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INTERDISCIPLINARY TOPICS

DETERMINATION OF ADDED MASS FOR EARTHQUAKE INDUCED HYDRODYNAMIC LOADINGS ON VERTICAL STRUCTURES

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ABSTRACT: Earthquakes can induce severe hydrodynamic loadings on dams and their equipment. The dynamic dam-reservoir interaction is usually modelled by the so-called added mass. There are two theoretical approaches in the literature, employing either momentum or compressibility theory. Both assume zero pressures at the top of the structure, leading to comparatively low design pressures for control structures. A new theory was developed for a simple approximation for added mass and hydrodynamic pressures. The added mass is assumed to be uniform over the dam height. The added mass becomes a linear function of acceleration and water depth. Comparison with 'classic' theories showed good agreement for accelerations up to $0.3g$, and higher pressures for higher accelerations.

KEY WORDS: dam-reservoir interaction, added mass, simplified theory.

1 INTRODUCTION AND LITERATURE REVIEW

1.1 OVERVIEW

Earthquake induced hydrodynamic loadings are an important design consideration for dams in seismically active zones. This is enhanced by the fact that the weight of the reservoir itself may induce earthquakes, e.g. Gupta [1]. The aspects of seismic loadings on dams are very variable, ranging from direct effects of the earthquake induced accelerations to rockfalls and damages to secondary elements such as power transmission infrastructure, e.g. Wieland [2]. In concrete dams, the seismic loadings are also important for the design of spillway gates located at the top of the structure. Figure 1 shows such gates.

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Fig. 1: Dam with typical control elements at the top (Picture: D. Kisliakov).

The control elements need to be designed for seismic loadings in order to remain functional even after a design earthquake.

1.2 Compressible water and momentum approach

The fact that a dam failure caused by an earthquake can have significant consequences, especially when the reservoir is full, led to investigations of seismically induced hydrodynamic pressures from the early 20th century onwards. The question of the physics and magnitude of such earthquake-induced hydrodynamic pressure, and of whether the compressibility of the water has to be taken into account, initiated a first systematic study of this phenomenon by Westergaard [3]. Here, the water was assumed to be a compressible fluid. One of the key findings was, that the interaction between the moving (rigid) dam due to its support excitation and the water in the reservoir can be represented by additional mass and damping terms in the governing equation of motion of the dam, which can be modeled as a Single Degree Of Freedom (SDOF) system undergoing horizontal harmonic oscillations. A formula was proposed to determine the added mass, and the arising pressures.

The results and conclusions allowed a simple modeling of the exceptionally complex physical phenomena in all related cases of practical application. Thus, the introduced concept of “added mass” - the inertia added to a system because the accelerating or decelerating dam structure must move some volume of the water adjacent to it - converted the problem of hydrodynamic fluid-structure interaction into a trivial one of solid mechanics only. The momentum-balance approach used by Kármán [4],

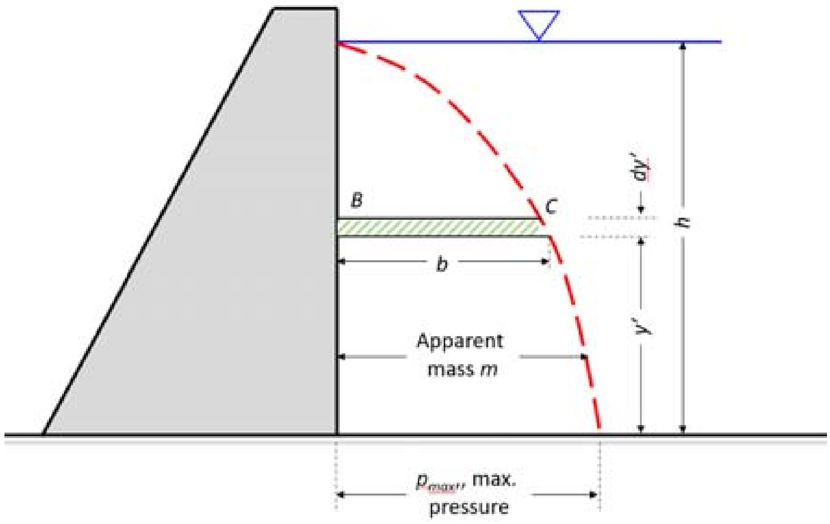


Fig. 2: Pressure distribution/added mass, [4].

