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SOLID MECHANICS

INFLUENCE OF THE GEOMETRICAL IMPERFECTIONS IN THE CYLINDRICAL STEEL SHELLS LOADED BY AXIAL COMPRESSION

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ABSTRACT: The classical analytical expression for critical compression stresses in the cylindrical shells in meridional direction $\sigma_{x,\mathrm{Rcr}}$ has been written by Lorenz, Timoshenko and Southwell at the beginning of XX century. During the natural experiments with those shells done later, it is obtained out that the stresses which cause the loss of stability are considerably different from those calculated by the analytical expression. In 1945 Koiter suggested that the primary reason for these differences are the imperfections in the shells. M. Tekleab attempted in her PhD thesis to determine the most unfavourable geometrical imperfection in the shells, pressured by their axis. Having encountered many incorrect results and conclusions in scientific publications, the author has decided to check the obtained conclusions.

KEY WORDS: thin-walled shells, axial (meridional) stresses, imperfections, buckling.

1 INTRODUCTION

The classical analytical expression of the critical compression stresses in the cylindrical shell in meridional direction $\sigma_{x,\mathrm{Rcr}}$ has been written by Lorenz [1], Timoshenko [2] and Southwell [3] and is

(1) $$\sigma_{x,\mathrm{Rcr}} = \frac{Et}{R\sqrt{3(1-\nu^2)}} \approx 0.605E\frac{t}{R}\,,$$

where is the modulus of elasticity of the steel; t – the thickness of the wall of the cylindrical shell; R – the radius of the cylindrical shell with a circular base in plan; $\nu = 0.3$ – Poisson's ratio for a carbon steel.

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From formula (1), it can be concluded that the critical normal stresses $\sigma_{x,\mathrm{Rcr}}$ in the classical shells without imperfection in them, does not depend on the yield strength of the steel nor its ultimate strength.

When natural experiments were done later for an accounting of critical stresses in axially loaded thin-walled cylindrical steel shells, the results were considerably different from those done according to the formula (1). Experimental values for $\sigma_{x,\mathrm{Rcr}}$ reached approximately 1/3 of calculated by analytical way [4]. In 1945 Koiter [5] suggested that the primary reason for the differences between the theoretically and experimentally accounted bearing capacity of the steel shells in case of local loss of stability are imperfections in them. These imperfections are unavoidable because they represent results of the production and mounting processes, necessary for the realization of the steel shells. Many analytical and experimental researches are conducted for an accounting of the influence of the initial imperfections. The conclusions of all of them are that the thin-walled cylindrical shells with circular base are very sensitive to the initial deviations from the ideal cylindrical shape. For instance, the results from the researches of Koiter [6, 7], show that if the amplitude of the initial radial displacement is equal to the thickness of the cylindrical shell, it decreases considerably critical normal stresses. Their values are only 20% from the determined for an ideal cylinder.

Unfortunately, the deviations from the ideal shape may have many forms and amplitudes, which makes determining their most unfavourable type and size a real challenge [8]. The imperfections should be realistic, taking into consideration the way of production of the steel shells. For instance, the current formulas for analytical calculation are obtained empirically, after conducting a lot of laboratory experiments, by which is determined the lowest bearing capacity of the shell before buckling. As the researched samples are produced in laboratories, their quality and imperfections are completely different from full-scaled, real shells [9].

Generally speaking, the imperfections considerably decrease the bearing capacity of the steel shells, but sometimes, the deep imperfections can cause an increase in the shell's stability. As an example, could be mentioned very deep imperfections that cause forms of loss of stability spanning the entire length of the shell [10].

The process for finding out which are the most unfavourable imperfections in the steel shells is additionally complicated due to many forms of buckling which influence each other, which leads to different sensitiveness of geometrical imperfections. For example, many cylindrical shells bear complex loading by axial /meridional/ pressure and radial pressure. The imperfections that lead to the lowest bearing capacity during the axial pressure are completely different to these for radial pressure. The first one is local, but the second must be defused on a large length. When we take into account that the real shells are loaded by non-equally distributed or concentrated loads, it becomes problematic to determine which their most unfavourable form is [9].

