 - s^2 - \frac{Ms}{1+m^2}, & A_{10} &= -\frac{\beta_3}{\beta_1}.
 \end{aligned}$$

The  $\pm\lambda_j$ , ( $j = 1, 2, 3$ ) are the roots of Eq. (29). The result of Eqs. (26) – (28) satisfying the radiation condition, i.e.  $\tilde{u}, \tilde{v}, \tilde{w} \rightarrow 0$  as  $z \rightarrow \infty$  yields

$$(30) \quad \hat{u}(\xi, z, s) = \sum_{j=1}^3 B_j e^{-\lambda_j z},$$

$$(31) \quad \hat{w}(\xi, z, s) = \sum_{j=1}^3 d_j B_j e^{-\lambda_j z},$$

$$(32) \quad \hat{T}(\xi, z, s) = \sum_{j=1}^3 l_j B_j e^{-\lambda_j z},$$

where  $B_j, j = 1, 2, 3$  being unknown constants and

$$d_j = \frac{\delta_2 A_4 \lambda_j^4 + (-A_5 A_4 + \delta_2 A_3) \lambda_j^2 + A_1 A_7 + A_5 A_3}{-\delta_3 \zeta_4 \lambda_j^4 + (\delta_3 A_3 - A_2 A_{10} - A_4 A_9) \lambda_j^2 + A_3 A_6},$$

$$l_j = \frac{\delta_2 \delta_3 \lambda_j^4 + (\delta_2 A_9 + A_5 \delta_3 - \delta_1^2 A_7^2) \lambda_j^2 + A_5 A_9 - A_6 A_8}{-\delta_3 \zeta_4 \lambda_j^4 + (\delta_3 A_3 - A_2 A_{10} - A_4 A_9) \lambda_j^2 + A_3 A_6}.$$

#### 4 BOUNDARY CONDITIONS

The half-space surface ( $z = 0$ ) is subjected to ramp type heat, therefore, we have

$$(33) \quad t_{zz}(x, z, t) = 0,$$

$$(34) \quad t_{xz}(x, z, t) = 0,$$

$$(35) \quad T(x, z, t) = G(t)\delta(x),$$

where  $G(t)$  is a function defined as

$$G(t) = \begin{cases} 0; & t \leq 0 \\ T_1 \frac{t}{t_0}; & 0 \leq t \leq t_0 \\ T_1; & t > t_0 \end{cases}.$$

Applying the transforms as defined by equations (24) – (25) to (35)

$$\hat{T}(\xi, 0, s) = \bar{G}(s),$$

where  $\bar{G}(s) = T_1(1 - e^{-st_0})/t_0 s^2$ .

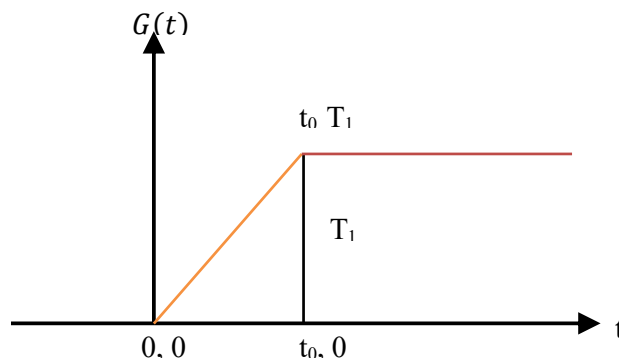


Fig. 2: Geometry of Ramp type heat.



