Journal of Theoretical and Applied Mechanics, Sofia, Vol.52 (2022) pp. 199-214
INTERDISCIPLINARY TOPICS

ESTABLISHMENT OF THE NATURAL FREQUENCY OF OSCILLATIONS OF THE TWO-DIMENSIONAL CONTINUOUS MEMBER OF THE VIBRATING TABLE

Pavlo Maistruk[1*], Oleksii Lanets[1],
Volodymyr Maistruk[2], Iryna Derevenko[3]

[1]*Department of Robotics and Integrated Mechanical Engineering Technologies, Institute of Mechanical Engineering and Transport, Lviv Polytechnic National University, Lviv, Ukraine*
[2]*Department of Designing and Operation of Machines, Institute of Mechanical Engineering and Transport, Lviv Polytechnic National University, Lviv, Ukraine*
[3]*Department of Strength of Materials and Structural Mechanics, Institute of Civil Engineering and Building Systems, Lviv Polytechnic National University, Lviv, Ukraine*

[Received: 10 June 2021. Accepted: 05 April 2022]

doi: https://doi.org/10.55787/jtams.22.52.3.199

ABSTRACT: In the article, using the approximate Rayleigh-Ritz method, the first natural frequency of oscillations of the continuous member (elastic plate) of the inter-resonance vibrating table with the electromagnetic drive is established. For this purpose, the kinetic and potential energies of the plate were presented in integral form, where the basic function of the plate surface deflections is given as the product of the functions of the transverse deflections along the x and y axes, taking into account the boundary conditions. The transverse deflection along the x-axis of the plate is described through Krylov-Duncan functions and along the y-axis – through circular functions. The article found the approximate value of the first natural frequency of oscillations of the continuous member. The reliability of the obtained data was confirmed experimentally.

KEY WORDS: continuous member, natural frequencies and mode shapes, Rayleigh-Ritz method, Krylov-Duncan functions.

*Corresponding author e-mail: pmaystruk@gmail.com

1 Introduction

Given the prevalence of vibrating machines in many industries, it is important to improve their technological efficiency. One of the most important indicators of the technological efficiency of vibratory machines is energy efficiency. Achieving the required amplitude of oscillations of the working body at the minimum power consumption of the drive can improve the energy efficiency of the vibratory machines. It is known that a significant increase in energy efficiency in vibrating machines can be achieved by implementing them as three-mass oscillating systems with inter-resonance regimes of work [1]. Actually, inter-resonance regimes of work allow the vibratory machines to achieve a significant increase in energy efficiency compared to resonant, which are based on two-mass oscillating systems. However, to implement highly efficient inter-resonance regimes of work, the reactive mass of the vibrating machine must be ultralight. In [2], to implement ultralight reactive masses, use a continuous member in the form of a rod driven by a crank mechanism. This allowed to effectively provide inter-resonance regimes of work.

The authors of this article are developing a discrete-continuous inter-resonance vibrating table with an electromagnetic drive, where they decided to use a plate as a reactive mass. In fact, the plate, which will be located between the wide poles of electromagnetic vibrators, will effectively perceive the electromagnetic forces of perturbation. The plate will play the role not only of the oscillating mass but also of the elastic unit, the calculation of which is given much attention in [3].

An important step in the design of a discrete-continuous inter-resonance vibrating table with an electromagnetic drive is to establish the first natural frequency of the continuous section in the form of a plate. To calculate the natural frequencies of oscillations of elastic plates with complex boundary conditions of attachment often use the approximate Rayleigh-Ritz method [4, 5]. The authors of [6] propose the method for calculating the natural frequencies of oscillations of a T-shaped plate. Jin et al. [7] developed an accurate modified solution of the Fourier series for the calculation of free oscillations of conical shells, where all coefficients of expansion were determined using the Rayleigh-Ritz method. Borković et al. [8] presented a new method for the spatial discretization of two-dimensional domains for calculating the natural frequencies of singly curved shells. Studies [9–12] performed free vibration analysis of elastic plates made of functionally graded materials (FGM).

The use of a continuous member perturbed in an electromagnetic field can cause beating the plate to the poles of the iron core of vibrators. This problem was considered in [13, 14]. It is possible to analyze the chance of beating both experimentally and with the help of analytical calculation methods or simulation modeling. Shishesaz et al. [15] using the generalized differential quadrature (GDQ) method calculated the effect of the magnetic field on the stresses and displacements in the annular FGM plate. This method F. Tornabene et al. [16] also used for calculating the natural oscillations of arbitrarily shaped laminated composite shells.

Based on the above methods for calculating elastic plates, we can draw the following conclusion. The most expedient method of calculation, which is quite suitable for solving our problem, is the approximate Rayleigh-Ritz method, which will allow establishing the natural frequency of oscillations of the plate with complex boundary conditions of attachment. This will require mathematically describing the shape of the oscillations of the plate at its natural frequency, calculating its kinetic and potential energies. The article proposes the solution to these tasks.

2 Methodology of Calculation the Natural Frequency of Oscillations of the Continuous Member

The three-dimensional model of a discrete-continuous inter-resonance vibrating table with an electromagnetic drive is shown in Fig. 1. The main components of the structure are the active mass, the intermediate mass, and the continuous section in the form of a plate hinged to the intermediate mass. The schematic diagram of the vibrating table is shown in Fig. 2, on which it is possible to see the conditions of attachment of the continuous member of the vibratory machine.

