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TORSIONAL WAVES PROPAGATION IN MICROPOLAR ELASTIC HALF-SPACE

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ABSTRACT: The possibility of propagation of torsional surface waves in a uniform micropolar elastic solid half-space is explored. For very poor value of a micropolar constant $(\mathcal{K})$, there exist three torsional modes propagating with distinct speeds. Out of these three torsional modes, only one mode is affected by the material parameters J, μ and γ. In the absence of micropolarity, the torsional surface waves transform into classical shear wave.

KEY WORDS: torsional surface wave, dispersion equation, micropolar, shear, speed.

1 INTRODUCTION

Waves that travel in elastic solid and remain confine near the boundary surface and penetrate very little into that solid are known as surface waves. Rayleigh, Love and Stoneley waves are the most popular surface waves and their literature is available in abundance. However, there exist another kind of surface waves called torsional waves, which are basically the horizontally polarized shear waves, but give remarkable twist to the medium through which they travel. These waves often propagate during earthquakes phenomena and become responsible to some extent for the destruction of the earth's crustal layer. The study of these waves is of great importance to seismologists due to their applications in possible prospects of geophysics and also, in understanding the cause and estimating the disastrous damages due to earthquakes.

Lord Rayleigh [1] was perhaps the first, who investigated torsional surface waves and concluded that these waves do not propagate in a homogeneous elastic solid half-space. Later, Meissner [2] showed that the torsional surface waves do exist in an inhomogeneous elastic half-space with shear modulus varying quadratically and density varying linearly with the depth. Since Meissner [2], the torsional surface waves did not get sufficient attention for a long time. Around half a century later,

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Vardoulakis [3] investigated the dispersion of torsional waves in two types of inhomogeneous elastic media with constant density, but shear modulus: (a) varying linearly with depth co-ordinate and (b) varying as the square-root of the depth coordinate. Dey et al. [4] explored the possibility of torsional surface waves in an elastic half-space with void pores and concluded that there might exist two torsional wavefronts. Later, Dey et al. [5, 6] investigated torsional waves in a non-homogeneous elastic and homogeneous viscoelastic half-spaces. Dey et al. [7] have also studied the propagation of torsional surface waves in an elastic layer with void pores overlying an elastic half-space with void pores. In the absence of superficial layer, they observed that the elastic half-space with void pores allows torsional waves, that does not do so for elastic half-space without voids. Gupta et al. [8] studied the effect of rigid boundary on the propagation of torsional surface waves in a porous elastic layer over a porous elastic half-space. Researches pertinent to the torsional surface waves propagation in various elastic media under different circumstances are contained in the papers by Georgiadis et al. [9], Dey and Sarkar [10], Selim [11], Davini et al. [12], Ozturk and Akbarov [13], Chattaraj [14], Akbarov et al. [15], Vishwakrma and Gupta [16], Gupta et al. [17–19], Kumari and Sharma [20], Gourgiotis and Georgiadis [21], Kundu and Kumari [22], Vaishnav et al. [24], Alam et al. [25], Manna et al. [26], Gupta et al. [27], Kakar and Kakar [28], Tomar and Kaur [29] among several others.

The underlying idea of classical theory of elasticity is that during the deformation process of elastic continua, the tiny particles are allowed to undergo relative displacement and the interaction between adjacent elements of the continua is governed by force stress tensor. The kinematic variable has three degrees of freedom to measure the deformation of the solid body. The stress-strain relation for such solids are governed by generalized Hooke's law. But the classical theory of elasticity is found to be inadequate to explain the experimental results obtained, particulary in the dynamical problems of granular solids under high frequency. To explain the discrepancies between theoretical and experimental results, several non-classical theories were proposed by the then researchers and they are lying in open literature. Micropolar theory of elasticity developed by Eringen [23] is one of the nice extensions of classical theory of elasticity, which is found to be the best suited to the materials having dumbbell shaped molecules. Eringen introduced body micro-inertia effects such that during the deformation process, the tiny molecules can undergo rotation independently without stretch about their center of mass, in addition to the rigid translation, thereby enhancing the degree of freedom from three to six, that is, three of translation plus three of rotation. The interaction between the neighboring elements of the continua are not only governed by the force stress tensor, but also by the couple stress tensor which is responsible for rotational degrees of freedom. The force stress and the couple stress tensors are asymmetric in nature. Based on Eringen's theory of micropolar elasticity, numerous problems of static and dynamical nature have been attempted by the researchers. Table 1 gives the comparison between the classical theory of elasticity and that of the Eringen's micropolar theory of elasticity.

