Journal of Theoretical and Applied Mechanics, Sofia, Vol.52 (2022) pp. 75-87
SOLID MECHANICS

A NEW DAMAGE DETECTION INDEX BASED ON BEAM MODE SHAPE SLOPE

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[Received: 28 November 2020. Accepted: 20 September 2021]

doi: https://doi.org/10.55787/jtams.22.52.1.075

ABSTRACT: A new damage index, called mode shape slope difference (MSSD), is proposed to predict the damage locations in beam structures. Unlike most vibration based damage detection methods, which use the undamaged state of the beam as reference, the MSSD index utilizes only the damaged state. It is derived by using two different approximation formulas for the first derivative of the damaged beam mode shape. To assess the effectiveness and the accuracy of the MSSD index method two test examples of finite element models are considered. The results obtained demonstrate that the proposed damage index correctly predicts the damage locations in beam structures with single or multiple damages.

KEY WORDS: damage detection, beam structure, mode shape slope, mode shape.

1 INTRODUCTION

The structures should be systematically checked for damages to reduce or eliminate the possibility of their sudden failure. In recent years vibration based damage detection methods have been considerably researched, because they are relatively cheap and quick. They have certain advantage compared to other nondestructive methods like visual inspecting, ultrasonic scanning, acoustic emission, etc. Vibration based damage detection methods could be applied to inaccessible components. Some of them could assess the severity of damages of the entire structure. The basic assumption of the vibration based damage detection methods is that the changes in structural parameters (mass, stiffness, flexibility) will induce reduction in the modal parameters (mode shapes and natural frequencies).

Detailed reviews of the damage indices based on modal parameters can be found in [1–5]. A significant part of the vibration-based methods uses natural frequency changes [6–9] as damage parameter. Salawu [10] proposed a detailed review of

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them. They are applicable to small simple structures with a single crack. Many authors proposed damage identification methods based on changes in mode shapes. Unlike the natural frequency, which is a global characteristic of the structure, the mode shape contains spatial information about the structure. Two commonly used mode shaped based methods are modal assurance criterion (MAC) [11] and coordinate modal assurance criterion (COMAC) [12]. Other methods based on mode shape data are structural translational and rotational error checking (STRECH) [13] and the methods based on mode shape derivatives [14–16]. The accuracy of all the mode shape-based methods depends on the number of measurement locations which has to be relatively large. There are damage identification methods based on dynamically measured flexibility of the structure [17, 18], but they are not applicable for small scale damages. Other vibration based damage detection methods utilize frequency response function (FRF) [19]. The main disadvantage of the FRF methods is their dependency on the measurement point location and number. There are damage detection methods based on updating physical parameters of the numerical model (in most cases finite element model) of the structure to match the measured data [20]. Some of the model updating methods can be used to assess the severity of the damage. The main drawback of these methods is the necessity of creating numerical model. They are inappropriate for locating small scale damages.

In this paper, a new damage detection index based on the change of the mode shape slope is proposed. The principle of the index is based on the comparison of the differences between mode shape slope from the left and from the right at considered points along the length of the beam. The two slopes are evaluated using a backward difference and a forward difference approximation to the first derivative of damaged beam mode shape, respectively. To assess the effectiveness and the accuracy of the proposed damage detection index two test examples with single and multiple damages are considered. The results obtained are compared with the ones obtained by well-known in the literature damage detection method. They demonstrate that the proposed damage index successfully predicts the damage locations in beam structures.

2 Damage Identification Methods Based on Mode Shape Data

The presence of damage changes the mode shape and its derivatives as the deviations are higher near the damaged cross section. These changes could be used to localize single and multiple damages. In this section the most common damage detection methods based on mode shape data are presented.

