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OPTIMAL CONTROL OF SIMULTANEOUS TOWER CRANE SLEWING AND TROLLEY MOVEMENT

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ABSTRACT: The article describes a mathematical model of tower crane slewing and trolley movement. Based on the model, a nonlinear problem of mechanisms' optimal control has been stated. A generalized optimization criterion was developed to cope with the constraints of the problem. In order to find an approximate solution of the problem, the metaheuristic method (ME-PSO) was applied.

The obtained results are illustrated via graphical dependencies of kinematic, dynamical, and energy characteristics for the two cases of the trolley movement: toward the tower and in the opposite direction. In order to analyze the obtained results numerical indicators of the specified characteristics have been calculated. They show the reduction of energy losses, dynamical forces, and elimination of the load oscillations. This result provides an increase of the crane's efficiency exploitation. The developed in the article methodology may be applied to similar optimal control problems.

KEY WORDS: tower crane, load oscillations, slewing, trolley movement, constraints, criterion, approximate solution.

1 INTRODUCTION

1.1 PROBLEM STATEMENT

Tower cranes are widely used in many sectors of the economy, including construction and civil engineering. Their performance, durability, and energy efficiency, as well as operational safety depend on the modes of crane mechanisms' movement. The rational and/or optimal modes of crane movement allow to improve the basic technical and operational indicators of the crane.

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Modern drive systems make it possible to implement the optimal laws of motion of tower crane mechanisms. Therefore, the relevance of scientific and applied works on the optimization of tower crane mechanisms movement is an important area of research.

1.2 Analysis of recent studies and publications

Most of the researches on optimal motion control of tower crane mechanisms involve duration criterion [1–8] and linear-quadratic criteria [4, 9, 10]. All these works are characterized by the requirement of elimination of the load oscillations on a flexible suspension.

In paper [1], the tower crane model is presented as a system of four nonlinear differential equations describing the tower slewing, the movement of the trolley along the boom, and the oscillation of the load on a flexible suspension in two planes (along the trolley motion and perpendicular to its direction). In the article, only a quasi-optimal solution of time-minimum control problem was found. The results of theoretical studies were tested via experimental laboratory studies. In [2] a nonlinear model of the several crane mechanisms (slewing, trolley movement, and hoisting) was used. Constraints were imposed on the boom, the trolley, and load acceleration. Implementing specialized software Makarevych, Shamardyna, and others [3] have obtained the optimal operating control for slewing, load hoisting, and trolley movement of the tower crane. In [4], the authors developed approaches to eliminate the load oscillations of rotating (including tower) cranes, using linear-quadratic optimization criterion. The paper elaborates the issues of control implementation, in particular, the structure of the controlled drive mechanism. The work [5] is devoted to the development of the optimal speed movement control of the crane slewing unit. The authors used the Pontryagin maximum principle [6]. Using nonlinear equations of system motion, the authors found the solution to the optimal control problem. The control function relied on a value proportional to the acceleration of the trolley, which, in turn, may complicate the practical implementation of the results. In paper [7], the same method was used to optimize the rotation speed of a tower crane. However, the linear motion model of the system was exploited there. An approximate solution to the minimum duration control problem of the tower crane slewing was found in the article [8]. The model used in the calculations is nonlinear with variable parameters (the length of flexible suspension). This approach greatly complicates the problem solution and ultimately does not allow to find its exact solution. The analysis of the approaches made in the work allowed the authors to conclude that the infeasibility of reducing the mathematical model of the system to the normal form made it impossible to apply the Pontryagin maximum principle. In the paper [11], based on the results obtained for the crane mechanisms of the trolley movement [12], which are grounded on Pontryagin's method, the authors suggest compensating Coriolis force. The article also provides recommendations for implementing optimal control by means of a controlled asynchronous electric drive. In [9], a mathematical model of a tower crane is obtained based on Lagrange equations, which describes the rotation of the tower, the movement of the trolley, the hoisting of the load, and its oscillations in two planes. The authors reduced the problem of minimizing the linear-quadratic functional to the system of Riccati's equations. Professor Galafshani [10] developed a mathematical model of a tower crane and formulated an optimization problem for eliminating load oscillations on a flexible suspension. The analysis of the results showed that some of them cannot be implemented in practice. Hanafy M. Omar and other researchers in [13] have considered the modes of tower crane operation movement controllers that may eliminate the load oscillations on a flexible suspension. In [14] to determine the discrete control values of the slewing mechanism a modified PSO method [15] was applied. As a result, the authors have found a control that minimizes the criterion, which reflected the RMS value of the driving torque and the speed of its rate.

Thus, most studies on optimal motion control of tower crane mechanisms use nonlinear mathematical models. Moreover, theoretical calculations and experimental research have been performed only for the parameters of laboratory models of tower cranes, which can complicate their implementation for real hoisting machines.