Fig. 2, gave a close approximation of the result of Westergaard's. Because of its significance, Westergaard's and Kármán's work resulted in a strong resonance in the professional community, see e.g. the discussion section in Kármán [4]. As expected, the proposed concept and relations (with their related assumptions and mathematics) immediately became subject of intensive further developments, aimed at both covering the most physically possible cases and relations for practical implementation.

For decades, the concept of the added mass and its implementation has been a subject of intensive research which resulted in a vast amount of dedicated works. In this regard, the papers presented by Chwang and Housner [5, 6] should be mentioned as some of the first ones giving related solution for dams with inclined slopes. Further work based of theory was based on the assumption of water being incompressible, Zangar [7]. It should be noted that under some particular conditions such as concrete gravity dams or similar massive structures, the influence of a water reservoir represented by added masses physically corresponds to an incompressible fluid, Bachmann [8].

1.3 Recent developments

If taken into account, frequency-dependent and complex added masses result from a detailed response analysis including the frequency range. A criterion for the influence of the compressibility of the water was proposed by Bachmann [8] and Hall [9]. If the ratio of the fundamental frequency of the reservoir (so called *cut-off-frequency*) and the frequency of the symmetrical fundamental oscillation of the dam (with empty reservoir) is greater than 1.5, then the water's compressibility may be neglected. In such cases, the added mass concept may be applied as realistic enough. For arch dams, there is an approximate formula for estimating the fundamental frequency of the reservoir, see e.g. Hall [9]. However, the large structural variety of the arch dams, the spatial character of the dynamic fluid-structure interaction, moreover, related to the parameters of the valley, do not allow for a direct application of this concept to arch dams. As a result of a related research project, a special computational procedure was developed for added masses at arch dams, Kuo [10]. A more recent development of the added mass model is based on both experimental and theoretical research, Maheri [11]. This modified method was validated with extensive numerical studies.

Here, van Brummelen [12] should especially be mentioned, where a sophisticated comparison of the added mass approximation for both compressible and incompressible fluid models was carried out. The fundamental difference between these two models was outlined with their consequences for staggered time-integration procedures, respectively.

Among the large amount of research works on the added mass approach, several review papers should finally be mentioned since they chronologically trace the development of this concept, presenting important theoretical and comparative studies, e.g. Bachmann [8], Hall [9], Finn et al. [13], Salamon [14, 15]. The most important results in these developments are the refined procedures for computation of the added masses, the extended boundary conditions for which this approach is valid as well as the much more realistic structural modeling of the dam or other hydraulic aspects. Another result of decisive importance is the more precise knowledge about the conditions of applicability of the added mass approach, i.e. for which types of dams and other hydraulic structures this approach can be applied without loss of safety.

It also should be noted that the application of the added mass approach is being constantly extrapolated to other types of hydraulic facilities and of fluid-structure interaction, respectively Hendrikse et al. [16], Maniaci and Ye [17]. Obviously, the extreme convenience of the problem transformation from complex physical interaction to a purely solid mechanics problem is here a factor of crucial importance.

Last but not least, the added mass concept is by no means the only approach to taking into account the role of the reservoir in the dynamic fluid-structure interaction of hydraulic structures and the water domain. Even a short review of the other methods implemented for modelling of these phenomena would go far beyond the frame of this short introduction. However, the review papers mentioned before as well as Shulman [18] could submit a very approximate impression of the development of these methods.

2 Motivation

The earthquake induced damages of control structures located in the upper third of dams (especially slender concrete dams) led to the assumption that actual pressures may be higher than those predicted by current added mass concepts. In fact, the observed accelerations and damages of facilities in the vicinity of the dam crest may be due to the considerable amplification of the input support excitation (including the change of the frequency content) in terms of kinematic quantities – acceleration, velocity and displacement, e.g. Wieland [2]. In addition, there is the question of earthquake induced surface waves and/or sloshing (seiches) – neglected by Westergaard [3] which led to the practical need of special free board requirements in the design regulations.

A key characteristics of all present solutions is the fact that the added mass and related hydrodynamic pressures near the surface are small, and maximum pressures occur at the toe of the structure.

The problem formulation is simple – what part of the water domain affects the externally induced motion of an adjacent rigid hydraulic structure in terms of additional load and/or impact, simply represented as an additional amount of mass.