According to the standard EN 1993-1-6:2007 [11], the maximum depth $\Delta\omega_{0x}$ of the imperfections depends on the length l_{gx} (Fig. 3). They could be calculated according to the formulae:

$$\Delta\omega_{0x} = U_{0x} l_{gx}\,, \tag{2}$$

$$l_{gx} = 4\sqrt{Rt}\,. \tag{3}$$

The dimple tolerance parameter U_{0x} depends on the quality of execution of the cylindrical shell and its maximum values could be seen in Table 1.

Table 1: Value of the parameter $U_{0x,\max}$ [11]

Fabrication tolerance quality class	Description	$U_{0x,\max}$
Class	excellent	0.006
Class	high	0.010
Class C	normal	0.016

In her thesis M. Tekleab [12] has attempted to determine the most unfavourable geometrical imperfections in a cylindrical shell that bear an axial compression. There, in a case when the depth of the imperfections is $\Delta\omega_{0x} = t$, the following conclusions are reached:

a) the most unfavourable are the imperfections with a length approximately $l_{gx} = 2\sqrt{Rt}$;

b) directed outward imperfections are more unfavourable than inward.

Also, the increase of the values of the depth of imperfections $\Delta\omega_{0x}$ leads to the lower bearing capacity at axial pressure. Here again more unfavourable are imperfections directed to outward and having a length $l_{gx} = 2\sqrt{Rt}$.

2 Analysis

Having encountered many incorrect results and conclusions in scientific publications, the author has decided to check the previously obtained conclusions. For this purpose, he used the data of conducted by laser scanning [13] geodesic survey of the deviation of the cylindrical shell of a tank in service for fuel oil with volume $V = 5,000$ m^3. The tank in question is produced and mounted by roll method, according to the Standard working documentation [14] and it is put into operation in

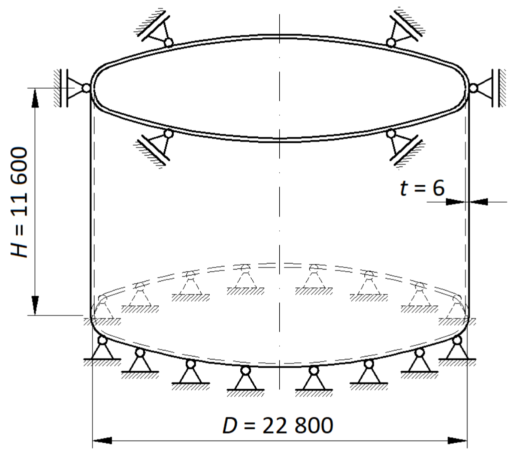


Fig. 1: Researched shell. Dimensions and boundary conditions.

1986. The facility is placed out of service, emptied and cleaned after a leak in the area of the foundation. A geodesic survey is done by laser scanning and deviations from the design cylindrical shape are determined. Data for 72 points, located in four vertical and nine horizontal planes, are provided.

For the current research, a model of the steel tank's shell is created, using the software product ANSYS [15]. Its parameters are as follow:

a) diameter up to the bottom $D = 22,800$ mm and height $H = 11,600$ mm (Fig. 1);

b) accepted thickness of the shell is constant and has a value $t = 6$ mm, as it is in the upper courses of this type of tank;

c) the restrains of edges are as follows:

- lower end – the movement of the edge is restrained in all directions, but with possibilities for free rotation;
- upper end – the movement of the edge is restrained in the horizontal plane, but with possibilities for free rotation and displacement on the vertical axis.

d) the design values of the vertical loads, accepted in the first step of the current research, are:

- snow on the roof – $s = 1.5$ kPa;

- negative pressure (vacuum) applied only on the roof, i.e. axially – $p_v = 0.5$ kPa.

The total load is applied on the upper edge of the shell and it is a uniform, equally distributed load q, with a value, determined by

(4) $$q = (s + p_v)\frac{D}{4} = (1.5 + 0.5)\frac{22.8}{4} = 11.4 \text{ kN/m}.$$

Meridional (axial) normal stress $\sigma_{x,\mathrm{Ed}}$ is

(5) $$\sigma_{x,\mathrm{Ed}} = \frac{q}{t} = \frac{11.4}{0.006} = 1,900 \text{ kN/m}^2.$$

e) 3D quad element SHELL281 is used for the model of the shell. The method of its creation is the "Quadrilaterals". The finite elements are quadrilateral with eight joints – in the edges and the middle of the side. The maximum dimension of the elements is 200 mm;

f) in separate, independent sub-models of the cylindrical shell are imported geometrical imperfections. They are:

- real – the measured through laser scanning deviation from the ideal cylindrical shape;
- synthetic – determined according to the instructions of the standard EN 1993-1-6:2007. They are symmetrically positioned toward the vertical axis on the entire circumference of the shells. They begin at a distance $0.1 = 1,160$ mm from the lower edge of the shell.

g) the lengths l_{gx} of the synthetic imperfections have two values:

- $l_{gx,1} = 4\sqrt{Rt} = 4\sqrt{11,400 \cdot 6} = 1,046$ mm,
- $l_{gx,2} = 2\sqrt{Rt} = 2\sqrt{11,400 \cdot 6} = 523$ mm.

h) the radial dimples $\Delta\omega_0$ of the synthetic imperfections are:

- $\Delta\omega_{0,1} = t = 6$ mm;
- $\Delta\omega_{0,2} = 0.5\Delta\omega_{0,4} = 0.5 \times 16.74 = 8.37$ mm;
- $\Delta\omega_{0,3} = 2t = 2 \times 6 = 12$ mm;
- $\Delta\omega_{0,4} = U_{0x} \times l_{gx,1} = 0.016 \times 1,046 = 16.74$ mm, according to EN1993-1-6:2007;
- $\Delta\omega_{0,5} = 3t = 3 \times 6 = 18$ mm.

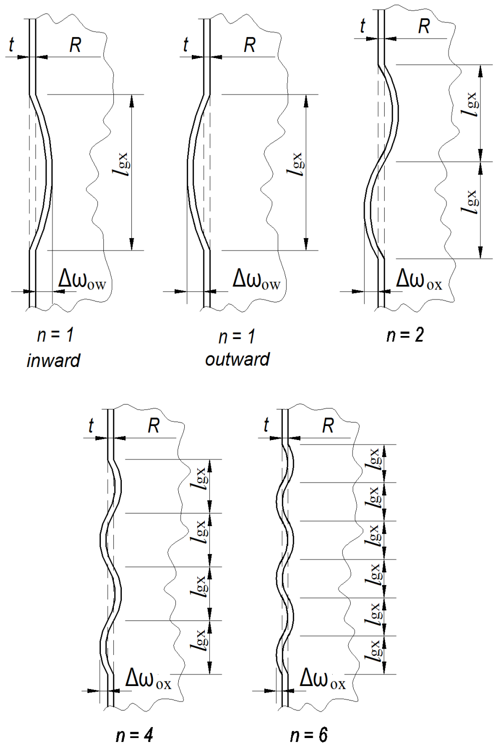


Fig. 2: Geometrical imperfections in the meridional direction of the shell. Shapes.

i) the synthetic geometrical imperfections have the shapes, shown in Fig. 2.

j) the thin-walled shells are sensitive to effects of change of the geometry during loading. On that reason, all models are researched considering the geometrical nonlinearity (GNA);

k) the material for the shell is steel S235 with mechanical properties according to the standard EN 10025-2:2004 [16], and precisely:

- yield strength – $f_y = 235$ MPa;
- modulus of elasticity – $= 210,000$ MPa;
- coefficient of Poisson – $\nu = 0.3$.

The current research is carried out for every modelled shell, having a various number of half-waves n and depth of imperfections $\Delta\omega_{ox}$. The study consists of two parts. In the first of them is activated the option "Buckling Analysis". With this