2.1 Methods based on direct change in mode shapes

The methods based on direct change in mode shapes compare two sets of series of mode shape (A and B). The sets could be analytical, experimental or analytical and

experimental [21]. The index of the modal assurance criterion (MAC) [11] is defined as

$$\text{MAC}_{(i,j)} = \frac{\left(\sum_{k=1}^{n} \Phi_{i,k}^{A} \Phi_{j,k}^{B}\right)^2}{\sum_{k=1}^{n} (\Phi_{i,k}^{A})^2 \sum_{k=1}^{n} (\Phi_{j,k}^{B})^2}, \tag{1}$$

where $i = 1, \ldots, N_A$ and $j = 1, \ldots, N_B$ are the considered numbers of investigated modes for the state A and B, respectively, $k = 1, \ldots, n$ is the number of considered coordinate of the mode shape, $\Phi_{i,k}^{A}$ is the k^{th} coordinate of i^{th} mode shape of state A, $\Phi_{j,k}^{B}$ is the k^{th} coordinate of j^{th} mode shape of state B.

The MAC value is in the range from 0 to 1. A value of 0 indicates that there is a mismatch between the two mode shapes. A value of 1 means a perfect match of the two mode shapes. The MAC criterion doesn't localize the places with differences in considered mode shapes. It only indicates presence of damages in the structure.

The coordinate modal assurance criterion (COMAC) [12] is based on MAC, but it gives information about the damage location. The COMAC takes into account the deviation between two sets of mode shapes for each considered coordinate

$$\text{COMAC}_{(k)} = \frac{\left(\sum_{i=1}^{N} \Phi_{i,k}^{A} \cdot \Phi_{i,k}^{B}\right)^2}{\sum_{i=1}^{N} (\Phi_{i,k}^{A})^2 \sum_{i=1}^{N} (\Phi_{i,k}^{B})^2}, \tag{2}$$

where N is the number of correlated mode shapes of the two sets, $\Phi_{i,k}^{A}$ and $\Phi_{i,k}^{B}$ are the k^{th} coordinate of i^{th} mode shape of state A and B, respectively. The COMAC value close to 0 indicates the possible location of damage. The COMAC value of 1 means perfect match of the modal displacements from the two sets.

2.2 Mode shape curvature method

The mode shape derivatives are commonly used to locate damage in structures. The curvature of a beam is inversely proportional to the flexural stiffness *EI* of the beam. The damage present at a certain cross section of the beam reduces the stiffness *EI* which increases the curvature. The change in curvature is local and indicates the damage location. Pandey, Biswas and Samman [14] used the absolute difference between mode shape curvature of damaged and undamaged beam. They named the proposed indicator mode shape curvature (MSC)

$$\text{MSC}_{(k)} = \sum_{i=1}^{N} \left| \left(\Phi_{i,k}^{d}\right)'' - \left(\Phi_{i,k}^{u}\right)'' \right|, \tag{3}$$

where N is the number of considered mode shapes, $(\Phi^d_{i,k})''$ and $(\Phi^u_{i,k})''$ are the mode shape curvatures at k^{th} coordinate of i^{th} mode shape of damaged and undamaged beam, respectively.

Pandey, Biswas and Samman [14] computed the mode shape curvature $(\Phi_{i,k})''$ at k^{th} coordinate of i^{th} mode shape using central difference approximation to second derivative [22]

$$(\Phi_{i,k})'' = \frac{\Phi_{i,k-1} - 2\Phi_{i,k} + \Phi_{i,k+1}}{h_m^2}\,, \tag{4}$$

where $\Phi_{i,k}$ is the modal displacement for the i^{th} mode at k^{th} coordinate, h_m is the distance between two successive measurement coordinates.

Abdel Wahab and De Roeck [23] proposed new index named curvature damage factor (CDF). It averages the absolute difference between mode shape curvature of damaged and undamaged beam for considered mode shapes

$$\text{CDF}_{(k)} = \frac{1}{N}\sum_{i=1}^{N}\left|\left(\Phi^d_{i,k}\right)'' - \left(\Phi^u_{i,k}\right)''\right|. \tag{5}$$

The large positive peak values of MSC and CDF indicate the possible damage locations.

3 The Proposed Damage Detection Index

In this study, a new damage detection index based on the mode shape slope changes is developed. The mode shape slope is computed through the forward difference and through the backward difference approximation to the first derivative of damaged beam mode shape. The damage index compares the differences between the results obtained using the two approximation formulae.

The mode shapes of an undamaged beam are smooth curves. The beam mode shape has a kink at cross section with a crack or damages. Therefore, at this place the mode shape slope from the left is different from those from the right.