After using (24) – (25) respectively on (33) – (35) and (14) – (16) by using (30) – (32), we obtained the components of displacement, current density and stress, as well as conductive temperature

$$(36) \quad [\hat{u}, \hat{w}, \hat{T}, \hat{t}_{xz}, \hat{t}_{zz}, \hat{J}_1, \hat{J}_3](\xi, z, s) = \frac{\bar{G}(s)}{\chi} \sum_{j=1}^3 [1, d_j, l_j, \chi_{2j}, \chi_{1j}, \psi_j, \varrho_j] \varsigma_j e^{-\lambda_j z},$$

where

$$\chi_{1j} = C_{13}i\xi - C_{33}d_j\lambda_j - \beta_3l_j, \quad \chi_{2j} = (\lambda_j + i\xi d_j), \quad \chi_{3j} = l_j, \quad j = 1, 2, 3$$

and

$$\begin{aligned} \chi &= \varsigma_1\chi_{11} - \varsigma_2\chi_{12} + \varsigma_3\chi_{13}, & \varsigma_1 &= \bar{G}(s)(\chi_{12}\chi_{23} - \chi_{13}\chi_{22}), \\ \varsigma_2 &= \bar{G}(s)(\chi_{11}\chi_{23} - \chi_{21}\chi_{13}), & \varsigma_3 &= \bar{G}(s)(\chi_{11}\chi_{23} - \chi_{21}\chi_{12}), \\ \psi_j &= \frac{\sigma_0 H_0 \mu_0}{(1 + m^2)} s(m - d_j), & \varrho_j &= \frac{\sigma_0 H_0 \mu_0}{(1 + m^2)} s(1 + md_j). \end{aligned}$$

### 5 INVERSION OF THE TRANSFORMATION

To obtain the results in physical domain, Fourier transform in Eq. (36) need be inverted by using

$$(37) \quad \tilde{f}(x, z, s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\xi x} \hat{f}(\xi, z, s) d\xi = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\cos(\xi x) f_e - i \sin(\xi x) f_0| d\xi.$$

The Laplace transform function  $\tilde{f}(x, z, s)$  following Honig and Hirdes [32], inverted to  $f(x, z, t)$  by

$$(38) \quad f(x, z, t) = \frac{1}{2\pi i} \int_{\nu-i\infty}^{\nu+i\infty} \tilde{f}(x, z, s) e^{-st} ds.$$

Romberg’s integration [33] is used for inversion integral in Eq. (37).

### 6 NUMERICAL RESULTS AND DISCUSSION

The physical data for cobalt material [34] is used to validate the theoretic outcomes and influence of fractional order parameter and Hall current, is given as

$$c_{11} = 3.07 \times 10^{11} \text{ Nm}^{-2}, \quad T_0 = 298 \text{ K}, \quad c_{44} = 1.510 \times 10^{11} \text{ Nm}^{-2},$$

$$\begin{aligned}\varepsilon_0 &= 8.838 \times 10^{-12} \text{ Fm}^{-1}, & \beta_1 &= 7.04 \times 10^6 \text{ Nm}^{-2}\text{deg}^{-1}, \\ \beta_3 &= 6.90 \times 10^6 \text{ Nm}^{-2}\text{deg}^{-1}, & c_{33} &= 3.581 \times 10^{11} \text{ Nm}^{-2}, & \rho &= 8.836 \times 10^3 \text{ kgm}^{-3}, \\ C_E &= 4.27 \times 10^2 \text{ Jkg}^{-1}\text{deg}^{-1}, & H_0 &= 1 \text{ Jm}^{-1}\text{nb}^{-1}, & K_1 &= 0.690 \times 10^2 \text{ Wm}^{-1}\text{K}^{-1}, \\ c_{13} &= 1.027 \times 10^{10} \text{ Nm}^{-2}, & K_3 &= 0.690 \times 10^2 \text{ Wm}^{-1}\text{K}^{-1}, & L &= 1.\end{aligned}$$

Dimensionless field variables are compared with distinct values of fractional order factor and constant Hall current factor  $m = 0.5$  is shown graphically. Figure 3 and Fig. 4 exhibit the variations in  $u$  and  $w$  w.r.t  $x$  respectively. For different values of  $\alpha$  the displacement variables  $u$  and  $w$ , first decreases when  $0 \leq x \leq 2$ , and thereafter exhibits oscillatory pattern.