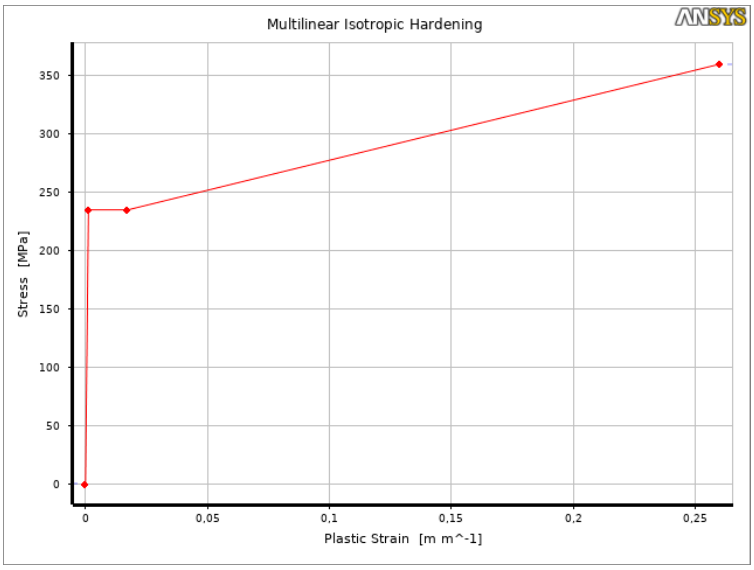


Fig. 3: Idealized diagram $\sigma - \varepsilon$. Shapes.

option is possible to calculate the reserve of the carrying capacity K (eigenvectors) in elastic (linear) behaviour of the steel. It gives us information about the influence of various geometric imperfections (GNA) on the bearing capacity of the axially loaded cylindrical shells and linear behaviour of the material.

In the second part of the study is included the material nonlinearity of the steel (MNA). For this purpose, the relation between stresses σ and strain ε is modelled as a multilinear (Fig. 3). On the upper edge of cylindrical shells are assigned vertical displacements and the resulting vertical forces in the bases of the shells are accounted. During the time of loading these base reactions looks like diagrams in Fig. 4. The biggest one is the carrying capacity of axially compressed shell with imperfections, before loss of stability.

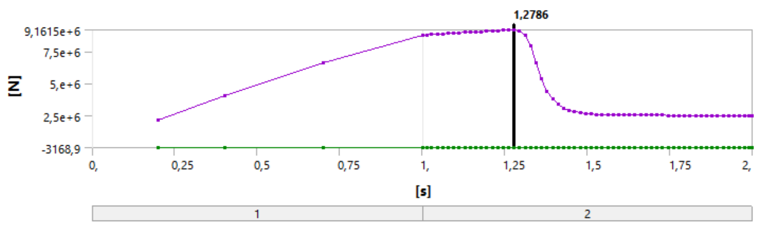


Fig. 4: Change of the vertical reaction in the base during the loading. Shapes.

3 RESULTS

First, the reserve of bearing capacity K (eigenvectors) of the shells before buckling in the first mode is reported (Fig. 5). The results of the research are shown in Tables 2 and 3.

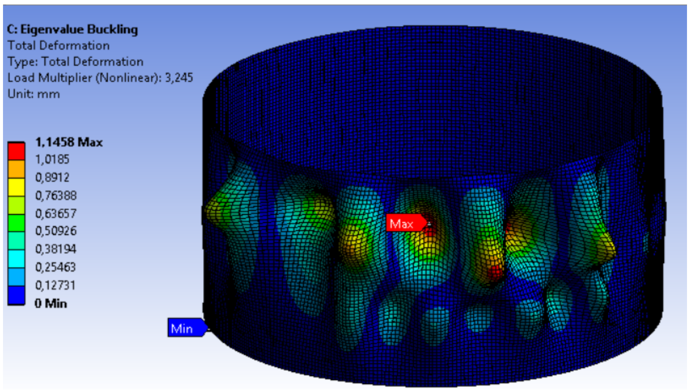


Fig. 5: Buckling of real cylindrical shell in the first mode.

Table 2: Value of the reserve K when the length of imperfection is $l_{gx,1} = 1,046$ mm

	$\Delta\omega_{0x}/t$				
Number of half-waves	1	1.395	2	2.79	3
1 outward	17.145	13.636	10.494	8.517	8.197
1 inward	12.306	10.605	10.771	13.01	13.709
2	9.28	7.830	7.968	9.505	10.049
4	6.95	5.151	4.001	3.57	3.58
6	6.123	4.338	3.139	2.516	2.302

Table 3: Value of the reserve K when the length of imperfection is $l_{gx,2} = 523$ mm

	$\Delta\omega_{0x}/t$				
Number of half-waves	1	1.395	2	2.79	3
1 outward	17.191	14.033	11.323	9.612	9.338
1 inward	14.235	13.905	15.289	17.777	18.388
2	11.182	11.163	11.889	12.767	12.983
4	7.434	6.283	5.627	5.391	5.344
6	6.613	6.069	4.935	5.023	4.066

When the values of the axial normal stresses $\sigma_{x,\mathrm{Ed}}$ within the shell and the reserve of bearing capacity K are known, through the formula (6) we could calculate the critical normal stresses $\sigma_{x,\mathrm{Rk,FEA}}$ in the axially loaded shell before buckling

$$\sigma_{x,\mathrm{Rk,FEA}} = K \cdot \sigma_{x,\mathrm{Ed}}. \tag{6}$$

The values of $\sigma_{x,\mathrm{Rk,FEA}}$ are shown in Table 4 and Table 5.