3 Uniform Added Mass Theory (UAM)

3.1 Analysis of existing theories

Westergaard's solution assumes a zero stress boundary condition at the water surface, the pressure then increases to reach a maximum at the toe of the structure. This can be criticised for two reasons:

A The horizontal displacement of the dam must lead to a vertical displacement of the water, which in turn creates an increased hydrostatic pressure at the original surface level. Westergaard's solution does predict a vertical increase, and actually has a singularity at the point where the surface meets the dam, with an infinite increase of the water level at that point. The resulting gravity induced forces are however neglected.

B The horizontal acceleration and compression of a mass of water which is constrained by a rigid vertical and a lower horizontal boundary must result in an upward vertical acceleration and expansion. This upward vertical acceleration, which increases the total inertia effect of the water, is not included.

Kármán [4] presented his momentum based solution, assuming incompressibility, which led to very similar results as Westergaard. However, there are two further points similar to those mentioned for Westergaard's theory where Kármán's work could be improved:

A Although a vertical acceleration term is introduced, the vertical upwards displacements and the resulting inertia forces are neglected. The vertical acceleration does not include a gravity related term.

B The vertical water surface distortion is assumed to be zero. The WL near the dam must however increase, due to the assumption of incompressibilty, and the horizontal displacement of volume. This would also lead to a non-zero stress condition at the original surface.

The question is of course, how important these factors can be. So, despite the fact that these solutions have been in used for nearly 90 years, there still is room for improvement.

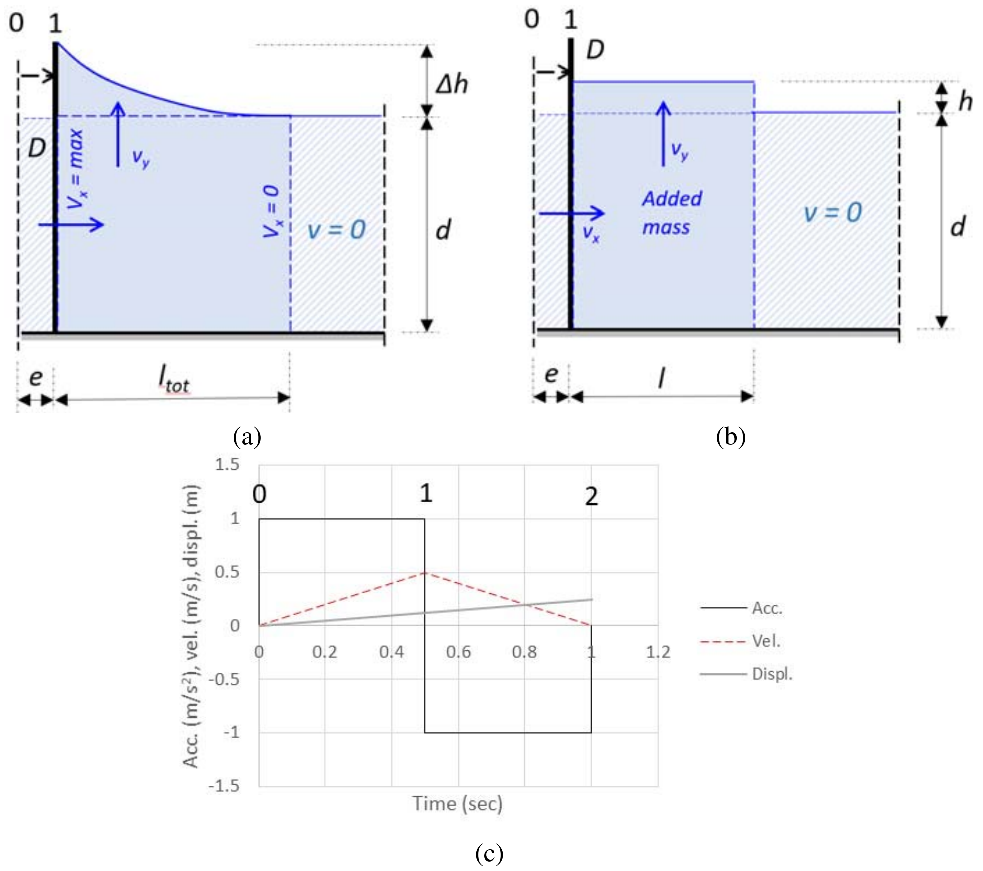


Fig. 3: Initial conditions for analysis: (a) dam displacement and water surface; (b) assumed dam displacement and uniform added mass; (c) assumed acceleration – time diagram.