1.3 The Purpose of the study

The purpose of the work is to synthesize optimal modes of simultaneous movement of the crane's slewing and trolley movement mechanisms, which eliminate load oscillations and minimize energy consumption during the start-up of the mechanisms. In order to achieve this goal, it is necessary to cope with the following tasks: 1) to develop a mathematical model of the simultaneous movement of the crane slewing and trolley movement mechanisms, suitable for the study of the optimal control problems; 2) to perform the statement of the optimal control problem; 3) to apply numerical optimization method and find an approximate solution to the problem; 4) to analyze the obtained results.

2 Main Material

In this investigation, the simultaneous movement of the trolley (along the boom) and slewing of the crane is represented by the dynamic model with four degrees of freedom (Fig. 1). The used in the investigation tower crane dynamic model reflects the low-frequency pendular oscillation of the load, slewing of the tower, and linear movement of the trolley along the boom. Model (Fig. 1) is similar to those, which presented in [1, 2, 5, 8–10, 13, 14]. We have chosen as generalized coordinates: linear

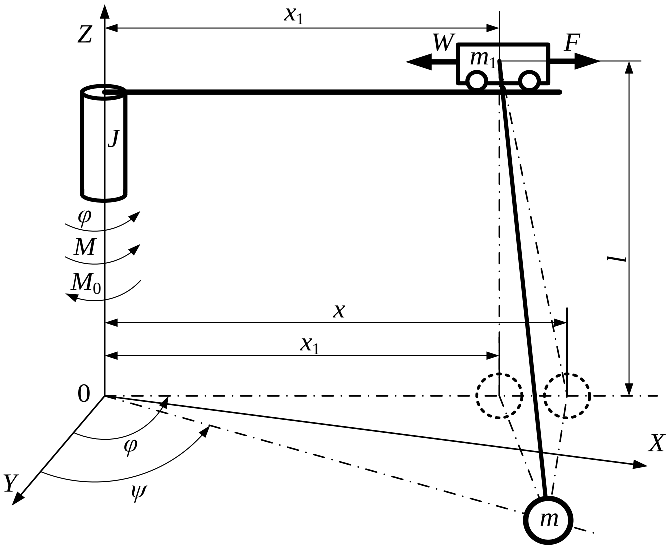


Fig. 1: A dynamic model of the simultaneous crane's slewing and trolley movement.

coordinates of the centers of mass of a trolley x_1 and a load x in the plane of a crane boom, as well as the angular coordinates of the boom slewing φ and the load ψ in the horizontal plane.

We assume that the length of the flexible suspension l is a constant (in practice l is varied. However, the speed of $l$variation is low. During regimens, which are subjects of the study, l may change slightly. This effect on the dynamics of the system is minor, and it provides the grounds for the assumption $l \quad = \text{const.}$). During the movement of the system, the load oscillates in two mutually perpendicular planes: in horizontal one (the deviation of the flexible suspension from the vertical is described by the expression $\varphi - \psi$) and in vertical one (here the angle of deviation is proportional to the remainder $x - x_1$). In the frame of the research small oscillations are under consideration. That is why they may be linearized. The accepted dynamic model corresponds to the system of differential equations of motion, which is obtained with using Lagrange's equations:

$$
\begin{aligned}
&m_1\ddot{x}_1 - m_1 x_1 \dot{\varphi}^2 = F - W + \frac{mg}{l}(x - x_1)\,;\\
&\ddot{x} - \dot{\psi}^2 x = \frac{g}{l}(x - x_1 + x(\psi - \varphi)^2)\,;\\
&(J + m_1 x_1^2)\ddot{\varphi} + 2m_1 x_1 \dot{x}_1 \dot{\varphi} = M - M_0 - \frac{mg}{l}x^2(\psi - \varphi)\,;\\
&\ddot{\psi}x + 2\dot{x}\dot{\psi} = \frac{g}{l}x(\psi - \varphi)\,,
\end{aligned}
\tag{1}
$$

where m_1 and m are reduced masses of the trolley and the load respectively; F is the reduced driving force of the trolley drive; W is reduced resistance of trolley movement; g is free-fall acceleration; l is the length of flexible load suspension; J is the moment of inertia of the slewing unit and crane pillar reduced to the axis of its rotation; M and M_0 are respectively the driving torque of the slewing unit drive and the torque of resistance forces reduced to the axis of crane rotation.

From the last equation of the system (1) we find the difference of angular coordinates

$$\psi - \varphi = \frac{l}{g}(\ddot{\psi} + 2\frac{\dot{x}\dot{\psi}}{x})\,. \tag{2}$$

The difference of linear coordinates x and x_1 we may obtain from the second equation of the system (1) and the expression (2)

$$x - x_1 = \frac{l}{g}\Big(\ddot{x} - x\Big(\frac{l}{g}\Big(\ddot{\psi} + 2\frac{\dot{x}\dot{\psi}}{x}\Big)^2 + \dot{\psi}^2\Big)\Big). \tag{3}$$