The coefficient for the reserve of bearing capacity of the really measured by laser scanning shell at the applied in this manner load is $K = 4.245$.

The critical normal stresses in the shell with really measured deformations through laser scanning is $\sigma_{x,\mathrm{Rk,FEA}} = 0.806$ kN/cm^2.

The changes of the bearing capacity of the shell with imperfections $\sigma_{x,\mathrm{Rk,FEA}}$, related to the bearing capacity of the ideal cylindrical shell $\sigma_{x,\mathrm{Rcr}}$, is shown in Fig. 6.

There, in steel shells with an elastic material and geometrical imperfections, could be seen:

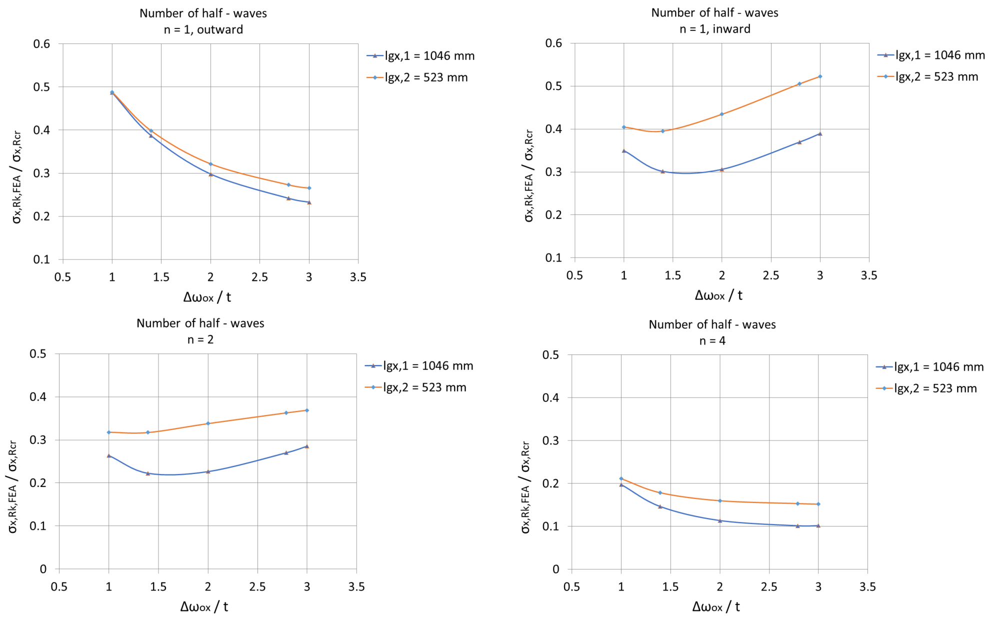
a) the imperfections with length $l_{gx,1} = 4\sqrt{Rt}$ are always more unfavourable than the imperfections with length $l_{gx,2} = 2\sqrt{Rt}$;

Table 4: Critical normal stresses in the shell $\sigma_{x,\mathrm{Rk,FEA}} = K \cdot \sigma_{x,\mathrm{Ed}}$ kN/cm^2, when the imperfection' length is $l_{gx,1} = 1,046$ mm

	$\Delta\omega_{ox}/t$				
Number of half-waves	1	1.395	2	2.79	3
1 outward	3.258	2.591	1.994	1.618	1.557
1 inward	2.338	2.015	2.046	2.472	2.605
2	1.763	1.488	1.514	1.806	1.909
4	1.320	0.979	0.760	0.678	0.680
6	1.163	0.824	0.596	0.478	0.437

Table 5: Critical normal stresses in the shell $\sigma_{x,\mathrm{Rk,FEA}} = K \cdot \sigma_{x,\mathrm{Ed}}$ kN/cm^2, when the length of the imperfection is $l_{gx,2} = 523$ mm