3.2 UNIFORM ADDED MASS THEORY

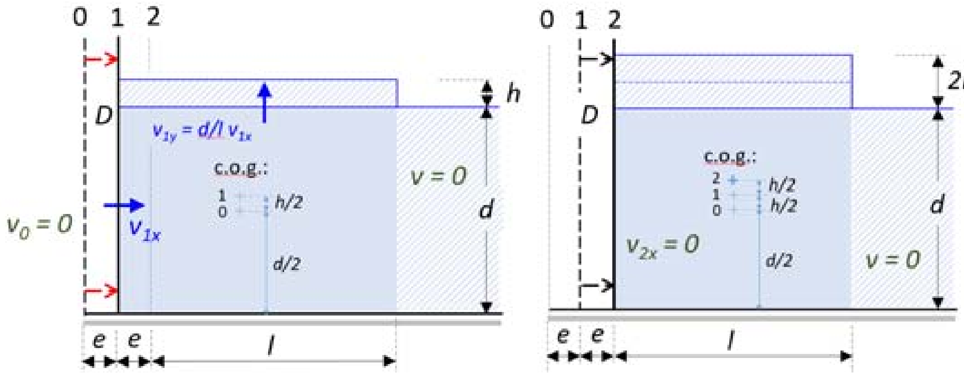
The theory was developed in order to determine a simple expression for a uniform added mass which takes part in the horizontal acceleration of a reservoir dam. Figure 3a shows the horizontal displacement (equal along its full height) of a dam D and the corresponding water level disturbance. Here, the fluid assumes a maximum velocity at the dam, and zero velocity at a distance 'l_{tot}' away from the dam. In the approach chosen, the actual accelerated mass of water is replaced with an added mass of length 'l'', where the velocity is assumed to be constant throughout. Outside the added mass, the fluid is assumed to be at rest, Fig. 3b. The relationship is then assessed between initial horizontal acceleration and the horizontal and vertical displacement of the water, as well as the additional pressures from inertia effects and an increased water level (WL).

For the analysis, a simplified acceleration – time diagram is assumed, Fig. 3c. The rigid dam D experiences only a horizontal displacement $2e$. It initially moves a distance e from point '0' to '1' during a time t, with a constant acceleration a_0, reaching a velocity v_1 at '1'. At point '1', the acceleration reverses and the dam moves forward with decreasing velocity through a distance $e$to point '2', where the displacement terminates. In Fig. 3c, the case is shown for a unit acceleration of $a_0 = 0.1g \approx 1$ m/s^2 for positive and negative acceleration, for a total duration $2t_e = 2$ s and a total displacement of $2e = 0.2$ m.

4 ASSUMPTIONS FOR THE ANALYSIS

For the theoretical model the following assumptions are made:

- The water is incompressible.
- The added mass m is an equivalent mass which experiences an initial uniform horizontal acceleration a_0. The rest of the fluid remains undisturbed. It follows, that the initial horizontal displacement $e$causes a uniform vertical displacement of height h.
- The added mass m has a length l, with $m = \rho dl$.
- The displacements e and h are small compared with l and d.
- The vertical dam D moves from point '0' to point '2' over a distance $2e$ during a time t_e. It experiences an initial positive acceleration a_0 for a time t with $0 \leq t \leq t_e/2$ from point '0' to '1', Fig. 4a, and a negative acceleration -a_0 for $t_e/2 \leq t \leq t_e$ from '1' to '2', Fig. 4b.



(a) (b)

Fig. 4: Horizontal and vertical displacement: (a) positive acceleration; (b) negative acceleration.

- At '1', the mass has a horizontal velocity v_{1x} and a vertical velocity v_{1y}, where $v_{1y} = v_{1x}d/l$.
- At the end of the horizontal displacement at '2', acceleration and velocity are zero. The WL has increased to a vertical distance $2h$ above the original location.