Table 1: Comparison between classical and Eringen's micropolar models

	Classical elasticity	Micropolar elasticity
Degrees of freedom	3 (displacement)	6 (3 of displacement + 3 of rotation)
Stress tensors	Symmetric force stress tensor	Asymmetric force and couple stress tensors
Strain measures	$e_{kl} = \frac{1}{2}(u_{l,k} + u_{k,l})$	$e_{kl} = u_{l,k} - \varepsilon_{klm}\phi_m$, $\gamma_{kl} = \phi_{k,l}$
Material moduli	λ, μ	λ, μ, $\mathcal{K}$, α, β, γ
Constitutive relations	$\sigma_{kl} = \lambda e_{rr}\delta_{kl} + 2\mu e_{kl}$	$\sigma_{kl} = \lambda e_{rr}\delta_{kl} + (\mu + \mathcal{K})e_{kl} + \mu e_{lk}$, $m_{kl} = \alpha\gamma_{rr}\delta_{kl} + \beta\gamma_{kl} + \gamma\gamma_{lk}$
Equations of motion	$\sigma_{kl,k} + \rho(f_l - \ddot{u}_l) = 0$	$\sigma_{kl,k} + \rho(f_l - \ddot{u}_l) = 0$, $m_{kl,k} + \epsilon_{lkm}\sigma_{lm} + \rho(l_l - J\ddot{\phi}_l) = 0$

In the present paper, we have investigated the possibility of propagation of torsional surface waves in an isotropic homogeneous micropolar elastic solid half-space. It is found that there may exist three torsional surface modes under the assumption that the micropolar constant $\mathcal{K}$ is so small that the terms containing its higher order are negligible. Out of the three torsional modes, one is a counterpart of the classical shear wave, while the remaining modes are new and have appeared due to the presence of micropolarity. The speeds of all the torsional modes are found to be affected by the material constants, namely, μ, γ and micro-inertia J. In the absence of micropolar parameters, one of the torsional modes transforms into classical shear mode (SV-mode) and the other two modes disappear. This is in agreement with Meissner [2] who showed that no torsional mode exists in classical elastic half-space.

2 Basic Equations and the Model

The constitutive relations and field equations for an isotropic homogeneous micropolar elastic solid are given by (see [23])

(1) $$\sigma_{kl} = \lambda e_{rr}\delta_{kl} + (\mu + \mathcal{K})e_{kl} + \mu e_{lk},$$

(2) $$m_{kl} = \alpha\gamma_{rr}\delta_{kl} + \beta\gamma_{kl} + \gamma\gamma_{lk},$$

and

$$(\lambda+\mu)\nabla\nabla\cdot\boldsymbol{u}+(\mu+\mathcal{K})\nabla^2\boldsymbol{u}+\mathcal{K}\nabla\times\boldsymbol{\phi}=\rho\ddot{\boldsymbol{u}}, \tag{3}$$

$$(\alpha+\beta)\nabla\nabla\cdot\boldsymbol{\phi}+(\gamma\nabla^2-2\mathcal{K})\boldsymbol{\phi}+\mathcal{K}\nabla\times\boldsymbol{u}=\rho J\ddot{\boldsymbol{\phi}}, \tag{4}$$

where σ_{kl} and m_{kl} are respectively, the force stress and couple stress tensors; $e_{kl}(=u_{l,k}-\varepsilon_{klm}\phi_m)$ denotes the relative distortion tensor, $\gamma_{kl}(=\phi_{k,l})$ is the curvature or wryness tensor. $\boldsymbol{u}$ and $\boldsymbol{\phi}$ represent the displacement and micro-rotation vectors respectively; ρ and J are the density and micro-inertia of the micropolar solid, respectively; λ, μ are the Lame's constants; $\mathcal{K}$, α, β and γ are the micropolar moduli with the following restrictions (page 112 of [23]):

$$3\lambda+2\mu+\mathcal{K}\geq 0,\quad 2\mu+\mathcal{K}\geq 0,\quad \mathcal{K}\geq 0,\quad 3\alpha+\beta+\gamma\geq 0,\quad \gamma+\beta\geq 0,\quad \gamma-\beta\geq 0.$$

The first Cosserat constant $\mathcal{K}$ is the micropolar couple modulus. Superposed dots represent the temporal derivatives and other symbols have their usual meanings. Note that in writing (3) and (4), the body force and body couple densities have been ignored.