	$\Delta\omega_{ox}/t$				
Number of half-waves	1	1.395	2	2.79	3
1 outward	3.266	2.666	2.151	1.826	1.774
1 inward	2.705	2.642	2.905	3.378	3.494
2	2.125	2.121	2.259	2.426	2.467
4	1.412	1.194	1.069	1.024	1.015
6	1.256	1.153	0.938	0.954	0.772



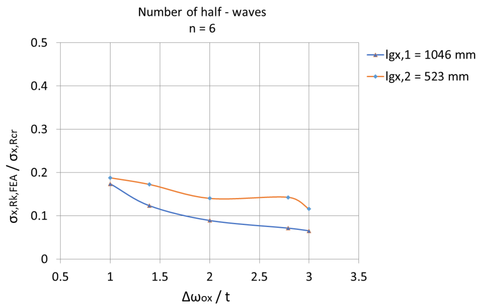


Fig. 6: Change in the ratio $\sigma_{x,\text{Rk,FEA}}/\sigma_{x,\text{Rcr}}$ depending on the depth $\Delta\omega_{0x}$ of the imperfection.

b) comparing the bearing capacity of the shells with really measured geometrical imperfections and those with synthetic imperfections, where $\Delta\omega_{0x} = U_{0x} l_{gx,1}$, the closest values are accounted when:

- the length of the imperfections is $l_{gx,1} = 4\sqrt{Rt}$ and $n = 4$ half-waves;
- the length of the imperfections is $l_{gx,2} = 2\sqrt{Rt}$ and $n = 6$ half-waves.

c) directed outward single half-wave imperfections are more unfavourable than inward when ratio $\Delta\omega_{ox}/t \geq 2$.

In the second part of the research, when the material non-linearity is included, the accounted results are shown in Table 6 and Table 7.

Table 6: Critical normal stresses in the shell $\sigma_{x,\mathrm{Rk,FEA},pl}$ kN/cm^2, when the imperfection' length is $l_{gx,1} = 1,046$ mm

	$\Delta\omega_{0x}/t$				
Number of half-waves	1	1.395	2	2.79	3
1 outward	2.627	2.581	2.360	2.085	2.026
1 inward	2.657	2.429	2.197	2.257	2.328
2	3.130	2.856	2.556	2.252	2.212
4	3.025	2.737	2.353	1.921	1.834
6	2.942	2.602	2.132	1.680	1.579

Table 7: Critical normal stresses in the shell $\sigma_{x,\mathrm{Rk,FEA},pl}$ kN/cm^2, when the imperfection' length is $l_{gx,1} = 523$ mm

	$\Delta\omega_{0x}/t$				
Number of half-waves	1	1.395	2	2.79	3
1 outward	2.726	2.425	2.082	1.723	1.636
1 inward	2.385	2.481	2.554	2.328	2.264
2	2.790	2.579	2.280	1.970	1.905
4	2.546	2.216	1.797	1.396	1.312
6	2.327	1.963	1.516	1.129	1.053

The critical normal stresses in the shell with really measured deformations through laser scanning is $\sigma_{x,Rk,FEA,pl} = 1.612$ kN/cm^2.

The changes of the bearing capacity of the shell with imperfections $\sigma_{x,Rk,FEA,pl}$, related to the bearing capacity of the ideal cylindrical shell $\sigma_{x,Rcr}$, is shown in Fig. 7.