From the formula (2) we express the angular coordinate of the crane slewing unit and find its first and second derivatives with time

$$\varphi = \psi - \frac{l}{g}\Big(\ddot{\psi} + 2\frac{\dot{x}\dot{\psi}}{x}\Big)\,; \tag{4}$$

$$\dot{\varphi} = \dot{\psi} - \frac{l}{g}\Big(\dddot{\psi} + 2\frac{\ddot{x}\dot{\psi} + \dot{x}\ddot{\psi}}{x} - 2\frac{\dot{x}^2\dot{\psi}}{x^2}\Big)\,; \tag{5}$$

$$\ddot{\varphi} = \ddot{\psi} - \frac{l}{g}\Big(\overset{\text{IV}}{\psi} + 2\frac{\dddot{x}\dot{\psi} + 2\ddot{x}\ddot{\psi} + \dot{x}\dddot{\psi}}{x} - 2\frac{\dot{x}(2\dot{x}\ddot{\psi} + 3\ddot{x}\dot{\psi})}{x^2} + 4\frac{\dot{x}^3\dot{\psi}}{x^3}\Big)\,. \tag{6}$$

From equation (3), we express the linear coordinate of the mass center of the trolley and take the derivatives with time up to the second order

$$x_1 = x + \frac{l}{g}\Big[x\Big(\frac{l}{g}\Big(\ddot{\psi} + 2\frac{\dot{x}\dot{\psi}}{x}\Big)^2 + \dot{\psi}^2\Big) - \ddot{x}\Big]\,; \tag{7}$$

$$\dot{x}_1 = \dot{x} + \frac{l}{g}\Bigg[\dot{x}\Big(\frac{l}{g}\Big(\ddot{\psi} + 2\frac{\dot{x}\dot{\psi}}{x}\Big)^2 + \dot{\psi}^2\Big) + 2x\Big(\frac{l}{g}\Big(\ddot{\psi} + 2\frac{\dot{x}\dot{\psi}}{x}\Big)\Big(\dddot{\psi} + 2\frac{\ddot{x}\dot{\psi} + \dot{x}\ddot{\psi}}{x} - 2\frac{\dot{x}^2\dot{\psi}}{x^2}\Big) + \dot{\psi}\ddot{\psi}\Big) - \dddot{x}\Bigg]\,; \tag{8}$$

$$\ddot{x}_1 = \ddot{x} + \frac{l}{g}\Bigg[\ddot{x}\Bigg(\frac{l}{g}\Big(\ddot{\psi} + 2\frac{\dot{x}\dot{\psi}}{x}\Big)^2 + \dot{\psi}^2\Bigg) +$$
$$+ 4\dot{x}\Bigg(\frac{l}{g}\Big(\ddot{\psi} + 2\frac{\dot{x}\dot{\psi}}{x}\Big)\Big(\psi + 2\frac{\ddot{x}\dot{\psi} + \dot{x}\ddot{\psi}}{x} - 2\frac{\dot{x}^2\dot{\psi}}{x^2}\Big) + \dot{\psi}\ddot{\psi}\Bigg)$$
$$+ 2x\Bigg(\frac{l}{g}\Big(\dddot{\psi} + 2\frac{\ddot{x}\dot{\psi} + \dot{x}\ddot{\psi}}{x} - 2\frac{\dot{x}^2\dot{\psi}^2}{x}\Big)^2 + \frac{l}{g}\Big(\ddot{\psi} + 2\frac{\dot{x}\dot{\psi}}{x}\Big)$$
$$\times\Big(\overset{IV}{\psi} + 2\frac{\dddot{x}\dot{\psi} + 2\ddot{x}\ddot{\psi} + \dot{x}\dddot{\psi}}{x} - 2\frac{2\dot{x}^2\ddot{\psi} + 3\dot{x}\ddot{x}\dot{\psi}}{x^2} + 4\frac{\dot{x}^3\dot{\psi}}{x^3}\Big)$$
$$+ \ddot{\psi}^2 + \dot{\psi}\psi\Bigg) - x^{IV}\Bigg]; \tag{9}$$

Taking into account the expression (3), from the first equation of the system (1) we express the driving force of the trolley drive

$$F = m_1(\ddot{x}_1 - \dot{\varphi}^2 x_1) + m\Big[x\Big(\frac{l}{g}\Big(\ddot{\psi} + 2\frac{\dot{x}\dot{\psi}}{x}\Big)^2 + \dot{\psi}^2\Big) - \ddot{x}\Big] + W\,. \tag{10}$$

Similarly, from the third equation of the system (1) we express the driving torque of the slewing mechanism, which, taking into account expression (2), has the next form:

$$M = (J + m_1 x_1^2)\ddot{\varphi} + 2m_1 x_1 \dot{x}_1 \dot{\varphi} + m x^2\Big(\ddot{\psi} + 2\frac{\dot{x}\dot{\psi}}{x}\Big) + M_0\,. \tag{11}$$

In the process of starting the mechanisms for the trolley movement and the crane slewing, we should find a mode of movement that provides the least energy losses. Therefore, as the optimization criterion, we choose the root mean square of the power of the driving mechanisms of the trolley and the crane slewing, which is determined by the following dependence:

$$P_{\text{ck}} = \sqrt{\frac{1}{t_n}\int_0^{t_n} (F\dot{x}_1)^2 + (M\dot{\varphi}_1)^2\, dt}\,, \tag{12}$$

where t_n is the duration of the transient start-up of both mechanisms.