5 Analysis

The problem will be analysed by considering the energy balance during a horizontal displacement. The analysis is divided into two steps:

A Fig. 4a, positive acceleration: the dam moves from '0' to '1' through a distance e with an acceleration a_0 during a time t, whereby $e = a_0t^2/2$. During this movement, the mass $m = \rho ld$ is accelerated to a horizontal velocity v_{1x}. and a vertical velocity $v_{1y} = d/lv_{1x}$. The displaced volume $V = ed$ leads to an increase in water level of +h. The centre of gravity c.o.g. moves upwards by a distance h/2.

B Fig. 4b, deceleration: the dam D moves from '1' to '2' with a negative acceleration -a_0 through a distance e. The displaced volume $V = ed$ leads to a further increase in water level of +h to a height $2h$ above the original level.

It is assumed that when moving from '1' to '2', the dam does not impart any more energy onto the added mass. At point '1', when the acceleration is zero for a moment, the kinetic energy E_{kin1} of the vertically moving mass of fluid m must –

for continuity – therefore be large enough to allow for the increase in WL from h to $2h$ when the dam moves from '1' to '2'. The potential energy ΔE_{pot2} obtained when lifting the WL from h to $2h$, to accommodate for the displaced volume, must therefore equal the vertical kinetic energy E_{kin1} available at '1'.

This condition will be employed to determine the length l of the added mass:

(1) $$E_{\text{kin1}} = \Delta E_{\text{pot2}} \,.$$

The kinetic energy E_{kin1} becomes:

(2) $$E_{\text{kin1}} = m\frac{v_{1y}^2}{2} = \rho(d+h)(l+e)\frac{v_{1y}^2}{2} \,.$$

When moving from '1' to '2', the centre of gravity c.o.g. of the added mass plus vertical WL increase h is raised by a vertical distance $\Delta y = h/2$.

(3) $$\Delta E_{\text{pot2}} = \rho g d(l+2h)\frac{h}{2} \,,$$

(4) $$\rho(d+h)(l+e)\frac{v_{1y}^2}{2} = \rho g l(d+2h)\frac{h}{2} \,.$$

With $h = ed/l$, $v_{1y} = v_{1x}d/l$ and assuming that $e \ll l$, Eq. (4) becomes:

(5) $$\rho\Big(d+\frac{ed}{l}\Big)(l+e)\frac{d^2}{l^2}\frac{v_{1x}^2}{2} = \rho g l\Big(d+2\frac{ed}{l}\Big)\frac{ed}{2l}$$

with only l as the unknown.

A simplification, equating the kinetic energy of the horizontally, and potential energy of the vertically displaced volume only, leads to the following formula:

(6) $$\rho\frac{ed}{l}l\frac{d^2}{l^2}\frac{v_{1x}^2}{2} = \rho g l 2\frac{ed}{l}\frac{ed}{2l} \,.$$

With $e = a_0t^2/2$ and $v_{1x} = a_o t$, this further simplifies to

(7) $$\frac{l}{d} = \frac{2a_0}{g} \,.$$

For a sinusoidal acceleration profile with a period T, Eq. (7) becomes:

(8) $$\frac{l}{d} = \frac{4}{\pi}\frac{a_0}{g} \,,$$

where a_0 is here the maximum horizontal acceleration. Figure 5 shows the linear relationship between acceleration and added mass length. The graph also shows the effective length l_{eff}, which takes the vertical acceleration into account. This is described in the next section.

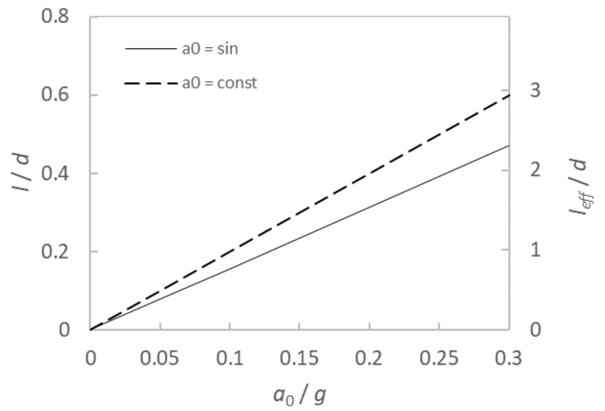


Fig. 5: Normalised added mass length l/d and effective length l_{eff}/d for constant and sinusoidally varying normalised acceleration.