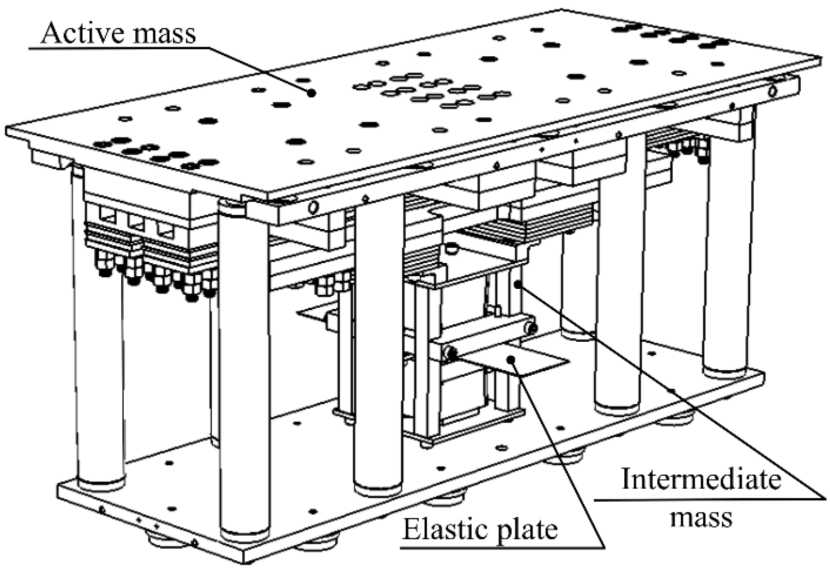

Fig. 1: The three-dimensional model of a discrete-continuous inter-resonance vibrating table with an electromagnetic drive.

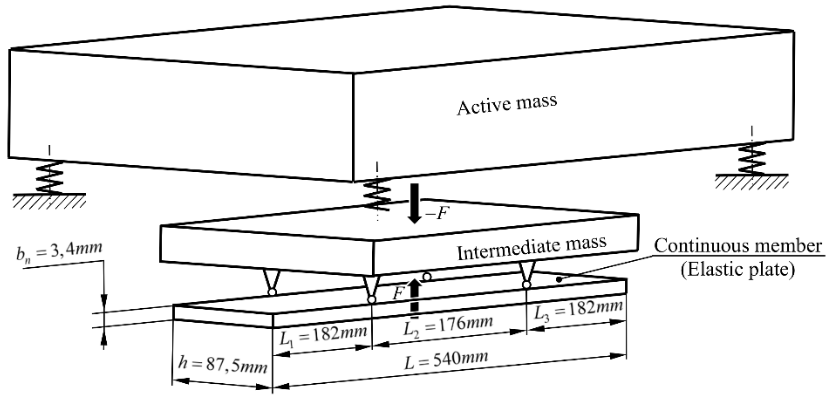

Fig. 2: The schematic diagram of the vibrating table with overall and fastening dimensions of the continuous member.

The object of research of this article is the continuous member with complex boundary conditions of attachment, for which it is necessary to set the natural frequency of oscillation. For this purpose, we will allocate a continuous member separately and consider that hinge support is motionless that is pertinent when establishing the natural frequency of oscillation of the plate. The harmonic force of perturbation drives the plate out of equilibrium. The shape of the deflections of the plate will look like in Fig. 3. The amplitude value F of the perturbation force is applied to the center of the plate. We assume that the plate oscillates in the presented mode shape at a

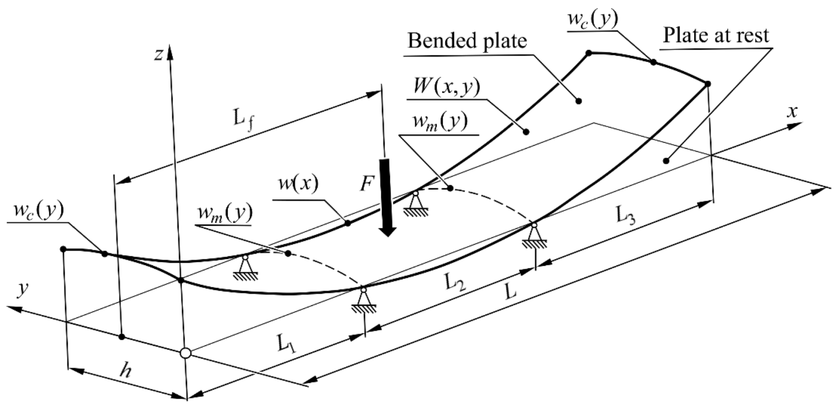

Fig. 3: The expected shape of the deflections of the plate at the first natural frequency of oscillation.

frequency close to its natural. To calculate the natural frequency of oscillations by the Rayleigh-Ritz method, we need to mathematically describe the presented mode shape.

The mode shape of the plate surface is mathematically given in the form of a basic function $W(x, y)$, as a function of its deflections by two Cartesian coordinates x and y (Fig. 3). We assume that the basis function $W(x, y)$ is formed as the product of the functions of the transverse deflections along x-axis $(w\,(x))$ and y-axis $(w\,(y))$ of mutually perpendicular conditionally separated beams. The function $w\,(x)$ is given in the form of Krylov-Duncan functions describing the deflection of a conditionally allocated beam along the axis Ox and the function $w\,(y)$ in the form of circular functions describing the deflection of a conditionally allocated beam along the axis Oy. The function $w\,(y)$ at the end of the console and on the hinge supports will have differences, and therefore it is presented in the form of functions $w_c(y)$ and $w_m(y)$. Functions $w(x)$, $w_c(y)$ and $w_m(y)$ describe the natural forms of oscillations of homogeneous beams. The deflection functions of the beams must satisfy the boundary conditions in accordance with the method of attaching the plate.