After substituting (10) and (11) into criterion (12), it takes the next form:

(13)
$$P_{\text{ck}} = \left\{ \frac{1}{t_n} \int_0^{t_n} \left[\left(m_1(\ddot{x}_1 - \dot{\varphi}^2 x_1) + m\left(x\left(\frac{l}{g}\left(\ddot{\psi} + 2\frac{\dot{x}\dot{\psi}}{x} \right)^2 + \dot{\psi}^2 \right) - \ddot{x} \right) + W \right) \dot{x}_1 \right]^2 \right.$$
$$\left. + \left[\left((J + m_1 x_1^2)\ddot{\varphi} + 2m_1 x_1 \dot{x}_1 \dot{\varphi} + m x^2 \left(\ddot{\psi} + 2\frac{\dot{x}\dot{\psi}}{x} \right) + M_1 \right) \dot{\varphi}_1 \right]^2 dt \right\}^{\frac{1}{2}} .$$

The integral functional (13) is nonlinear with respect to the unknown functions $x = x(t)$ and $\psi = \psi(t)$ and their derivatives up to the fourth order.

The minimum of the criterion (13) must be determined under boundary conditions

$$(14)\qquad \begin{cases} x_1(0) = x(0) = x_0\,; & \varphi(0) = \psi(0) = 0\,; \\ \dot{x}_1(0) = \dot{x}(0) = 0\,; & \dot{\varphi}(0) = \dot{\psi}(0) = 0\,; \\ x_1(t_n) = x(t_n) = x_{t_n}\,; & \varphi(t_n) = \psi(t_n) = \psi_{t_n}\,; \\ \dot{x}_1(t_n) = \dot{x}(t_n) = v\,; & \dot{\varphi}(t_n) = \dot{\psi}(t_n) = \omega\,, \end{cases}$$

where x_{t_n} and ψ_{t_n} are linear and angular positions, respectively, of the trolley and the load (in the radial direction) and the crane towers and the load (in the tangential direction) at the end of crane acceleration.

We reduce the boundary conditions (14) to the functions $x = x(t)$ and $\psi = \psi(t)$ and their derivatives with time

$$(15)\qquad \begin{aligned} &x(0) = x_0\,;\ \dot{x}(0) = \ddot{x}(0) = x(0) = 0\,; \\ &\psi(0) = \dot{\psi}(0) = \ddot{\psi}(0) = \psi(0) = 0\,; \\ &x(t_n) = x_{t_n}\,;\ \dot{x}(t_n) = v\,;\ \ddot{x}(t_n) = (x_0 + \Delta x)\omega^2\,;\ x(t_n) = -3v\omega^2\,; \\ &\psi(t_n) = \psi_{t_n}\,;\ \dot{\psi}(t_n) = \omega\,;\ \ddot{\psi}(t_n) = -\frac{2v\omega}{x_0 + \Delta x}\,;\ \dddot{\psi}(t_n) = \frac{6v^2\omega}{(x_0 + \Delta x)^2} - 2\omega^3\,, \end{aligned}$$

where Δx is the distance that the trolley should cover during the duration of start t_n.

The stationary value of the integral criterion of the functional (13) under boundary conditions (15) is the Poisson equation.

In addition, in the problem statement of optimal motion control of both mechanisms, we use constraints on the overload capacity of their engines

$$(16)\qquad \begin{aligned} &\frac{M_{\max}}{M_n} \le \lambda_1\,; \\ &\frac{F_{\max}}{F_n} \le \lambda_2\,. \end{aligned}$$

where M_{max} and M_n are maximum and nominal torques of the crane mechanism; F_{max} and F_n are maximum and nominal driving force of the engine of the trolley movement mechanism; λ_1 and λ_2 are the overload capacities of the slewing mechanism engine and the trolley movement mechanism engine respectively.

Taking into account the constraints (16) and the requirement of minimizing criterion (13), a generalized criterion was developed:

$$Cr = P_{\text{ck}} + t_n \delta_t \delta_1 + (\tilde{C}r_1 + \tilde{C}r_2)\delta_2 \,, \tag{17}$$

where δ_1, δ_2 are weight coefficients, which show the importance of minimizing the corresponding components (this study assumes that $\delta_1 = 10^4$ and $\delta_2 = 10^6$, which made it possible to meet the conditions (16) and minimize the duration of the transient mode of the system motion); $\tilde{C}r_1$ and $\tilde{C}r_2$ are criteria that take into consideration the first and second constraints (16). They are determined by the following dependencies:

$$\begin{aligned}
&\tilde{C}r_1 = \begin{cases} \dfrac{\text{max}}{n} & \text{if } \dfrac{\text{max}}{n} > \lambda_1 \,; \\ 0 & \text{if } \dfrac{\text{max}}{n} \le \lambda_1 \,; \end{cases} \\
&\tilde{C}r_2 = \begin{cases} \dfrac{F_{\text{max}}}{F_n} & \text{if } \dfrac{F_{\text{max}}}{F_n} > \lambda_2 \,; \\ 0 & \text{if } \dfrac{F_{\text{max}}}{F_n} \le \lambda_2 \,. \end{cases}
\end{aligned} \tag{18}$$

The essence of the criteria $\tilde{C}r_1$ and $\tilde{C}r_2$ is that they are quite big in the case when conditions (16) are not met. If constraints (16) are met, then the criteria are abruptly reduced and become zero. Thus, the topology of criterion (17) is quite complex.