5.1 MAXIMUM HORIZONTAL PRESSURE, INCLUDING THE ADDITIONAL HYDROSTATIC PRESSURE

The maximum pressure $p_{\max}$ acting on the dam D is the sum of the acceleration pressure p_{acc} and the hydrostatic pressure $p_{\text{hyd}} = \rho g h$ at point '1'. The pressure is independent of the depth. The maximum pressure occurs when the initial acceleration combines with the hydrostatic pressure at '1'. For the determination of the acceleration pressure p_{acc} it must be considered that the horizontal acceleration a_0 also causes a vertical acceleration $a_y = 2h/t^2$, and a subsequent displacement of the water. The total acceleration of the fluid a_1 which resists the motion is therefore taken as the sum of horizontal and vertical acceleration, $a_1 = a_0 + 2h/t^2$:

(9) $$p_{\max} = a_1 \rho l + \rho g h \,,$$

where for constant acceleration a_0:

(10) $$h = e\frac{d}{l} = a_0\frac{t^2}{2}\frac{g}{2a_0} = \frac{gt^2}{4} \,.$$

This allows to simplify the expression for the effective acceleration to $a_1 = a_0 + g/2$. With this expression, the effective length l_{eff} of the added mass can be determined using Eq. (7) for a rectangular acceleration – time graph as:

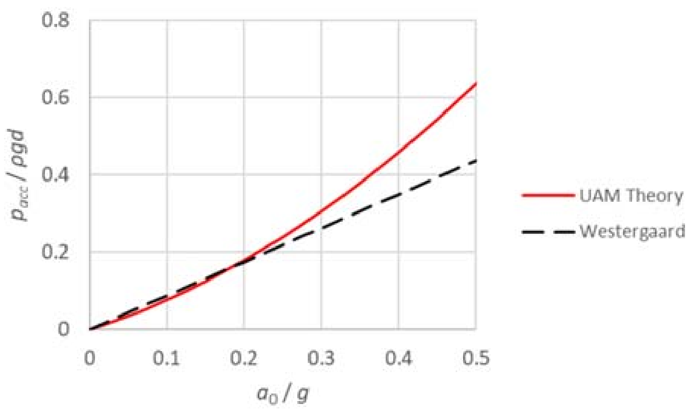
(11) $$\frac{l_{\text{eff}}}{d} = \frac{2}{g}a_1 \,,$$

l_{eff} is shown in Fig. 5 as function of a_0/g. The maximum pressure $p_{\max}$ becomes:

$$p_{\max} = a_1 \rho \frac{a_0}{g} d + \rho g \frac{gt^2}{4} \; . \tag{12}$$

Figure 6a shows the normalised pressures as function of the normalised acceleration. For comparison, the figure also includes the maximum pressure for Westergaard's approximate solution as presented in Salamon [14]. These are the pressures at the toe of the structure, see Fig. 2. The normalised pressures $p_{\max}/\rho g d$ are a function of the acceleration ratio a_0/g only. The values for $p_{\max}/\rho g d$ from Westergaard's equation are close to those from the UAM for normalised accelerations of $a_0/g < 0.3$, for larger values the new theory gives larger results. It should be noted

(a)



(b)

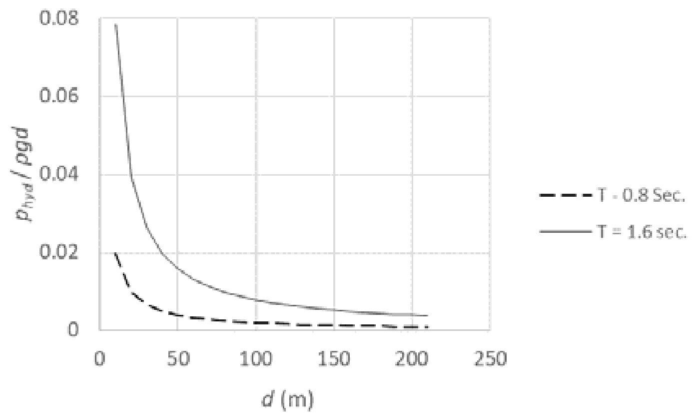


Fig. 6: (a) Normalised maximum acceleration pressure $p_{\text{acc}}/\rho g d$, as function of normalised acceleration. (b) Normalised additional hydrostatic pressure $p_{\text{hyd}}/\rho g d$ as function of water depth and excitation period T.