The mode shape slope at any given point along the length of the beam is the first derivative of the mode shape at the same location. The most common approximation formulae to the first derivative involve three or five evaluation points [22]. They are relative to the central point for a central difference approximation, to the leftmost point for a forward difference approximation and to the rightmost point for a backward difference approximation. The mode shape slope $(\Phi_k)'$ at k^{th} coordinate of the mode shape using three-point forward difference approximation formula for the first derivative is

$$(\Phi_{k,f})' = \frac{-3\Phi_k + 4\Phi_{k+1} - \Phi_{k+2}}{2h_m}\,; \tag{6}$$

and using three-point backward difference approximation formula for the first derivative is

$$(\Phi_{k,b})' = \frac{\Phi_{k-2} - 4\Phi_{k-1} + 3\Phi_k}{2h_m}\,, \tag{7}$$

where Φ_k is the modal displacement at k^{th} coordinate, h_m is the distance between two successive measurement coordinates, $(\Phi_{k,f})'$ and $(\Phi_{k,b})'$ are mode shape slopes at k^{th} coordinate of the mode shape using forward and backward difference approximation, respectively.

The difference $\Delta(\Phi_k)'$ between slopes calculated using equations (6) and (7) at k^{th} coordinate of the considered mode shape is

$$\Delta\,(\Phi_k)' = (\Phi_{k,b})' - (\Phi_{k,f})' = \frac{\Phi_{k-2} - 4\Phi_{k-1} + 6\Phi_k - 4\Phi_{k+1} + \Phi_{k+2}}{2h_m}\,. \tag{8}$$

The difference $\Delta(\Phi_k)'$ increases at peaks and valleys of the mode shape, but the larger values of the difference are obtained at the mode shape kinks. The relative difference between the mode shape slopes calculated using backward and forward difference approximation is the proposed new damage index. It is computed considering only the first mode shape. The proposed damage index is named mode shape slope difference (MSSD) and is expressed by the following relation:

$$\text{MSSD}_{(k)} = \frac{\left|\Delta\,(\Phi_k)'\right|}{\max\left|\Delta\,(\Phi_{k=1,\ldots,n})'\right|}\,, \tag{9}$$

where $\max\|(\Delta(\Phi_k = 1,\ldots,n)'\|$ is the maximum absolute value of the difference $\Delta(\Phi_k)'$ between slopes calculated using backward and forward difference approximation.

The points with large peak MSSD value indicate the possible locations of damage. If there is a single damage in the beam, at this location the MSSD value is equal to 1.

4 Numerical Examples

In order to demonstrate the effectiveness and robustness of the proposed damage detection index two test examples are considered. The numerical analysis is carried out for a simply supported beam and for a two-span continuous beam.

The mode shape displacements for the different cases are calculated by the finite element method, using computer program SAP2000 [24]. The beam is discretized by 8-node solid elements with equal length (0.15 m), as shown in Fig. 1. The cracks are modelled by disconnecting joints of certain solid elements. The damage indices are calculated using MATLAB [25].

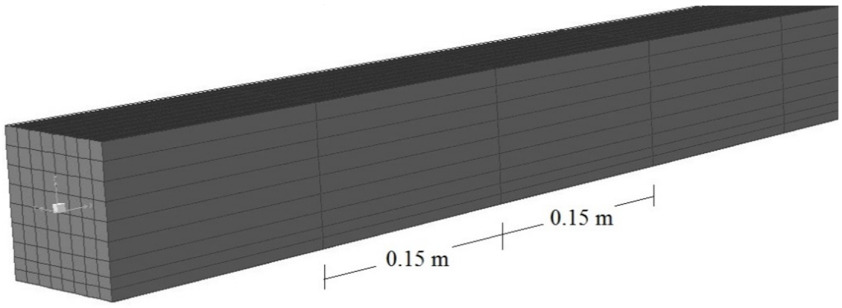

Fig. 1: A part of the beam finite element model.