Figure 5 shows the variations in  $t_{zx}$  w.r.t.  $x$  and Fig. 6 illustrates the  $t_{zz}$  w.r.t.  $x$ . For  $\alpha = 1.0$  the tangential stress  $t_{zx}$  shows an oscillatory behaviour. However, when  $\alpha = 0.5$ ,  $t_{zx}$  and  $t_{zz}$  shows an oscillatory behaviour with declining amplitude with rise in  $x$  and the normal stress  $t_{zz}$  shows oscillatory behaviour with larger amplitude with increase in  $x$ . For  $\alpha = 1.5$ ,  $t_{zx}$  abruptly increases when  $0 \leq x \leq 2$  and then become oscillatory while normal stress  $t_{zz}$  abruptly rises when  $0 \leq x \leq 2$  and then shows oscillatory behaviour with declining amplitude with rise in  $x$ . Figure 7 shows the variations in  $T$  with  $x$ . Initially, it sharply decrease and then become oscillatory for rest of the length. Figure 8 illustrates the  $J_1$  and Fig. 9 demonstrates the  $J_3$  w.r.t.

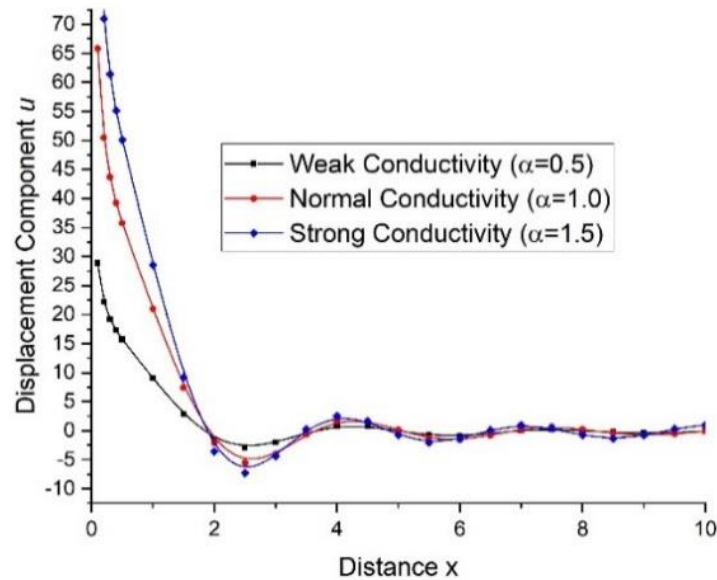


Fig. 3: Deviations of  $u$  w.r.t.  $x$ .

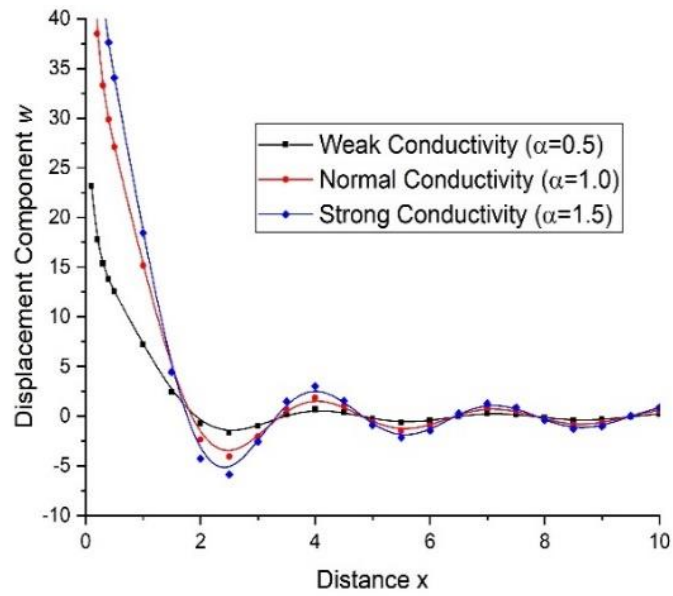


Fig. 4: Variations of  $w$  w.r.t.  $x$ .