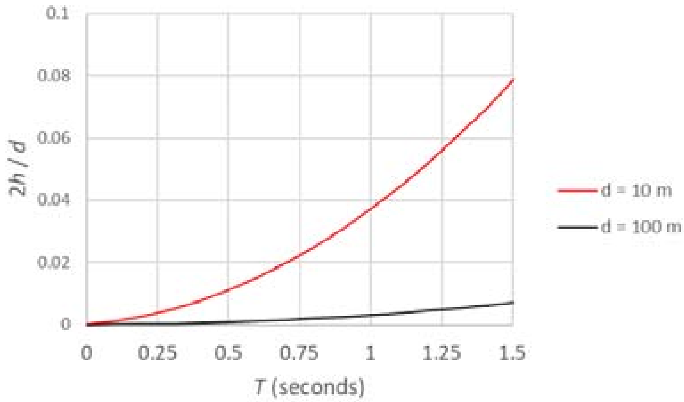


Fig. 7: Normalised WL increase $2h/d$ as function of acceleration period T, and water depth d.

that Westergaard's formula results in a zero pressure at the water surface, so that it will lead to: (a) lower pressures in the upper section of the dam and on the control gates; and (b) to a lower total force acting on the dam.

Figure 6b shows the normalised additional hydrostatic pressure $p_{\text{hyd}}/\rho g d$ as function of the water depth for two different excitation periods T. The additional hydrostatic pressure p_{hyd} is independent of the acceleration, since with reducing acceleration the length of the added mass l also reduces. It becomes more significant with reducing water depth d, and increasing acceleration period T.

Figure 6b already showed that the effect of the relative WL increase is small for larger water depths d. For smaller water depths however, the increase in WL may be of importance due to the increased quasi-hydrostatic pressure, and the possibility of overtopping. In Fig. 7, the normalised WL increase $2h/d$ is plotted as function of acceleration period T. For a water depth of $d = 10$ m, and an acceleration period of $T = 1.5$ seconds, $2h/d$ reaches 0.08, so that a water level increase of 0.8 m can be expected. The model assumes a simplified constant WL increase. In reality, the increase in WL at the dam will be significantly higher, probably 2 to 3 times this value, see Fig. 3a. If overtopping is an issue, this would need to be considered.

6 Comparison with Westergaard's Solution

6.1 Example from the literature

In Salamon [15], an example of the calculation of an added mass and additional pressures is given:

Water depth: $d = 240$ m

Acceleration: $a_0 = 0.1g$

Period: $T = 1.33$ s, so that $t = T/4 = 0.33$ s.

The maximum pressure occurs at the toe of the structure with $p_{\max} = 210$ kN/m^2. The length of the added mass at the top of the structure is zero, at the bottom $l = 210$ m, see Fig. 2, with an average value of $l_{\text{av}} = 140.7$ m. The resultant force per meter width is $F_{\text{res}} = 11.01$ MN/m. This value appears to be faulty, the integral of a parabolic pressure distribution with a maximum of 210 kN/m at the bottom and zero at the top gives a value of 32.8 MN. The pressure at $y = 30.3$ m below WL is $p_y = 70$ kN/m^2.

The values can now be compared with the results from the approach developed in this paper. The acceleration is assumed to be sinusoidal, so that $a_0 = 2/\pi \times 1.0$ m/s$^2 = 0.64$ m/s^2.

The calculations then give the following results:

$$l = 31.3 \text{ m}, \qquad h = 0.27 \text{ m}$$
$$p_{\max} = (1 + 9.81/2)31.3 + 9.81 \times 0.27 = 187.5 \text{ kN/m}^2\,.$$

Resultant force $F_{\text{res}} = 45.0$ MN/m.

These results are compared with the results from the uniform added mass theory in Table 1.

Table 1: Comparison of results

Parameter	Westergaard	Uniform added mass
Av. length l (l_{eff}) of added mass (m)	140.7	31.3 (184.8^a)
Vertical acceleration (m/s^2)	0	4.905
Vertical displacement $2h$ (m)	Not given	0.54
Pressure 30.3 m below WL (kN/m^2)	70	187.5
Max. pressure $p_{\max}$ at bed (kN/m^2)	210	187.5
Resultant force F_{res} (MN/m)	11.01 (32.8^b)	45.0

^{a}: including vertical acceleration; ^{b}: corrected value

Pressures and forces generated by the new solution are significantly larger than those from Westergaard.