To solve this optimization problem, we use the ME-PSO method [15]. To do this, we should set the basis functions that will be used to find an approximate solution to the optimization problem. The basis function for the trolley movement mechanism movement is a solution of the boundary problem

$$\begin{aligned}
&\overset{\text{IV}}{x} = 0 \,; \\
&x(0) = x_0 \,; \quad \dot{x}(0) = \ddot{x}(0) = \dddot{x}(0) = 0 \,; \\
&x(t_n/2) = x_{t_n/2} \,; \\
&x(t_n) = x_0 + \Delta x \,; \quad \dot{x}(t_n) = v \,; \quad \ddot{x}(t_n) = (x_0 + \Delta x)\omega^2 \,; \quad \dddot{x}(t_n) = -3v\omega^2 \,,
\end{aligned} \tag{19}$$

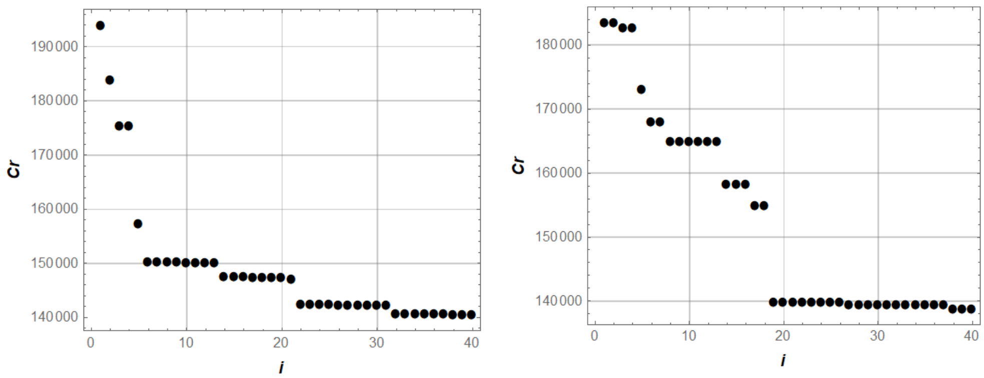
where $x_{t_n/2}$ is the radial position of the load at the moment t_n/2.

The basis function for the coordinate ψ is the solution similar to (19) boundary value problem

$$(20)\quad \begin{aligned} &\overset{\text{IV}}{\psi} = 0; \\ &\psi(0) = 0;\ \dot{\psi}(0) = \ddot{\psi}(0) = \dddot{\psi}(0) = 0; \\ &\psi(t_n/2) = \psi_{t_n/2}; \\ &\psi(t_n) = \psi_{t_n};\ \dot{\psi}(t_n) = v;\ \ddot{\psi}(t_n) = -\frac{2v\omega}{x_0 + \Delta x};\ \dddot{\psi}(t_n) = \frac{6v^2\omega}{(x_0 + \Delta x)^2} - 2\omega^3, \end{aligned}$$

where $\psi_{t_n/2}$ is the tangential position of the load at the moment $t_n/2$. The final conditions for the load movement (in both boundary-value problems) are set in a manner that its oscillations on the flexible suspension at the time t_n must be absent.

The ME-PSO method parameters for solving the problem and argument search area for both basis functions are: number of iterations – 40; swarm population – 30; criterion's minimization acceptable rate – 0.05; $\Delta\psi = 4\omega, \ldots, 10\omega$; $\psi_{t_n/2} = 0, \ldots, 5\omega$; $\Delta x = 4v, \ldots, 10v$; $x_{t_n/2} = 0, \ldots, 5v$; $t_n = 2, \ldots 10$ s. Approximate solutions were obtained for two variants: the trolley moves from the tower and in the opposite direction. All calculations were performed on the basis of system parameters: $l = 3$ m; $J = 5.5 \times 10^6$ kg m^2; $m_1 = 300$ kg; $m = 5{,}000$ kg; $v = 0.84$ m/s (in case of movement towards the tower $v = -0.84$ m/s); $\omega = 0.066$ rad/s; the starting position of the trolley: for the first case $x_0 = 3$ m, for the second case $x_0 = 30$ m. The parameters correspond to the Liebherr 140 hc tower crane [16]. As a result of this method application such parameter values were obtained for the first variant: $\Delta x = 1.68$ m; $x_{t_n/2} = 1.61$ m; $\Delta\psi = 0.322$ rad; $\psi_{t_n/2} = 0.04$ rad and the acceleration duration $t_n = 6.49$ s. For the second case, the following values of



(a) (b)