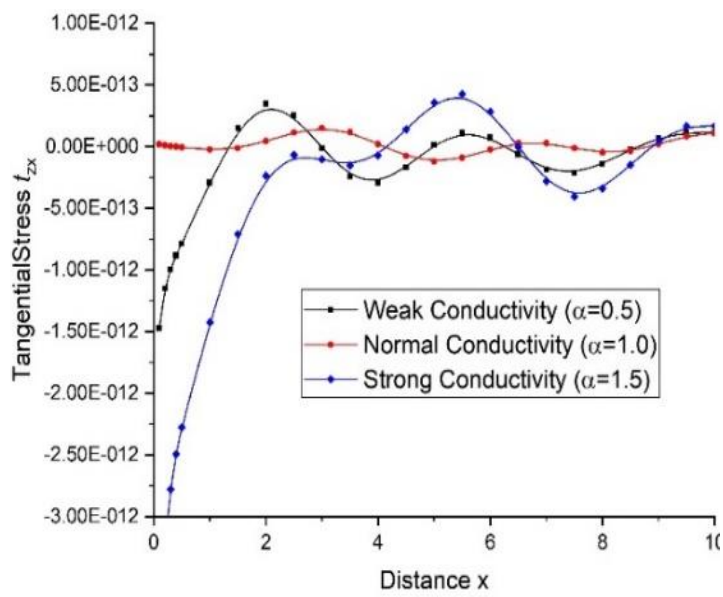


Fig. 5: Deviations of  $t_{zx}$  w.r.t.  $x$ .

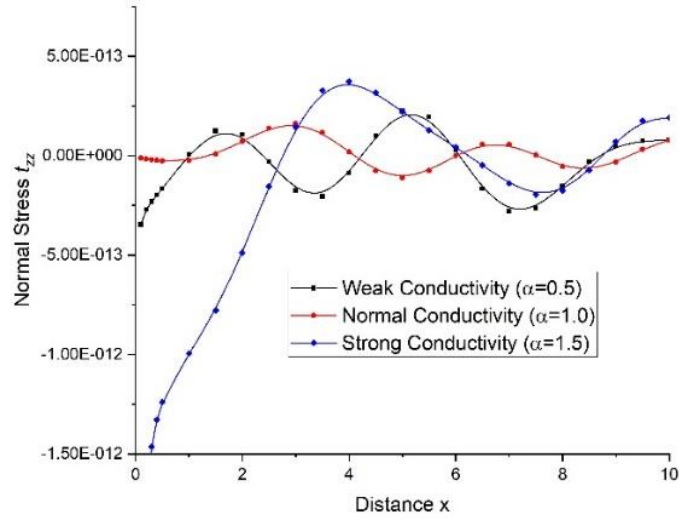


Fig. 6: Deviations of  $t_{zz}$  w.r.t.  $x$ .

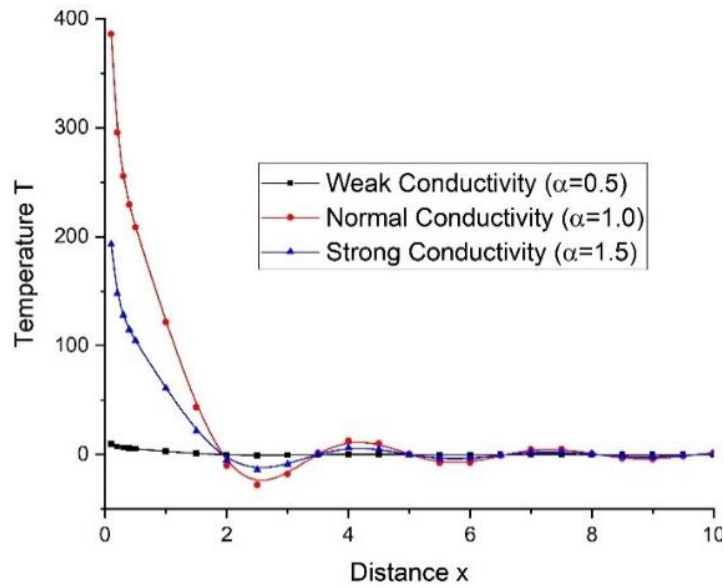


Fig. 7: Deviations of  $T$  with  $x$ .