To assess the efficiency and the accuracy of the proposed damage location index the results of MSSD given by equation (9) are compared with the ones of CDF given by equation (5). The CDF index is calculated considering the first three mode shapes. For better comparison the CDF index is normalized and is given as

$$\text{nCDF}_{(k)} = \frac{\frac{1}{N}\sum_{i=1}^{N}\left|\left(\Phi_{i,k}^{d}\right)'' - \left(\Phi_{i,k}^{u}\right)''\right|}{\max\left(\frac{1}{N}\sum_{i=1}^{N}\left|\left(\Phi_{i,k}^{d}\right)'' - \left(\Phi_{i,k}^{u}\right)''\right|\right)}. \tag{10}$$

4.1 Example 1: A simply supported beam

The dimensions of the simply supported beam are presented in Fig. 2: length $L = 3$ m, height $h = 0.1$ m, width $b = 0.1$ m. The material properties are: modulus of elasticity $E = 2.1 \times 10^{11}$ N/m^2, Poisson's coefficient $\nu = 0.3$, density $\rho = 7,850$ kg/m^3.

In order to demonstrate the potential of the proposed index for single and multiple damage detection fourteen damage cases are considered. The main crack parameters are crack location l_c, depth d_c and width w_c, as illustrated in Fig. 2. The cracks are

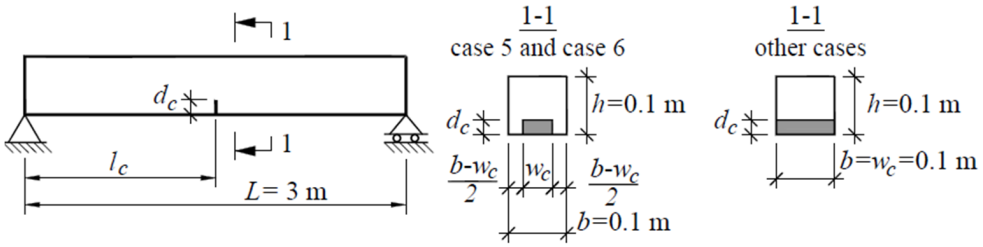

Fig. 2: Geometry of the cracks and the simply supported beam.

Table 1: Parameters of cracks for different damage cases of the simply supported beam

Case number	Crack location l_c [m]	Crack depth d_c [mm]	Crack width w_c [mm]
1	1.2	15.00	100
2	1.2	10.00	100
3	1.2	7.50	100
4	1.2	5.00	100
5	1.2	2.50	75
6	1.2	1.25	50
7	0.3	10.00	100
8	0.6	10.00	100
9	0.9	10.00	100
10	1.5	10.00	100
11	0.3 0.6	10.00 10.00	100 100
12	0.3 0.9	10.00 10.00	100 100
13	0.3 1.2	10.00 10.00	100 100
14	0.3 0.9 1.5	10.00 10.00 10.00	100 100 100

edge and have the width of the cross section, except in case 5 and case 6 (Table 1). In these two cases the cracks are edge and symmetrically positioned with respect to the cross section width. The parameters of the cracks for the different cases are given in Table 1.

Plots of damage indices MSSD and nCDF along the length of the beam for different crack depth at crack location $l_c = 1.2$ m are presented in Fig. 3. As shown in the damage identification charts the two indices precisely localize the damage. The two damage indices at cross section with crack have value equal to 1. For case 1 and case 2 the plots of MSSD and nCDF indices have single peak at crack location. For the rest of the cases the plots of MSSD have valleys at locations nearest to damage and sharp peak only at crack location.

The graphs of MSSD and nCDF indices for cases of single identical damage at different locations along the length of the beam are presented in Fig. 4. They have sharp peak at crack location, where the values of both damage indices are equal to 1.