The surface of the perturbed plate will be mathematically described by the dependence:

$$W(x, y) = w(x)w(y)\,. \tag{1}$$

To establish the function $w(x)$, we consider a beam mounted on two hinged supports, which is affected by the amplitude value of the harmonic perturbation force F at a frequency close to its natural (Fig. 4). The two supports conditionally divide the beam into the left end, the middle and the right end, the deflection of which is respectively described by the functions $w_l(x)$, $w_m(x)$, and $w_r(x)$.

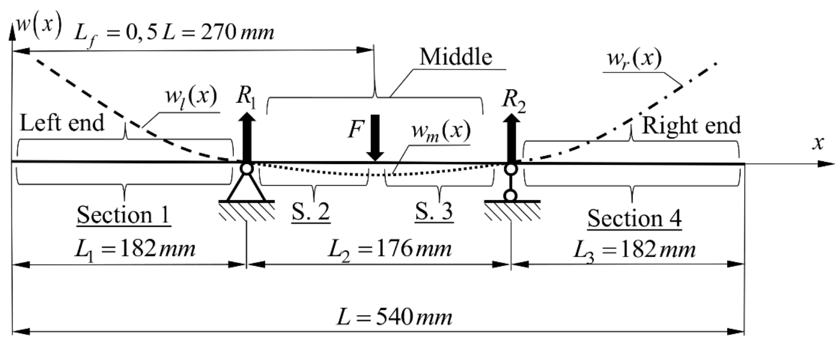

Fig. 4: The calculation scheme of the conditionally allocated transverse beam along x-axis divided into sections.

To determine the deflection of the beam, we use the known differential equation [17]:

$$EJ_z \frac{d^4 w(x)}{dx^4} - \omega^2 m_{rm} w(x) = 0\,, \tag{2}$$

where E is Young's modulus. For steel it is equal $E = 2.1 \times 10^{11}$ N/m^2, J_z – the moment of inertia of the area of the cross section of the beam, ω – the forced circular frequency of oscillations of the beam, m_{rm} – the mass per unit length of the beam.

If we denote $\xi^4 = \omega^2 m_{rm}/(EJ_z)$, the equation (2) will be:

$$\frac{d^4 w(x)}{dx^4} - \xi^4 w(x) = 0\,. \tag{3}$$

The solution of the differential equation (3) is generally written as:

$$w(x) = AS(\xi x) + BT(\xi x) + CU(\xi x) + DV(\xi x)\,, \tag{4}$$

where A, B, C, D denote coefficients, determined through boundary conditions

$$S(\xi x) = (\cosh(\xi x) + \cos(\xi, x))/2\,, \quad T(\xi x) = (\sinh(\xi x) + \sin(\xi x))/2\,,$$
$$U(\xi x) = (\cosh(\xi x) - \cos(\xi x))/2\,, \quad V(\xi x) = (\sinh(\xi x) - \sin(\xi x))/2$$

– Krylov-Duncan functions.

Take the left end of the beam as the starting point. According to the boundary conditions of attachment at this end, we form the equation of deflections of the left section (Fig. 4). The left end of the section is free, where there is no bending moment and transverse force. Therefore, the initial boundary conditions at this end will be:

$$\left.\frac{\mathrm{d}^2 w_l(x)}{\mathrm{d}x^2}\right|_{x=0} = \xi^2 C = 0\,, \tag{5}$$

$$\left.\frac{\mathrm{d}^3 w_l(x)}{\mathrm{d}x^3}\right|_{x=0} = \xi^3 D = 0\,. \tag{6}$$

Therefore, the constants $C = D = 0$. In this case, imposing conditions (5), (6) on expression (4), the equation of deflections on the left section will have the form:

$$w_l(x) = AS(\xi x) + BT(\xi x)\,. \tag{7}$$

For the middle and right sections of the beam, the corresponding equations are:

$$w_m(x) = w_l(x) + \frac{R_1}{\xi^3 EJ_z} V(\xi(x - L_1)) + \frac{F}{\xi^3 EJ_z} V(\xi(x - L_1 - L_f))\,, \tag{8}$$

$$w_r(x) = w_m(x) + \frac{R_2}{\xi^3 EJ_z} V(\xi(x - L_1 - L_2))\,, \tag{9}$$

where

$$V(\xi(x-L_1)) = \frac{\sinh(\xi(L-L_1)) - \sin(\xi(L-L_1))}{2} = V_2\,,$$
$$V(\xi(x-L_1-L_f)) = \frac{\sinh(\xi(L-L_1-L_f)) - \sin(\xi(L-L_1-L_f))}{2} = V_f\,,$$
$$V(\xi(x-L_1-L_2)) = \frac{\sinh(\xi(L-L_1-L_2)) - \sin(\xi(L-L_1-L_2))}{2} = V_3\,.$$

Substituting expression (7) in (8), the general equation of deflections of the middle section of the beam on the $L_1 \leq \ \leq L_1 + L_2$ will be:

$$\text{(10)}\quad w_m(x) = AS(\xi x) + BT(\xi x) + \frac{R_1}{\xi^3 EJ_z}V(\xi(x-L_1)) + \frac{F}{\xi^3 EJ_z}V(\xi(x-L_1-L_f)).$$