Within the context of cylindrical polar coordinate system (r,θ,z), we consider a micropolar elastic solid half-space bounded by $r-\theta$ plane and $z-$ axis pointing vertically downwards. In order to explore the possibility of torsional surface waves in the micropolar solid half-space, we take the components of displacement vector $\boldsymbol{u}=(u,v,w)$, and the micro-rotation vector $\boldsymbol{\phi}=(\phi_1,\phi_2,\phi_3)$ such that

$$u=w=\phi_1=\phi_3=0,\quad v=v(r,z,t),\quad \phi_2=\phi_2(r,z,t),\quad \text{and}\quad \frac{\partial}{\partial\theta}\cdot\equiv 0.$$

With these considerations, equations (3) and (4) reduce to

$$(\mu+\mathcal{K})D^2_{r,z}v+\mathcal{K}D^1_{r,z}\phi_2=\rho\ddot{v}, \tag{5}$$

$$\gamma\, D^2_{r,z}\phi_2-2\mathcal{K}\phi_2+\mathcal{K}D^1_{r,z}v=\rho J\ddot{\phi}_2, \tag{6}$$

where

$$D^1_{r,z}=\frac{\partial}{\partial r}+\frac{1}{r}+\frac{\partial}{\partial z}\quad \text{and}\quad D^2_{r,z}=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}.$$

Introducing the quantities

$$s=\frac{r}{L},\quad p=\xi+\frac{z}{L},\quad \nu=\frac{v}{L},\quad \text{and}\quad \tau=\frac{t}{T}, \tag{7}$$

representing the dimensionless radial coordinate, depth coordinate, transverse displacement and temporal coordinates, respectively. Here, the quantity ξ is non-dimensional constant, the quantity L^{-1} has the dimension of wavenumber, and the quantity

T^{-1} has the dimension of frequency. Employing (7) into equations (5) and (6), we obtain

$$D^2_{s,p}\nu + \alpha_1 D^1_{s,p}\phi_2 = \alpha_2 \frac{\partial^2 \nu}{\partial \tau^2}, \tag{8}$$

$$D^2_{s,p}\phi_2 - 2\alpha_3\phi_2 + \alpha_3 D^1_{s,p}\nu = \alpha_4 \frac{\partial^2 \phi_2}{\partial \tau^2}, \tag{9}$$

where

$$\alpha_1 = \frac{c_3^2}{c_2^2 + c_3^2}, \quad \alpha_2 = \frac{L^2T^{-2}}{c_2^2 + c_3^2}, \quad \alpha_3 = \frac{\mathcal{K}L^2}{\gamma},$$
$$\alpha_4 = \frac{L^2T^{-2}}{c_4^2}, \quad c_2^2 = \frac{\mu}{\rho}, \quad c_3^2 = \frac{\mathcal{K}}{\rho}, \quad c_4^2 = \frac{\gamma}{\rho J}.$$

For the advancement of time harmonic torsional waves, the relevant components of displacement and micro-rotation vectors can be taken as (see Dey et al. [4])

$$\{\nu, \phi_2\} = \{V(p), \Phi(p)\} J_0(s) e^{\iota\omega\tau}, \tag{10}$$

where $J_0(s)$ is a Bessel's function of first kind and order zero, ω is the non-dimensional frequency defined by $\omega = \Omega T$, Ω is the circular frequency and is connected with the speed of the propagating torsional wave (c_t) through the relation $c_t = \Omega L$. Now, inserting the expressions given in (10) into equations (8) and (9), we obtain

$$\left[\frac{d^2}{dp^2} - (1 - \alpha_2\Omega^2T^2)\right] V + \alpha_1 \left[\frac{J_0'(s)}{J_0(s)} + \frac{1}{s} + \frac{d}{dp}\right] \Phi = 0, \tag{11}$$

$$\left[\frac{d^2}{dp^2} - (1 + 2\alpha_3 - \alpha_4\Omega^2T^2)\right] \Phi + \alpha_3 \left[\frac{J_0'(s)}{J_0(s)} + \frac{1}{s} + \frac{d}{dp}\right] V = 0, \tag{12}$$

where $J_0'(s)$ represents the first order derivative of $J_0(s)$ with respect to s.