$x$ . for  $\alpha = 0.5$  and  $\alpha = 1.0$  the  $J_1$  is remain similar, while  $J_3$  for  $\alpha = 0.5$  the value remains similar and for  $\alpha = 1.0$  the value of of  $J_3$  first increases when  $0 \leq x \leq 2$  and then remains identical as for  $\alpha = 0.5$ .

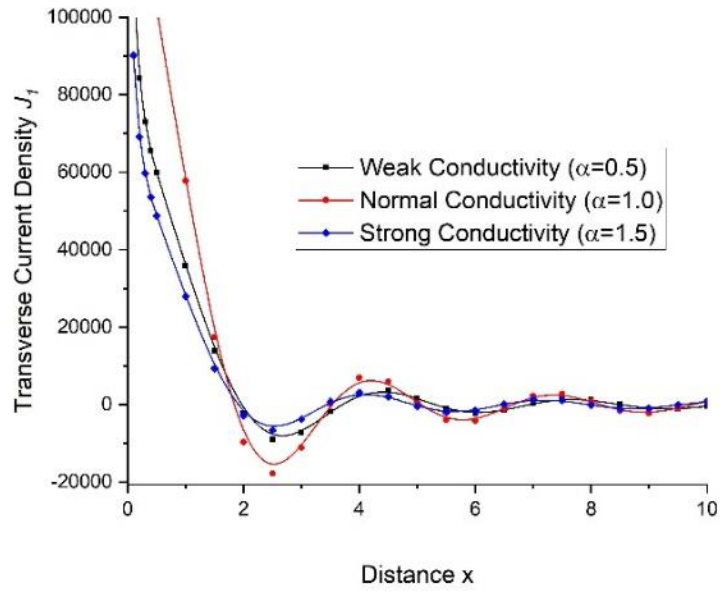


Fig. 8: Deviations of  $J_1$  w.r.t.  $x$ .

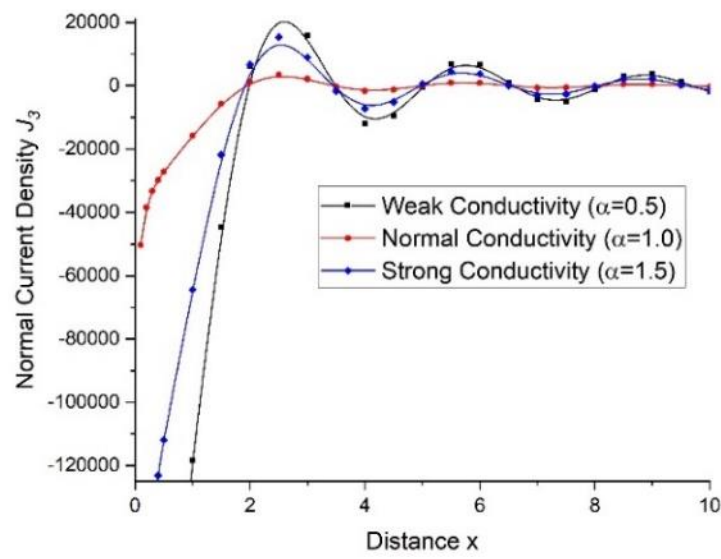


Fig. 9: Deviations of  $J_3$  w.r.t.  $x$ .

## 7 CONCLUSION

- A novel mathematical model is purposed for the study of HTIMT material with FOT of thermal stresses due to ramp type heat and Hall current effect
- Fractional order parameter  $\alpha$  has a substantial impact on the various parameters of HTIMT medium. As the value of  $\alpha$  increases, it shows the major effect on the various components.
- The effect of ramp heat in HTIMT medium with Hall Effect and fractional order factor plays a significant role in the analysis of the deformed media.
- The graphs shows the effect of  $\alpha$  with constant value of  $m$  on the media and also fulfil research motive.

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