For the right section $(L_1 + L_2 \leq \ \leq L)$, substituting expression (10) in (9), we obtain the following equation of deflections:

$$\text{(11)}\quad w_r(x) = AS(\xi x) + BT(\xi x) + \frac{R_1}{\xi^3 EJ_z}V(\xi(x-L_1)) + \frac{F}{\xi^3 EJ_z}V(\xi(x-L_1-L_f)) + \frac{R_2}{\xi^3 EJ_z}V(\xi(x-L_1-L_2)).$$

Equations (7) – (11) are written in a general form. To establish constants , and reactions in the hinge supports R_1, R_2, it is necessary to synthesize a system of four equations. We form the first two equations using expression (11). The right end of the beam is free. The bending moment and the transverse force that are respectively the second and third derivatives of the deflection function at the right end are equal to zero. Imposing these conditions on equation (11) we can write two necessary equations:

$$\text{(12)}\quad \frac{\mathrm{d}^2 w_r(x)}{\mathrm{d}x^2}\Big|_{x=L} = A\xi^2 U(\xi L) + B\xi^2 V(\xi L) + \frac{R_1}{\xi EJ_z}T(\xi(L-L_1)) + \frac{R_2}{\xi EJ_z}T(\xi(L-L_1-L_2)) + \frac{F}{\xi EJ_z}T(\xi(L-L_f)) = 0\,,$$

$$\text{(13)}\quad \frac{\mathrm{d}^3 w_r(x)}{\mathrm{d}x^3}\Big|_{x=L} = A\xi^3 T(\xi L) + B\xi^3 U(\xi L) + \frac{R_1}{EJ_z}S(\xi(L-L_1)) + \frac{R_2}{EJ_z}S(\xi(L-L_1-L_2)) + \frac{F}{EJ_z}S(\xi(L-L_f)) = 0\,,$$

where

$$U(\xi L) = \frac{\cosh(\xi L) - \cos(\xi L)}{2} = U_1 \,,$$
$$V(\xi L) = \frac{\sinh(\xi L) - \sin(\xi L)}{2} = V_1 \,,$$
$$T(\xi(L - L_1)) = \frac{\sinh(\xi(L - L_1)) + \sin(\xi(L - L_1))}{2} = T_2 \,,$$
$$T(\xi(L - L_1 - L_2)) = \frac{\sinh(\xi(L - L_1 - L_2)) + \sin(\xi(L - L_1 - L_2))}{2} = T_3 \,,$$
$$T(\xi(L - L_f)) = \frac{\sinh(\xi(L - L_f)) + \sin(\xi(L - L_f))}{2} = T_f \,,$$
$$T(\xi L) = \frac{\sinh(\xi L) + \sin(\xi L)}{2} = T_1 \,,$$
$$S(\xi(L - L_1)) = \frac{\cosh(\xi(L - L_1)) + \cos(\xi(L - L_1))}{2} = S_2 \,,$$
$$S(\xi(L - L_1 - L_2)) = \frac{\cosh(\xi(L - L_1 - L_2)) + \cos(\xi(L - L_1 - L_2))}{2} = S_3 \,,$$
$$S(\xi(L - L_f)) = \frac{\cosh(\xi(L - L_f)) + \cos(\xi(L - L_f))}{2} = S_f \,.$$

In the presence of four unknowns (constants , , and reactions in the supports R_1 and R_2) we need two more equations besides (12) and (13). The third equation is formed from the condition that the deflection of the beam in the hinge support R_1 is equal to zero. Using equation (7) for the left section of the beam, we obtain the dependence:

$$w_l(x)_{x=L_1} = AS(\xi L_1) + BT(\xi L_1) = 0 \,, \tag{14}$$

where

$$S(\xi L_1) = (\cosh(\xi L_1) + \cos(\xi L_1))/2 = S_4 \,,$$
$$T(\xi L_1) = (\sinh(\xi L_1) + \sin(\xi L_1))/2 = T_4 \,.$$

The fourth equation is formed from the condition that there is no deflection in the hinge support R_2. Applying equation (10) for the middle section, we obtain:

$$w_m(x)_{x=L_1+L_2} = AS(\xi(L_1 + L_2)) + BT(\xi(L_1 + L_2)) + \frac{R_1}{\xi^3 EJ_z} V(\xi L_2) + \frac{F}{\xi^3 EJ_z} V(\xi L_f) = 0 \,, \tag{15}$$

where

$$S(\xi(L_1 + L_2)) = \frac{\cosh(\xi(L_1 + L_2)) + \cos(\xi(L_1 + L_2))}{2} = S_5 \,,$$

$$T(\xi(L_1+L_2)) = \frac{\sinh(\xi\,(L_1+L_2)) + \sin(\xi(L_1+L_2))}{2} = T_5\,,$$
$$V(\xi L_2) = \frac{\sinh(\xi L_2) - \sin(\xi L_2)}{2} = V_5\,,$$
$$V(\xi L_f) = \frac{\sinh(\xi\,L_f) - \sin(\xi L_f)}{2} = V_f\,.$$