Eliminating V and Φ from these equations and using $Z \equiv \frac{d}{dp}$, we obtain

$$(Z^4 + AZ^2 + BZ + C)\{V, \Phi\} = 0, \tag{13}$$

where

$$A = (\alpha_2 + \alpha_4)\Omega^2T^2 - 2(1 + \alpha_3) - \alpha_1\alpha_3,$$
$$B = -2\alpha_1\alpha_3 \left(\frac{J_0'(s)}{J_0(s)} + \frac{1}{s}\right),$$

$$C = (1 - \alpha_2\Omega^2T^2)(1 + 2\alpha_3 - \alpha_4\Omega^2T^2) + \frac{B}{2}\left(\frac{J_0'(s)}{J_0(s)} + \frac{1}{s}\right).$$

The roots of equation (13) are not easy to compute analytically. However, under the assumption that the micropolar constant $\mathcal{K}$ of the medium is so small that its higher powers are negligible, one can find the analytical expressions of the roots. Thus, for small value of $\mathcal{K}$, one can ignore the product of quantities α_1 and α_3, and the equation (13) reduces to

$$(Z^4 + A_1Z^2 + A_2)\{V, \Phi\} = 0, \tag{14}$$

where

$$A_1 = (\alpha_2+\alpha_4)\Omega^2T^2 - 2(1+\alpha_3) \quad \text{and} \quad A_2 = (1-\alpha_2\Omega^2T^2)(1+2\alpha_3-\alpha_4\Omega^2T^2).$$

We see that equation (14) is a quadratic equation in Z^2, whose discriminant is

$$[(\alpha_2 - \alpha_4)\Omega^2T^2 + 2\alpha_3]^2 > 0.$$

Hence, this equation possesses real and distinct roots given by

$$Z_i^2 = r_i, (i = 1, 2),$$

where $r_1 = 1 + 2\alpha_3 - \alpha_4\Omega^2T^2$ and $r_2 = 1 - \alpha_2\Omega^2T^2$. Thus, all the four roots of equation (14) are given by

$$\mp\sqrt{r_1}, \quad \mp\sqrt{r_2}.$$

The general solution of the differential equation (14) is therefore, given by

$$\begin{cases} V(p) = B_1e^{-\sqrt{r_1}p} + B_2e^{-\sqrt{r_2}p} + B_3e^{\sqrt{r_1}p} + B_4e^{\sqrt{r_2}p}, \\ \Phi(p) = E_1B_1e^{-\sqrt{r_1}p} + E_2B_2e^{-\sqrt{r_2}p} + E_3B_3e^{\sqrt{r_1}p} + E_4B_4e^{\sqrt{r_2}p}, \end{cases} \tag{15}$$

where B_i's $(i = 1, 2, 3, 4)$ are arbitrary constants, and E_i's $(i = 1, 2, 3, 4)$ are the coupling coefficients given by

$$\begin{cases} E_{1,3} = \dfrac{1 - \alpha_2\Omega^2T^2 - r_1}{\alpha_1\left(\dfrac{J'_0(s)}{J_0(s)} + \dfrac{1}{s} \mp \sqrt{r_1}\right)}, \\ E_{2,4} = \dfrac{1 - \alpha_2\Omega^2T^2 - r_2}{\alpha_1\left(\dfrac{J'_0(s)}{J_0(s)} + \dfrac{1}{s} \mp \sqrt{r_2}\right)}. \end{cases} \tag{16}$$

In view of the expression of r_2 and $(16)_2$, it can be noticed that $E_{2,4} = 0$.