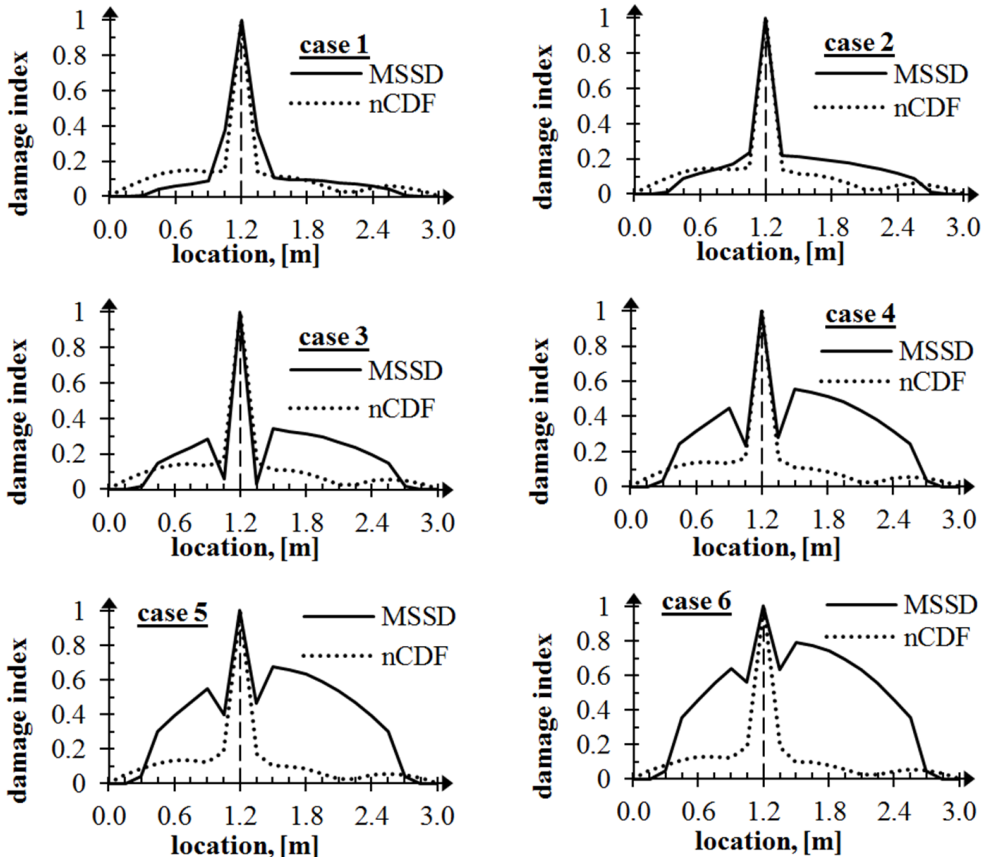

Fig. 3: Damage identification charts for cases of single damage located at 1.2 m from the left end of the simply supported beam.

As can be seen in Fig. 4 the values of MSSD indices at undamaged points increase when the crack location comes closer to the end of the beam. At crack location $l_c = 0.3$ m (case 7) the maximum value of MSSD at undamaged location is 0.80 and at crack location $l_c = 1.5$ m (case 10) the maximum value is 0.23.

Figure 5 displays damage localization for multiple crack cases. Both damage indices correctly identify the damage locations. The only exception is case 11, where the MSSD plot has a kink, not a peak, at crack location $l_c = 0.3$ m. Although all the cracks have equal parameters the MSSD indices and the nCDF indices have different values at different damage locations. They vary from 0.24 to 1 for MSSD indices and from 0.57 to 1 for nCDF indices.

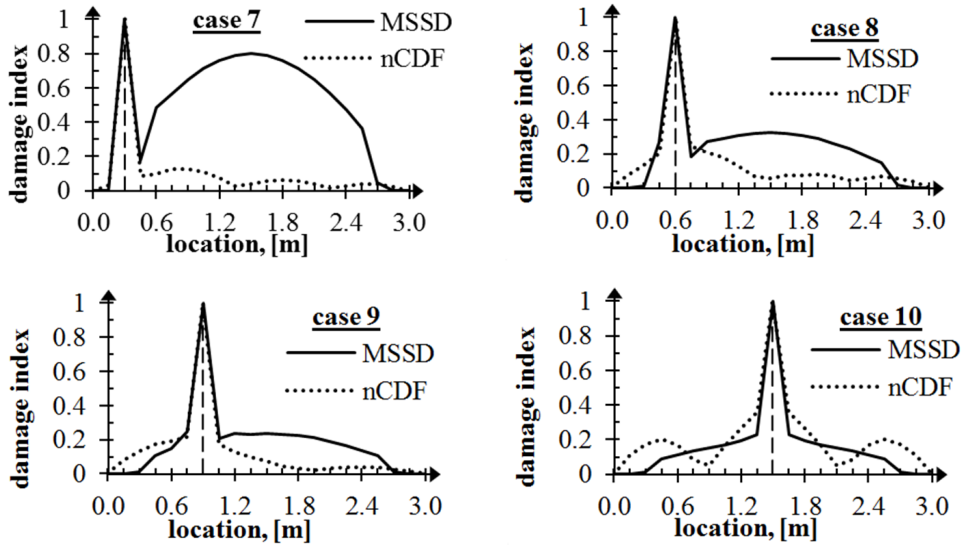

Fig. 4: Damage identification charts for cases of single identical damage at different locations along the length of the simply supported beam.