Therefore, taking into account expressions (12) – (15), we obtain a system of four equations of forced oscillations of the beam:

$$\begin{aligned}
&AU_1 + BtV_1 + \frac{R_1}{\xi EJ_z}T_2 + \frac{R_2}{\xi EJ_z}T_3 + \frac{F}{\xi EJ_z}T_f = 0\,,\\
&AT_1 + BU_1 + \frac{R_1}{EJ_z}S_2 + \frac{R_2}{EJ_z}S_3 + \frac{F}{EJ_z}S_f = 0\,,\\
&AS_4 + BT_4 = 0\,,\\
&AS_5 + BT_5 + \frac{R_1}{\xi^3 EJ_z}V_5 + \frac{F}{\xi^3 EJ_z}V_f = 0\,.
\end{aligned} \tag{16}$$

From the system of equations (16), analytical expressions for establishing all four unknowns will be:

$$A = \frac{-FT_4(-S_3T_2V_f + S_3T_fV_5 + S_2T_3V_f - S_fT_3V_5)}{\xi^3 EJ\begin{pmatrix} -S_3T_2S_5T_4 + S_3T_2S_4T_5 + S_3T_4U_1V_5 - S_3S_4V_1V_5 \\ +S_2S_5T_3T_4 - S_2S_4T_3T_5 - T_1T_3T_4V_5 + S_4T_3U_1V_5 \end{pmatrix}}\,, \tag{17}$$

$$B = \frac{FS_4(-S_3T_2V_f + S_3T_fV_5 + S_2T_3V_f - S_fT_3V_5)}{\xi^3 E\,J\cdot\begin{pmatrix} -S_3T_2S_5T_4 + S_3T_2S_4T_5 + S_3T_4U_1V_5 - S_3S_4V_1V_5 \\ +S_2S_5T_3T_4 - S_2S_4T_3T_5 - T_1T_3T_4V_5 + S_4T_3U_1V_5 \end{pmatrix}}\,, \tag{18}$$

$$R_1 = \frac{-F\begin{pmatrix} -S_3S_5T_4T_f + S_5S_fT_3T_4 + S_3S_4T_5T_f - S_4S_fT_3T_5 \\ +S_3T_4U_1V_f - S_3S_4V_1V_f - T_1T_3T_4V_f + S_4T_3U_1V_f \end{pmatrix}}{\begin{pmatrix} -S_3S_5T_2T_4 + S_3S_4T_2T_5 + S_3T_4U_1V_5 - S_3S_4V_1V_5 \\ +S_2S_5T_3T_4 - S_2S_4T_3T_5 - T_1T_3T_4V_5 + S_4T_3U_1V_5 \end{pmatrix}}\,, \tag{19}$$

$$R_2 = \frac{-F\begin{pmatrix} S_2S_5T_4T_f - S_2S_4T_5T_f - S_2T_4U_1V_f + S_2S_4V_1V_f \\ +T_1T_2T_4V_f - T_1T_4V_5T_f - S_4T_2U_1V_f + S_4T_fU_1V_5 \\ -S_5S_fT_2T_4 + S_4S_fT_2T_5 + S_fT_4U_1V_5 - S_4S_fV_1V_5 \end{pmatrix}}{\begin{pmatrix} -S_3S_5T_2T_4 + S_3S_4T_2T_5 + S_3T_4U_1V_5 - S_3S_4V_1V_5 \\ +S_2S_5T_3T_4 - S_2S_4T_3T_5 - T_1T_3T_4V_5 + S_4T_3U_1V_5 \end{pmatrix}}\,. \tag{20}$$

Considering the conditionally allocated beam along x-axis (Fig. 3) with the dimensions of the plate, which are shown in Fig. 2, its moment of inertia of the area of the cross section will be:

$$J = h, b_n^3/12 = 2.635 \times 10^{-10} \text{ m}^4 \, , \tag{21}$$

where h and b_n are respectively, the width and thickness of the area of the cross section of the plate.

The mass per unit length of the conditionally allocated beam will be:

$$m_{rm} = hb_n\rho = 2.28 \text{ kg/m} \, , \tag{22}$$

where $\rho = 7850$ kg/m^3 is the density of steel.

Assuming that the amplitude value of the perturbation force $F = 300$ N, the frequency of forced oscillations $\omega = 314$ rad/s, given the lengths of the sections shown in Fig. 3, according to expressions (17) – (20) we set the values of the unknowns:

$$A = 0.022; \quad B = -0.017; \quad R_1 = R_2 = -519932 \, . \tag{23}$$

Using the experience obtained above, we construct a transverse deflection of the plate along x-axis, dividing the plate into 4 sections (Fig. 4). Using expression (7) and entering a variable x_1 lying within $0 < x_1 < L_1 = 0.182$ m, the function that describe the transverse deflection of the plate along x-axis in the first section will take the following form:

$$w_1(x_1) = AS_S(x_1) + BT_T(x_1) \, , \tag{24}$$

where

$$S_S(x_1) = (\cosh(\xi x_1) + \cos(\xi x_1))/2 \, ,$$
$$T_T(x_1) = (\sinh(\xi x_1) + \sin(\xi x_1))/2 \, .$$