Now, looking at the expressions of solution (15), we see that as $z \to \infty$, the terms containing negative sign of exponential would go on diminishing, while those having positive sign would blow up unlimitedly provided r_1 and r_2 are positive quantities. Owing to the radiation condition for the waves to be torsional surface waves, the appropriate solutions from (15) are given by

$$\{\nu,\ \phi_2\} = \left\{\left(B_1 e^{-\sqrt{r_1}p} + B_2 e^{-\sqrt{r_2}p}\right),\ E_1 B_1 e^{-\sqrt{r_1}p}\right\} J_0(s) e^{\iota\omega\tau}, \tag{17}$$

where the quantities r_1 and r_2 are positive. For these quantities to be positive, both the roots of equation (14) must be positive. But, the nature of these roots depends on the coefficients A_1 and A_2, whose expressions are real valued. The possibility of occurring two positive real and distinct roots of equation (14) is that $A_1 < 0$ and $A_2 > 0$. The other possibilities of A_1 and A_2 have been ruled out[1].

3 Dispersion Equation

The appropriate boundary conditions are vanishing of stresses on the boundary surface $z = 0$, that is, $\sigma_{p\theta} = m_{p\theta} = 0$, which are written in non-dimensional form as

$$\mu \frac{\partial \nu}{\partial p} = 0, \quad \gamma \frac{\partial \phi_2}{\partial p} = 0 \quad \text{at} \quad p = \xi. \tag{18}$$

Plugging (17) into the boundary conditions given in (18), we obtain the following matrix equation at $p = \xi$

$$\begin{bmatrix} \sqrt{r_1} & \sqrt{r_2}\, e^{(\sqrt{r_1}-\sqrt{r_2})\xi} \\ \sqrt{r_1} E_1 & 0 \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{19}$$

The condition for non-trivial solution of B_1 and B_2, from this matrix equation yields the following dispersion relations for the propagation of torsional waves in a micropolar elastic solid half-space, as:

$$r_1 = 0 \quad \text{or} \quad r_2 = 0 \quad \text{or} \quad E_1 = 0. \tag{20}$$

[1]The possibilities other than $A_1 < 0$ and $A_2 > 0$ are given as follows:

(i) $A_1 > 0$ and $A_2 > 0$;

(ii) $A_1 < 0$ and $A_2 < 0$;

(iii) $A_1 > 0$ and $A_2 < 0$.

By Descarte's rule, in (i) there does not exist any real root. However, in (ii) and (iii), there exist two real and two complex conjugate roots.

Each of these three dispersion relations will provide the speed of a torsional mode and are analysed as follows:

(i) Equation (20)$_1$ yields the speed of torsional mode (c_{t_1}) of the first kind, given as

$$c_{t_1}^2 = c_4^2\left[1 + \frac{1}{(Rk_1)^2}\right], \quad R = \sqrt{\frac{\gamma}{2\mathcal{K}}}, \quad k_1 = \frac{1}{L}. \tag{21}$$

(ii) Equation (20)$_2$ yields the speed of torsional mode (c_{t_2}) of second kind, given as

$$c_{t_2}^2 = c_2^2 + c_3^2. \tag{22}$$

(iii) Equation (20)$_3$ provides the speed of torsional mode (c_{t_3}) of third kind, given as

$$c_{t_3}^2 = \frac{c_4^2}{(Rk_1)^2(1-S)}, \quad S = \frac{c_4^2}{c_2^2 + c_3^2}. \tag{23}$$

It is clear from the formulae (21) and (23) that the speeds of these torsional modes depend on k_1 and hence the corresponding modes are dispersive in nature. While, the formula (22) shows that the speed of this torsional mode does not depend upon wavenumber and hence, the corresponding mode is non-dispersive in nature. It is also clear from this formula that $c_{t_2} \approx c_2$. This is because the contribution of c_3^2 is almost negligible, since the quantity $\mathcal{K}$ is too small.

4 Special Case

If we ignore the micropolarity parameters from the medium, then one shall obtain the corresponding results of classical elasticity. For this purpose, we substitute $\gamma = \mathcal{K} = 0$, into (21)–(23), we obtain $c_{t_1} = c_{t_3} = 0$, and

$$c_{t_2} = c_2 = \sqrt{\frac{\mu}{\rho}}.$$

This is the speed of shear wave in $r - z$ plane and hence this cannot be the speed of SH-wave. It is certainly the speed of SV-wave in the classical elastic solid half-space. Since, c_{t_2} cannot act as a torsional mode as there does not exist any torsional surface wave in uniform elastic solid half-space (see Meissner [2]). Thus, in the absence of micropolarilty, the torsional mode transforms into shear mode of classical elasticity.