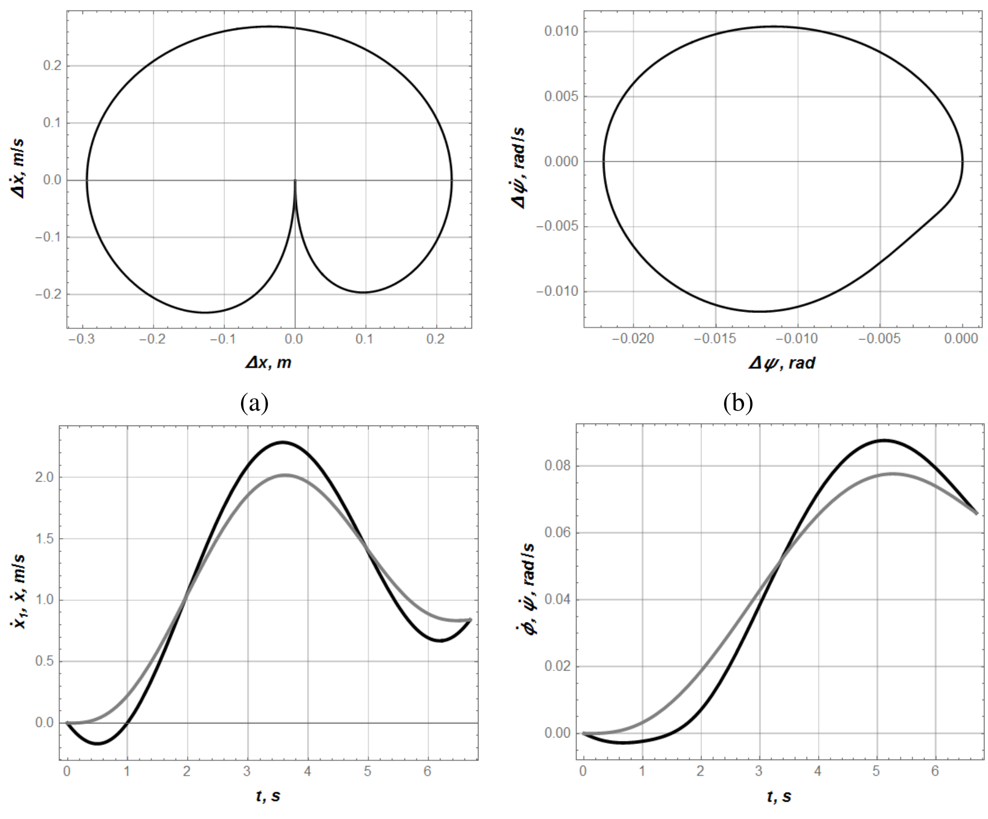
Fig. 2: Plots of reduction the *Cr* criterion for the first (a) and the second (b) cases.

the basic function parameters were obtained: $\Delta x = -6.15$ m; $x_{t_n/2} = 25.80$ m; $\Delta\psi = 0.219$ rad; $\psi_{t_n/2} = 0.04$, and the acceleration duration $t_n = 6.56$ s. The efficiency of the algorithm is confirmed by plots showing the decrease of the Cr criterion when using the ME-PSO algorithm (Fig. 2).

At the first iterations of the algorithm, a rapid minimization of the *Cr* criterion value is seen. The generalized criterion is being reduced over almost all iterations.

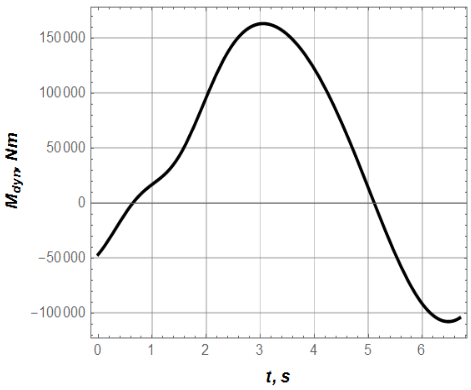
In order to illustrate the results obtained in Fig. 3 and Fig. 4, the graphical dependencies of the kinematic, dynamic, and energy characteristics of the trolley and slewing mechanisms movement of the Liebherr 140 hc tower crane are shown below.

The analysis of the data given in Table 1, as well as the plots in Fig. 3 and Fig. 4 allows us to state, that the applied technique makes it possible to find the approximate solution to the optimization problem of the simultaneous work of both mechanisms, taking into account the constraints caused by their drives.

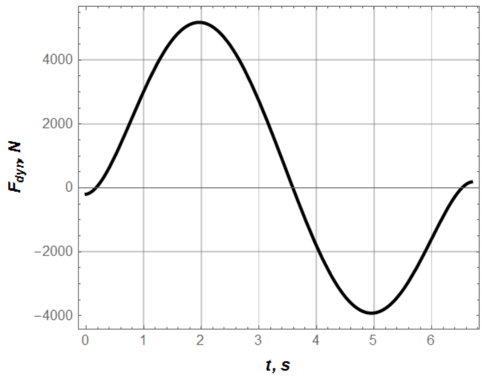


(a) (b)

(c) (d)