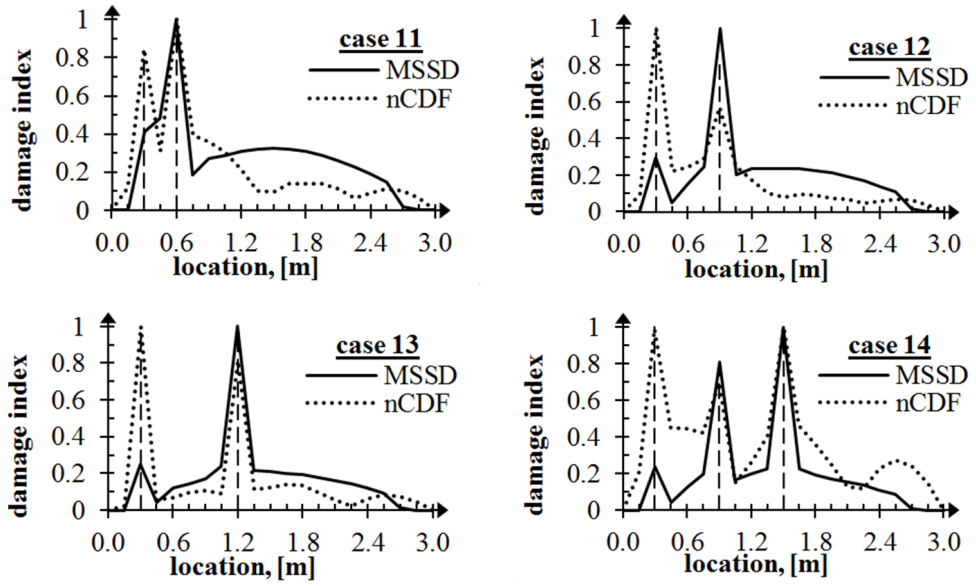

Fig. 5: Damage identification charts for cases of multiple identical damages at different locations along the length of the simply supported beam.

4.2 EXAMPLE 2: A TWO-SPAN CONTINUOUS BEAM

The dimensions of the continuous beam of two equal spans are presented in Fig. 6: length $L = 6$ m, height $h = 0.1$ m, width $b = 0.1$ m. The material properties are: modulus of elasticity $E = 2.1 \times 10^{11}$ N/m^2, Poisson's coefficient $\nu = 0.34$, density $\rho = 7,850$ kg/m^3.

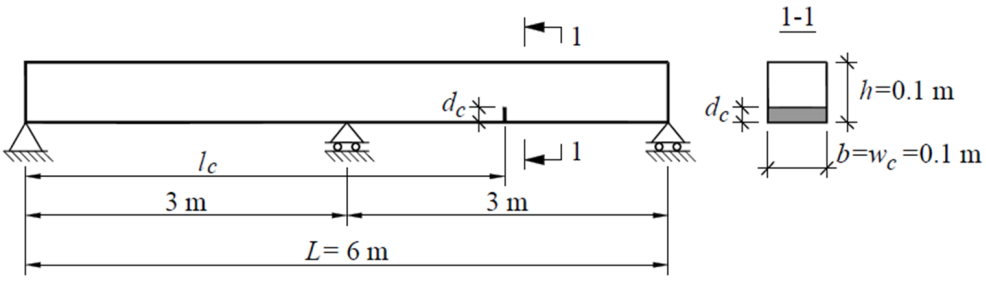

Fig. 6: Geometry of the cracks and the two-span continuous beam.