5 Numerical Results

To compute the speeds of various torsional modes, a specific model with the following values of material parameters is considered: $\mu = 6.46 \times 10^{11}$ N/m^2, $\gamma = 0.365 \times 10^6$ N, $\rho = 1900$ kg/m^3, and $\mathcal{K} = 0.0125 \times 10^{-3}$ N/m^2. In order to see the effect of micro-inertia J on the speeds of torsional surface modes, we have taken its two different values, namely, $J = 0.0212 \times 10^{-4}$ m^2 and $J = 0.0212 \times 10^{-3}$ m^2. With this numerical data, the non-dimensional speeds of torsional modes are computed against the non-dimensional wavenumber from the formulae given through (21)–(23).

Figure 1 depicts the variation of non-dimensional speeds (c_{t_i}/c_2; $i = 1, 2, 3$) of torsional modes as well as their comparison with respect to distinct values of J, against non-dimensional wavenumber (Rk_1). Here, dotted curves correspond to torsional modes for $J = 0.0212 \times 10^{-3}$ m^2, while solid curves correspond to the torsional modes for $J = 0.0212 \times 10^{-4}$ m^2, and the curves in blue, red and black colors correspond to torsional surface modes of first, second and third kinds, respectively. The speeds of torsional modes of first and third kinds are extremely high in magnitude in the vicinity of zero wavenumber, which falls off very fast continuously with increase of wavenumber and then slowly decrease with further increase of wavenumber. For very high wavenumber, i.e., when $Rk_1 \to \infty$, the first mode becomes constant at the value equal to c_4/c_2 and the third mode diminishes and finally

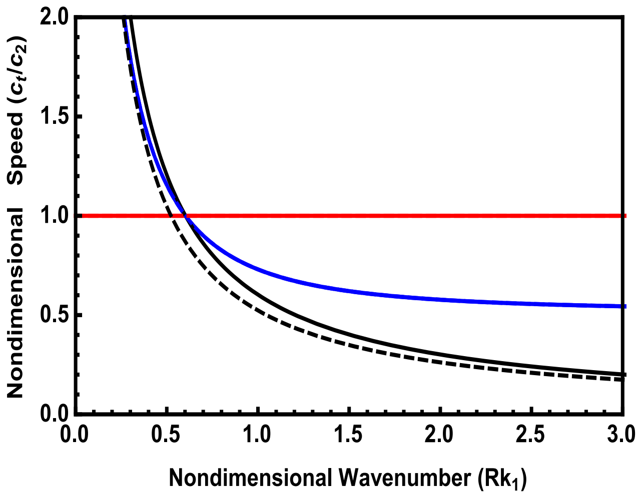

Fig. 1: Effect of J on the plots of dispersion curves of torsional modes for two different values of $J = \{0.0212 \times 10^{-4}$ m^2 (solid curve), 0.0212×10^{-3} m^2 (dotted curve)$\}$. Blue curve – c_{t_1}/c_2; red curve – c_{t_2}/c_2; black curve – c_{t_3}/c_2.

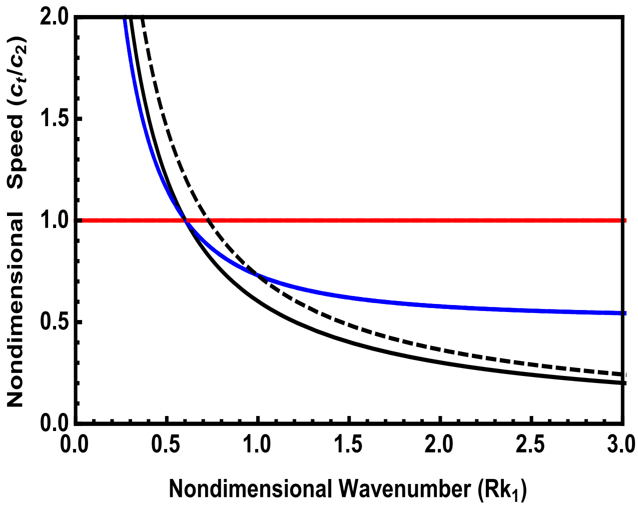

Fig. 2: Effect of μ on the plots of dispersion curves of torsional modes for two different values of $\mu = \{6.46 \times 10^{11}$ N/m^2 (solid curve), 3.46×10^{11} N/m^2 (dotted curve)$\}$. Blue curve – c_{t_1}/c_2; red curve – c_{t_2}/c_2; black curve – c_{t_3}/c_2.