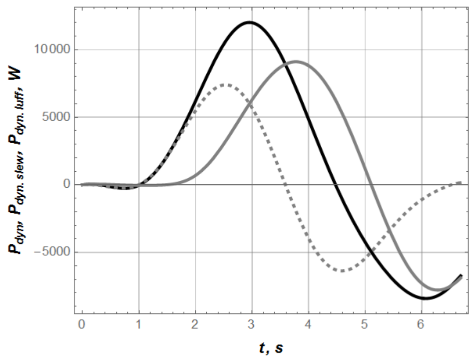


(e)



(f)

Fig. 3: Plots of kinematic, dynamical, and energy characteristics of trolley movement and crane slewing mechanisms for the case of trolley movement from the tower: the phase trajectory of the load oscillation in the radial direction (a); the phase trajectory of the load oscillation in the tangential direction (b); the speed of trolley movement (black line) and load speed in the radial direction (gray line) (c); the speed of crane slewing (black line) and the load slewing in the tangential direction (gray line) (d); the dynamic component of the driving torque of the crane slewing mechanism (e); the dynamic component of the driving force of the trolley movement mechanism (e); the dynamic component of the drive power of trolley movement mechanism (gray dashed line), the dynamic component of the drive power of the crane slewing mechanism (gray line) and the sum of these powers (black line) (g).



(g)

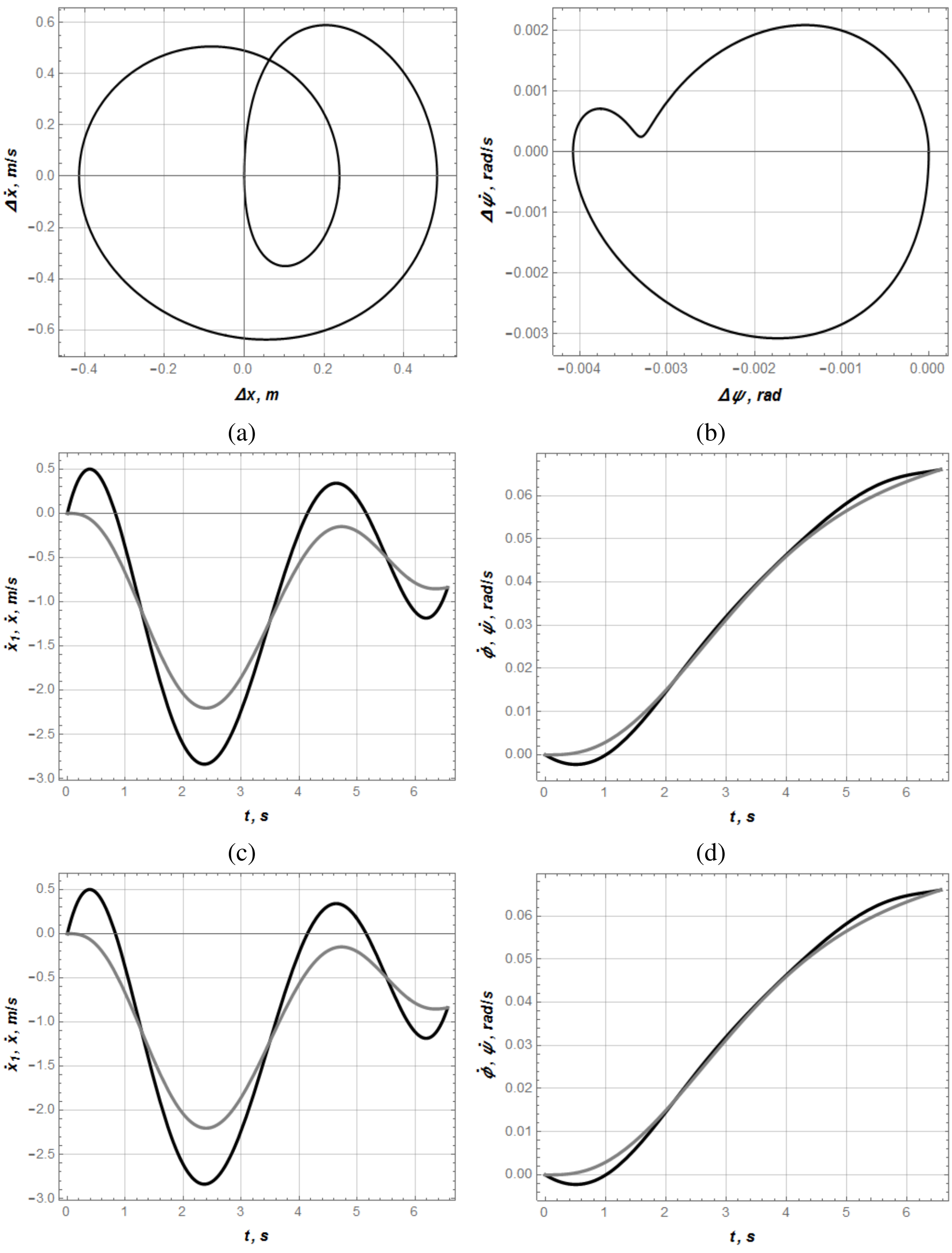
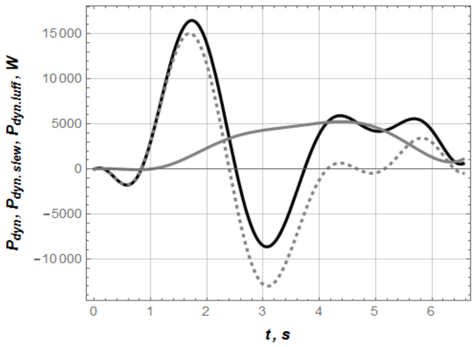