For the second section, enter a variable x_2 that varies within $L_1 = 0.182 \text{ m} < x_2 < L_f = 0.27$ m. At these limits, expression (10) as a function $w_2(x_2)$ describing the transverse deflection of the plate along x-axis in the second section will take the form:

$$w_2(x_2) = AS_S(x_2) + BT_T(x_2) + \frac{R_1}{\xi^3 EJ_z} \cdot V_{V1}(x_2) \, , \tag{25}$$

where

$$S_S(x_2) = (\cosh(\xi\, x_2) + \cos(\xi x_2))/2 \, ,$$
$$T_T(x_2) = (\sinh(\xi x_2) + \sin(\xi x_2))/2 \, ,$$
$$V_{V1}(x_2) = (\sinh(\xi(x_2 - L_1)) - \sin(\xi(x_2 - L_1)))/2 \, .$$

For the third section, enter a variable x_3 that varies within $L_f = 0.27$ m $< x_3 < L_1 + L_2 = 0.358$ m. Using expression (10), we describe the function $w_3(x_3)$ of the transverse deflection of the plate along x-axis in the third section:

$$w_3(x_3) = AS_S(x_3) + BT_T(x_3) + \frac{R_1}{\xi^3 EJ_z} V_{V1}(x_3) + \frac{F}{\xi^3 EJ_z} V_{VF}(x_3)\,, \tag{26}$$

where

$$\begin{aligned}
S_S(x_3) &= \frac{\cosh(\xi x_3) + \cos(\xi x_3)}{2}\,, \\
T_T(x_3) &= \frac{\sinh(\xi x_3) + \sin(\xi x_3)}{2}\,, \\
V_{V1}(x_3) &= \frac{\sinh(\xi(x_3 - L_1)) - \sin(\xi(x_3 - L_1))}{2}\,, \\
V_{VF}(x_3) &= \frac{\sinh(\xi(x_3 - L_f)) - \sin(\xi(x_3 - L_f))}{2}\,.
\end{aligned}$$

Applying expression (11) to describe the deflection of the fourth section, which is within the limits $L_1 + L_2 = 0.358$ m $< x_4 < L = 0.54$ m, we find the function $w_4(x_4)$ that describes the transverse deflection of the plate along x-axis in the fourth section:

$$\begin{aligned}
w_4(x_4) = {} & AS_S(x_4) + BT_T(x_4) \\
& + \frac{R_1}{\xi^3 EJ_z} V_{V1}(x_4) + \frac{R_2}{\xi^3 EJ_z} V_{V2}(x_4) + \frac{F}{\xi^3 EJ_z} V_{VF}(x_4)\,,
\end{aligned} \tag{27}$$

where

$$\begin{aligned}
S_S(x_4) &= \frac{\cosh(\xi x_4) + \cos(\xi x_4)}{2}\,, \\
T_T(x_4) &= \frac{\sinh(\xi x_4) + \sin(\xi x_4)}{2}\,, \\
V_{V1}(x_4) &= \frac{\sinh(\xi(x_4 - L_1)) - \sin(\xi(x_4 - L_1))}{2}\,, \\
V_{V2}(x_4) &= \frac{\sinh(\xi(x_4 - L_1 - L_2)) - \sin(\xi(x_4 - L_1 - L_2))}{2}\,, \\
V_{VF}(x_4) &= \frac{\sinh(\xi(x_4 - L_f)) - \sin(\xi(x_4 - L_f))}{2}\,.
\end{aligned}$$

Substituting all the necessary parameters in expressions (24) – (27), the compared functions of the deflections in the four sections that form the transverse deflection of the plate along x-axis on its entire length, will take the form as in Fig. 5.

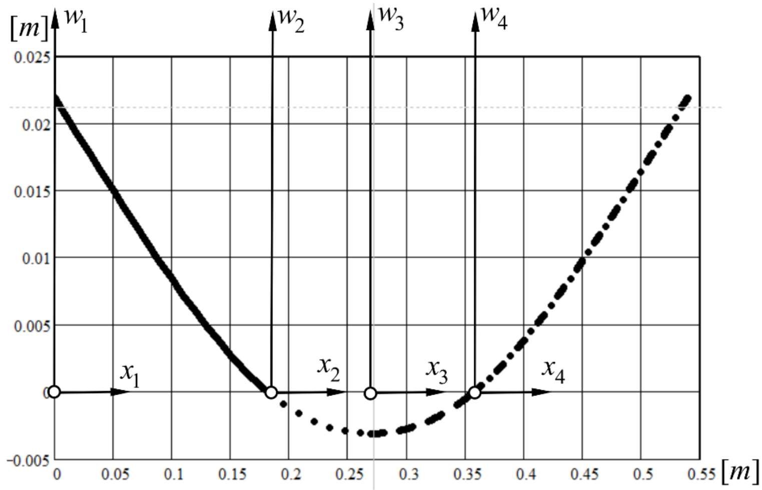

Fig. 5: The transverse deflection of the plate along x-axis.

As can be seen from Fig. 5, section 1 (Fig. 4) is symmetrical to section 4, and section 2 is symmetrical to section 3, so in the future calculations, we consider only section 1 (console) and section 2 (left part of the middle).