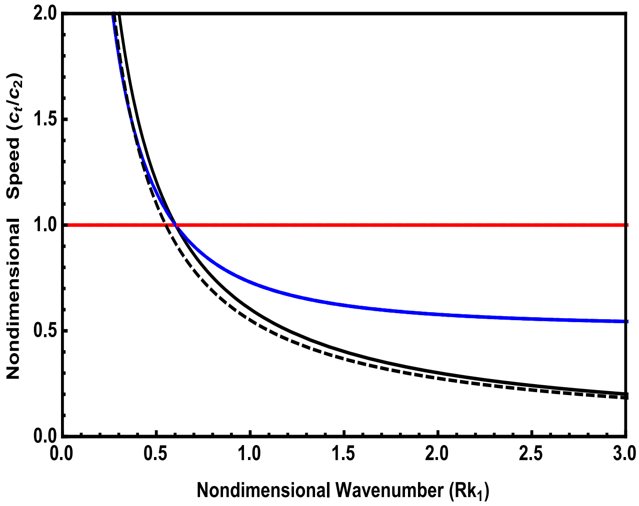

Fig. 3: Effect of γ on the plots of dispersion curves of torsional modes for two different values of $\gamma = \{0.365 \times 10^6$ N (solid curve), 0.165×10^6 N (dotted curve)$\}$. Blue curve – c_{t_1}/c_2; red curve – c_{t_2}/c_2; black curve – c_{t_3}/c_2.

disappears. However, the torsional mode of second kind remains constant throughout the entire range of wavenumber and is nearly equal to unity. It is seen that the dispersive behavior of torsional modes of first and third kinds are more prominent in the domain $0 < Rk_1 < 1$, while the torsional mode of second kind is non-dispersive in nature. From the analytical formulae (21) and (23), it is clear that the torsional modes of first and third kinds are functions of micro-inertia parameter J, while the torsional mode of second kind given by (22), does not depend on J.

Figure 2 depicts the effect of material parameter μ on the torsional modes for two different values of μ, namely, $\mu = 6.46 \times 10^{11}$ N/m^2 (solid curve) and $\mu = 3.46 \times 10^{11}$ N/m^2 (dotted curve). We note that by decreasing the value of μ, the speed of third torsional mode increases, while the speeds of other two modes remain unaffected.

Figure 3 depicts the variation of torsional modes for two different values of γ, namely, $\gamma = 0.365 \times 10^6$ N (solid curve) and $\gamma = 0.165 \times 10^6$ N (dotted curve). Here, we note that by decreasing the value of γ, the speed of third torsional mode decreases slightly, but the speeds of other torsional modes remain uninfluenced by change in the parameter γ.

It is clear from Figs. 1–3, that only the speed of torsional mode of third kind is affected by the parameters J, μ and γ.

6 Conclusions

Propagation of torsional modes in an isotropic micropolar solid half-space has been explored. The formulae for the speeds of torsional modes are obtained in closed form under the assumption that a particular micropolar constant, namely, $\mathcal{K}$ is very small so that the terms containing its higher order are negligible. From the investigation, we conclude that

1. There may exist three torsional modes in an isotropic micropolar solid half-space. Out of these modes, one is predominantly classical shear wave, while the remaining modes are new and arise due to the micropolarity of the medium.

2. The predominantly torsional mode is non-dispersive, while the other two modes are dispersive in nature.

3. Out of the three torsional modes, two depend on the micropolar parameters $\mathcal{K}$, γ and J, while the remaining one depends on $\mathcal{K}$ only.

4. In the absence of micropolarity, one of the torsional modes transforms into classical shear wave, while the other two modes disappear.

5. The non-dimensional speed of only one torsional mode is found to be affected by the parameters μ, J and γ for the considered model.

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