Δẋ, m/s
Δx, m
(a)
Δψ̇, rad/s
Δψ, rad
(b)
ẋ1, ẋ, m/s
t, s
(c)
φ̇, ψ̇, rad/s
t, s
(d)
ẋ1, ẋ, m/s
t, s
(e)
φ̇, ψ̇, rad/s
t, s
(f)

Fig. 4: Plots of kinematic, dynamical, and energy characteristics of trolley movement and slewing mechanisms for the case of trolley movement toward the tower: the phase trajectory of the load oscillation in the radial direction (a); the phase trajectory of the load oscillation in the tangential direction (b); the speed of trolley movement (black line) and load speed in the radial direction (gray line) (c); the speed of crane slewing (black line) and load slewing in the tangential direction (gray line) (d); the dynamic component of the driving torque of the crane slewing mechanism (e); the dynamic component of the driving force of the trolley movement mechanism (e); the dynamic component of the drive power of trolley movement mechanism (gray dashed line), the dynamic component of the drive power of the crane slewing mechanism (gray line) and the sum of these powers (black line) (g).



(g)

For the numeral evaluation of the optimal modes of crane slewing and trolley movement, the data given in Table 1 were calculated.

The difference in the values of the evaluation indexes (Table 1) is due to the action of centrifugal force acting on the trolley and the load in the radial direction. The dynamical, energy and kinematic characteristics of the found laws of mechanisms motion are smooth, which confirms the possibility of their practical implementation by means of controlled asynchronous electric drive.

Negative values of power (Table 1) may be explained in the following way: kinetic energy of the system transforms into the electric form (motor is exploited as a generator). Negative power shows a decrease in system's energy. There are no technical problems in such transformation in the case when a return inverter is used in the structure of a controlled electrical drive.

Table 1: Numerical values of estimation indexes of the optimal motion laws

Parameter	Trolley movement	
	from the tower	toward the tower
Maximum torque of the slewing mechanism, Nm	163.193	159.790
Minimum torque of the slewing mechanism, Nm	-107.844	-47848
Maximum (reduced) force of the trolley movement mechanism, N	5.177	7.410
Minimum (reduced) force of the trolley movement mechanism, N	-3.917	-8.690
Maximum of the total power of both mechanisms, W	11.351	13.633
Minimum of the total power of both mechanisms, W	-8.701	-6.668
Maximum deviation in positions of the trolley and the load in the radial direction, m	0.2941	0.4842
Maximum deviation in positions of the boom and the load in the tangential direction, rad	0.0218	0.0041
RMS value of the driving torque of the crane slewing mechanism, Nm	96.529	102.942
RMS value of the driving force of the trolley movement mechanism, N	3.103	4.934
RMS of the total power of both mechanisms, W	6.643	6.010
RMS value of the deviation in the position of the trolley and the load, m	0.0128	0.0028
RMS value of the deviation in the position of the boom and the load, rad	0.1760	0.2752

3 Conclusions

1. There has been carried out the synthesis of the mathematical model of the simultaneous operations of the tower crane slewing and trolley movement, which is represented by the system of second-order nonlinear differential equations.

2. The problem of the mechanism movement optimal control has been formulated. It includes the criterion of energy consumption during the start-up of the mechanisms; the requirement to ensure the overload capacity of the mechanism drives; boundary conditions corresponding to the elimination of the load oscillations in the radial and tangential directions at the end of the mechanism's acceleration.

3. A generalized criterion was developed to ensure that the constraints were met. It includes the penalty coefficients required to ensure that the system movement was

constrained. In addition, the basic functions were selected to search for approximate solutions to the optimization problem.

4. Using the ME-PSO metaheuristic method, the parameters of the basic functions and the duration of the transition mode were found. These parameters minimize the value of the generalized criterion. All the calculations were made for the cases of the trolley moving from the tower rotation axis and toward it.

5. Based on the graphical dependencies, and a complex of estimation indexes, which refer to the Liebherr 140 hc tower crane, the analysis of the obtained results has been carried out. Furthermore, there has been established the smoothness of the change of system motion characteristics. It improves the reliability of the mechanisms. For instance, during starting process RMS of the total power of both mechanisms varies in the range 6.010–6.643 W; the maximum of the total power of mechanisms is in the range 11.351–13.633 W. In addition, eliminating the load oscillation in radial and tangential directions allows increasing the crane productivity, reduces the hoistman's workload, and provides the possibility of automating load movement operations.

6. Further investigations in this direction are connected with the confirmation of the theoretical results by carrying out experimental studies. The results of experiments and their comparison with the data in the current investigation will be given in the next author's works.

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