To describe the transverse deflections of the console along y-axis (Fig. 3) on the region $0 < y < h = 0.0875$ m, we propose the following dependence:

$$w_c(y) = \sin(\xi y/(2h))0.003 + 1\,. \tag{28}$$

Transverse deflections of the middle of the plate along y-axis, given that the deflection in this area will be slightly smaller than the deflection of the console, are described by the function:

$$w_m(y) = \sin(\xi y/(2h)0.001 + 1\,. \tag{29}$$

onsidering the plate console (Fig. 3), we construct the basic functions on individual sections of the plate. Deflection occurs simultaneously in the transverse directions along the x and y axes. Operating by dependencies (24) and (28), the basic function of deflections of this section will be:

$$W_c(x_1 y) = w_1(x_1)w_c(y)\,. \tag{30}$$

Multiplying expressions (25) and (29), we can find the basic function for establishing the deflection of the surface of the left part of the middle of the plate. The obtained function will be following:

$$W_m(x_2, y) = w_2(x_2)w_m(y)\,. \tag{31}$$

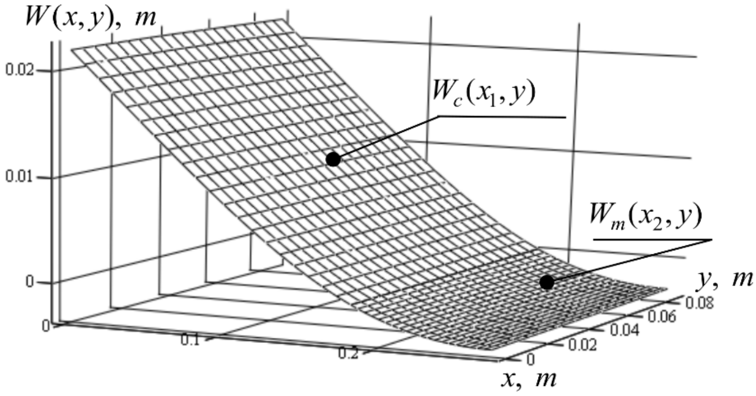

Fig. 6: Three-dimensional deflection of the surface of half of the plate.

The graphical dependences of functions (30), (31), which describe the deflection of the surface of half of the plate in 3D, are shown in Fig. 6. The deflection of the second half of the plate will be symmetrical. This condition can be seen in Fig. 5.

Figure 6 shows that the transverse deflection along y-axis is small compared to the transverse deflection along x-axis. Therefore, to simplify the calculations of the natural frequencies of oscillations of rectangular plates with similar fastening conditions, we can consider their deflection only along x-axis.

According to the Rayleigh-Ritz method [17], the kinetic energy of the plate is equal to the potential. So we can write the following equation:

$$2P_c + 2P_m = 2K_c + 2K_m \,, \tag{32}$$

where

$$P_c = \frac{1}{2}\int_0^{L_1}\int_0^{h} D\Bigg[\Big(\frac{d^2W_c(x_1,y)}{dx_1^2} + \frac{d^2W_c(x_1,y)}{dy^2}\Big)^2 + 2(1-\mu) \times \Big[\Big(\frac{d^2W_c(x_1,y)}{dx_1 dy}\Big)^2 - \frac{d^2W_c(x_1,y)}{dx_1^2}\frac{d^2W_c(x_1,y)}{dy^2}\Big]\Bigg])\mathrm{d}y\,\mathrm{d}x_1 \,, \tag{33}$$

$$P_m = \frac{1}{2}\int_{L_1}^{L_f}\int_0^{h} D\Bigg[\Big(\frac{d^2W_m(x_2,y)}{dx_2^2} + \frac{d^2W_m(x_2,y)}{dy^2}\Big)^2 + 2(1-\mu) \times \Big[\Big(\frac{d^2W_m(x_2,y)}{dx_2 dy}\Big)^2 - \frac{d^2W_m(x_2,y)}{dx_2^2}\cdot\frac{d^2W_m(x_2,y)}{dy^2}\Big]\Bigg]\mathrm{d}y\,\mathrm{d}x_2 \tag{34}$$

— respectively, the potential energies of the console and the left side of the middle of the plate:

$$K_c = \int_0^{L_1} \int_0^h \rho b_n \omega_n^2 (W_c(x_1, y))^2 \, \mathrm{d}y \, \mathrm{d}x_1 \,, \tag{35}$$

$$K_m = \int_{L_1}^{L_f} \int_0^h \rho b_n \omega_n^2 (W_m(x_2, y))^2 \, \mathrm{d}y \, \mathrm{d}x_2 \tag{36}$$

— kinetic energies of analogous parts of the plate.

In expressions (33), (34) the multiplier D can be obtained from:

$$D = E b_n^3 / (12(1 - \mu^2)) \,, \tag{37}$$

where $\mu = 0.26$ – Poisson's ratio.

Given expressions (33) – (36), from equation (32) we can write an analytical expression to establish the first natural frequency of the plate:

$$\omega_n = \sqrt{\frac{2P_c + 2P_m}{2 \int_0^{L_1} \int_0^h \rho b_n (W_c(x_1, y))^2 \, \mathrm{d}y \, \mathrm{d}x_1 + 2 \int_{L_1}^{L_f} \int_0^h \rho b_n (W_m(x_2, y))^2 \, \mathrm{d}y \, \mathrm{d}x_2}} \,. \tag{38}$$

Substituting the necessary parameters into expression (38), the first natural circular frequency of the plate oscillations will be $\omega_n = 311.8$ rad/s (cyclic frequency $\upsilon_n = 49.6$ Hz).

To verify the reliability of the proposed method for calculating the natural frequency of oscillations of a plate with complex boundary conditions, experimental investigation was performed on a discrete-continuous inter-resonance vibrating table with an electromagnetic drive.