In addition, to check the performance of the proposed damage detection index to other static scheme of beam, four different damage cases are considered, All the cracks are edge and have the width of the cross section ($w_c = b$). The crack location l_c and depth d_c, shown in Fig. 6, are listed in Table 2.

Table 2: Parameters of cracks for different damage cases of the continuous beam

Case number	Crack location l_c [m]	Crack depth d_c [mm]
1	1.20	10
2	1.20 4.20	10 10
3	0.30 1.20 4.80	10 10 10
4	0.30 1.20 2.10 3.90 4.80	10 10 10 10 10

As can be seen in Fig. 7, the two indices localize the damages for the cases of single and multiple cracks. Although all the cracks have equal parameters the MSSDF indices at damage location have different values. The values of MSSD and nCDF indices depend on crack location as they are greater near the middle of the beam span.

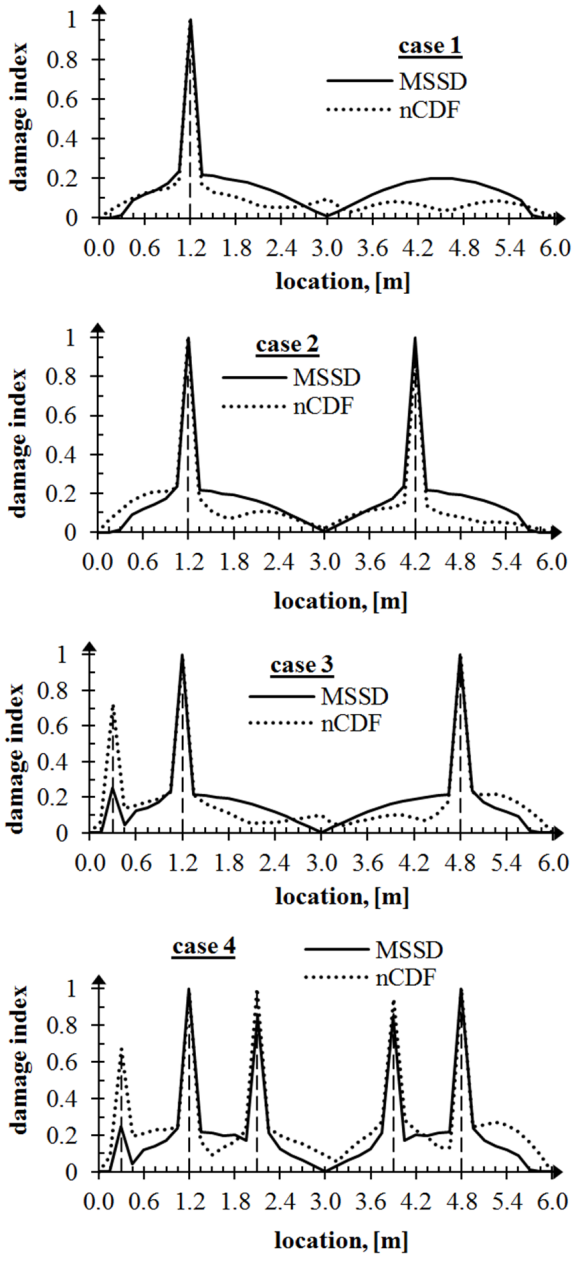

Fig. 7: Damage identification charts for cases of single and multiple identical damages at different locations along the length of the continuous beam.

5 Conclusion

In this paper, a new damage detection index based on the change in the mode shape slope is proposed. Two test examples are used to evaluate the validity and limitation of the method. The results obtained by method based on MSSD indices are compared with those of other well-known damage detection method in the literature, CDF. Both methods identify the location of the single or multiple damages with a precision for the considered cases. For multiple damage cases the damage index charts have peak at each damage location as the value of the peak doesn't correspond to crack parameters. The two damage indices demonstrate the same efficiency for damage localization. The major advantage of the proposed damage index over CDF index is that MSSD index utilizes the mode shape data of only damaged state of the beam. The CDF index needs the data from the undamaged state of the beam, which could be complicated to obtain or even impossible. Another advantage is that the MSSD indices consider only the first mode shape unlike the CDF indices, which use the data from three and more vibration modes. The numerical results demonstrate that MSSD index is efficient tool for locating single or multiple damages in beam structures.

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