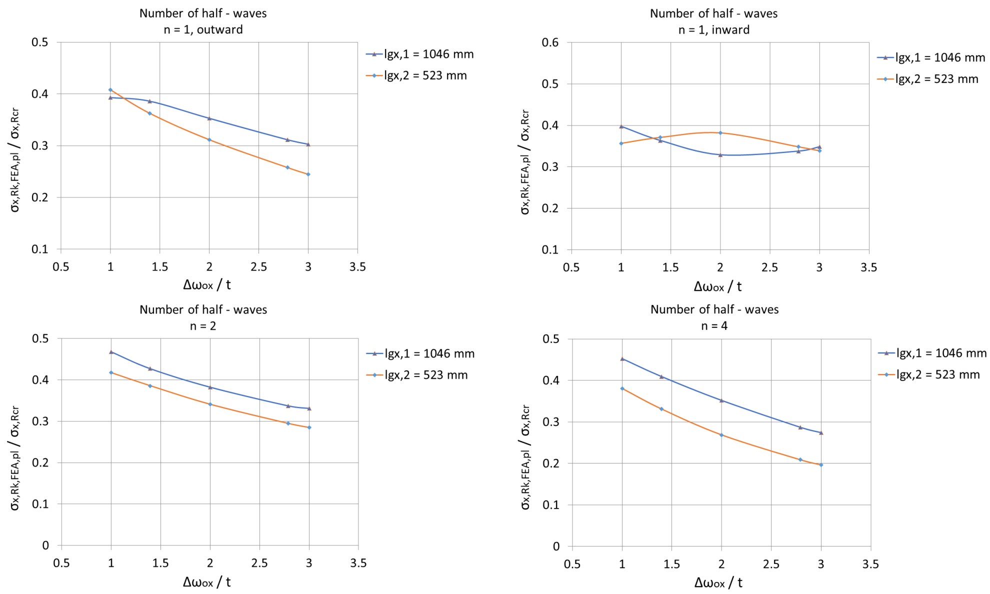
In the second part of the analysis, in steel shell with a nonlinear material and geometrical imperfections, could be seen:

a) the imperfections with length $l_{gx,2} = 2\sqrt{Rt}$ in the most cases are more unfavourable than the imperfections with length $l_{gx,1} = 4\sqrt{Rt}$;

b) comparing the bearing capacity of the shells with really measured geometrical imperfections and those with synthetic imperfections, where $\Delta\omega_{0x} = U_{0x}l_{gx,1}$, the closest values are accounted when:

   - the length of the imperfections is $l_{gx,1} = 4\sqrt{Rt}$ and $n = 6$ half-waves;
   - the length of the imperfections is $l_{gx,2} = 2\sqrt{Rt}$ and $n = 4$ half-waves.

c) directed outward single half-wave imperfections are more unfavourable than inward when ratio $\Delta\omega_{0x}/t \geq 2.5$.



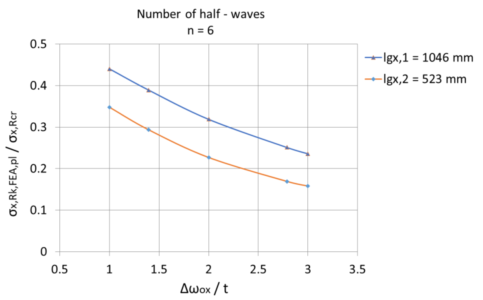


Fig. 7: Change in the ratio $\sigma_{x,\text{Rk,FEA,pl}}/\sigma_{x,\text{Rcr}}$ depending on the depth $\Delta\omega_{0x}$ of the imperfection.

4 Conclusions

From the current research of axially loaded steel shells with diameter $D = 22,800$ mm, height $H = 11,600$ mm, constant thickness $t = 6$ mm and geometrical imperfections, the following conclusions could be done:

a) the imperfections with length $l_{gx,1} = 4\sqrt{Rt}$ are more unfavourable than the imperfections with length $l_{gx,2} = 2\sqrt{Rt}$ when the ratio $\sigma - \varepsilon$ is linear (elastic material);

b) the imperfections with length $l_{gx,2} = 2\sqrt{Rt}$ are more unfavourable than the imperfections with length $l_{gx,1} = 4\sqrt{Rt}$ when the nonlinear behaviour of the steel is included into analysis;

c) generally, the increase of the imperfections' depth $\Delta\omega_{0x}$ leads to decrease of the bearing capacity in case of meridional compression, but here in some of the cases it is exactly contrary;

d) the increase of the number of half-waves in the synthetic imperfections causes a decrease of the bearing capacity of the shell;

e) directed outward single half-wave imperfections are more unfavourable than inward when ratio $\Delta\omega_{0x}/t \geq 2 \div 2.5$.

It could be seen that some of the results of this research, carried on upon a shell with relatively large D and H, are in controversy with the conclusions mentioned in the thesis of M. Tekleab [12]. Therefore, if there are no data from real measurement and the shell has to be researched with synthetic imperfections, it should be checked for several imperfections' lengths and a number of half-waves.

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