3 Experimental Investigation

In Fig. 7(a), an experimental sample of a discrete-continuous inter-resonance vibrating table during its work at the first natural frequency of the plate oscillations is shown. The voltage on the electromagnetic vibrators was supplied from a 100-watt harmonic signal generator. The perturbation of the plate occurred from electromagnets according to the sinusoidal law. The dimensions of the plate in the experimental setup are similar to the dimensions of the theoretically considered plate.

The obtained value of the frequency sensor, shown in Fig. 7(b), indicates that the first natural frequency of the plate corresponds to the results obtained using the

(a) (b)

Fig. 7: Experimental sample of a discrete-continuous inter-resonance vibrating table with a plate (a) perturbed at the first natural oscillation frequency, and sensor parameters (b) that show the values of voltage, current, and frequency.

proposed method of calculating the natural frequency of the plate (the results obtained according to expression (38)).

4 Conclusion

Considering a plate with complex boundary conditions used in an inter-resonance vibrating table with an electromagnetic drive, its first natural oscillation frequency was established by the approximated Rayleigh-Ritz method. This value is necessary for further synthesis of the structure. The feature of this calculation is that the kinetic and potential energies of the plate were established through the basic functions of the deflections of its surface, presented as the product of the functions of the transverse deflections along the x and y axes, taking into account the boundary conditions. The transverse forced deflection along the x-axis of the plate is described by Krylov-Duncan functions, and transverse forced deflection along the y-axis by circular functions. Due to this, it was possible to find the approximate value of the first natural frequency of the plate oscillations quite accurately, which was confirmed experimentally.

References

[1] O. Lanets, P. Maistruk, V. Borovets, I. Derevenko (2019) Adjustment of parameters of three–mass interresonant vibrating machines with an inertial exciter. *Industrial Process Automation in Engineering and Instrumentation* **53** 13-22.

[2] O. Lanets, O. Kachur, V. Korendiy (2019) Classical approach to determining the natural frequency of continual subsystem of three–mass inter-resonant vibratory machine. *Ukrainian Journal of Mechanical Engineering and Materials Science* **5**(3-4) 77-87.

[3] V. GURSKY, I. KUZIO (2016) Strength and durability analysis of a flat spring at vibro–impact loadings. *Eastern-European Journal of Enterprise Technologies* **5** 4-10.

[4] A. LEISSA (2005) The historical bases of the Rayleigh and Ritz methods. *Journal of Sound and Vibration* **287**(4-5) 961-978.

[5] Y. KUMAR (2018) The Rayleigh–Ritz method for linear dynamic, static and buckling behavior of beams, shells and plates: A literature review. *Journal of Vibration and Control* **24**(7) 1205-1227.

[6] X. SHI, D. SHI (2018) Free and forced vibration analysis of T–shaped plates with general elastic boundary supports. *Journal of Low Frequency Noise Vibration and Active Control* **37**(2) 355-372.

[7] G. JIN, X. MA, S. SHI, T. YE, Z. LIU (2014) A modified Fourier series solution for vibration analysis of truncated conical shells with general boundary conditions. *Applied Acoustics* **85** 82-96.

[8] A. BORKOVIĆ, G. RADENKOVIĆ, D. MAJSTOROVIĆ, S. MILOVANOVIĆ, D. MILAŠINOVIĆ, R. CVIJIĆ (2020) Free vibration analysis of singly curved shells using the isogeometric finite strip method. *Thin-Walled Structures* **157** 107-125.

[9] K. ŻUR, M. AREFI, J. KIM, J. REDDY (2020) Free vibration and buckling analyses of magneto–electro–elastic FGM nanoplates based on nonlocal modified higher–order sinusoidal shear deformation theory. *Composites Part B: Engineering* **182** 107-601.

[10] T. TRAN, P. NGUYEN, Q. PHAM (2021) Vibration analysis of FGM plates in thermal environment resting on elastic foundation using ES–MITC3 element and prediction of ANN. *Case Studies in Thermal Engineering* **24** 100-852.

[11] M. LI, C. SOARES, R. YAN (2021) Free vibration analysis of FGM plates on Winkler/Pasternak/Kerr foundation by using a simple quasi–3D HSDT. *Composite Structures* **264** 113-643.

[12] D. LIU, Y. ZHOU, J. ZHU (2021) On the free vibration and bending analysis of functionally graded nanocomposite spherical shells reinforced with graphene nanoplatelets: Three–dimensional elasticity solutions. *Engineering Structures* **226** 111-376.

[13] X. XU, Q. HAN, F. CHU (2018) Review of Electromagnetic Vibration in Electrical Machines. *Energies* **11**(7) 1779.

[14] K. YIM, J. JANG, G. JANG, M. KIM, K. KIM (2012) Forced Vibration Analysis of an IPM Motor for Electrical Vehicles due to Magnetic Force. *IEEE Transactions on Magnetics* **48**(11) 2981-2984.

[15] M. SHISHESAZ, A. ZAKIPOUR, A. JAFARZADEH (2016) Magneto–Elastic Analysis of an Annular FGM Plate Based on Classical Plate Theory Using GDQ Method. *Latin American Journal of Solids and Structures* [on-line] **13**(14) 2736-2762.

[16] F. TORNABENE, N. FANTUZZI, M. BACCIOCCHI (2016) The GDQ method for the free vibration analysis of arbitrarily shaped laminated composite shells using a NURBS–based isogeometric approach. *Composite Structures* **154** 190-218.

[17] S. PONOMAREV (1959) “Strength calculations in mechanical engineering”. Mashgiz, Moscow, USSR, pp. 294-302.