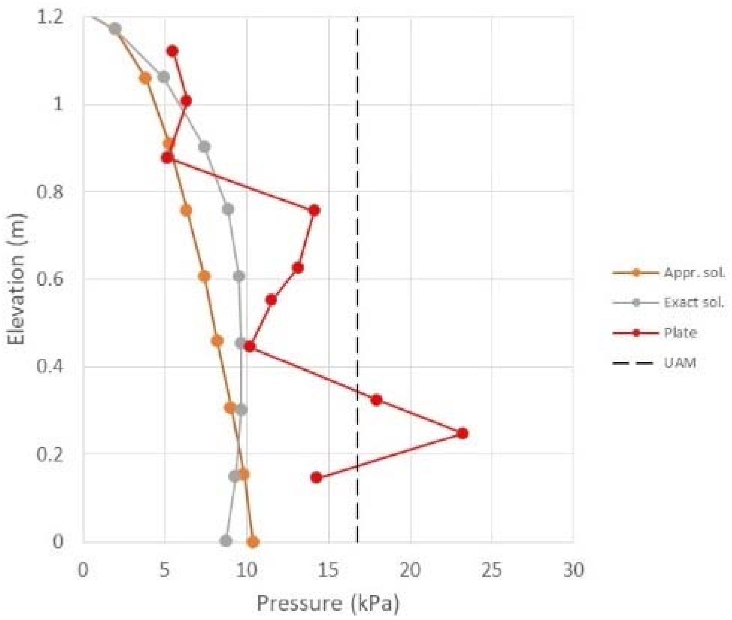


Fig. 8: Test results from Mortensen [19], and comparison with UAM theory.

6.2 Comparison with model tests

Mortensen [19] describes laboratory tests where the pressures on a spillway gate were investigated. The water depth was 1.18 m, excitation frequencies were $f = 5$, 10 and 50 Hz. Figure 8 shows the results of a test run with $a_0 = 1.02g$, $f = 10$ Hz with a calculated horizontal displacement of ± 1 mm (0.04 inch), and the theoretical pressures from Westergaard's solution.

The measured pressures were significantly larger than those predicted by Westergaard's theory, this was attributed to the structural response of the instrumented plate. The theoretical results from the UAM theory were also added, taking into account that the excitation during the tests was sinusoidal so that for the theory $a'_0 = 4/\pi a_0$. The added mass had a length $l = 1.50$ m, the maximum pressure $p_{\max} = 16.8$ kPa. The UAM theory provides a simplified vertical distribution, but matches the measured higher pressures reasonably well.

7 Discussion

A new theory was developed to assess the added mass at a rigid vertical dam under an earthquale excitation. The purpose of the work was, to develop a simple heory which allows for an initial estimate of added mass and pressures without having to revert to complex nnumerical models.

The theory includes the effect of a vertical acceleration and a vertical displacement of the water surface. The assumptions of a uniform added mass and a simple forward motion means that the results can only be an approximation, Nevertheless, the inclusion of the vertical acceleration and gravity forces led to significant differences to the two 'classic' theoretical approaches by Westergaard and Kármán'. The added mass is now uniformly distributed, and therefore larger near the top of the dam. This will increase accelerations at the top of the dam.

The pressures in the upper section of the dam are susequently higher and, for horizontal accelerations larger than approximately $0.3g$, the pressures and total forces on the dam are larger as well. The relative vertical surface distortion h/d increases for reducing d, so that the additional hydrostatic load (neglected by Westergaard and Karman) can become significant for shallow water depths.

The model presented here is consistent regarding horizontal and vertical accelerations and velocities as well as continuity and conservation of mass. The elastic deformation of the water is considered as negligible.

The assumption of a uniform added mass, and a rectangular vertical displacement, is very simple. A more realistic assumption would be a triangular shape of the vertical displacement, with a maximum near the dam. This would also enforce a zero acceleration condition at the reservoir boundary of the added mass.

8 CONCLUSIONS

A simple theory was developed to determine the added mass of a reservoir dam under earthquake excitation. The theory assumes a uniform added mass and takes horizontal and verzical accelerations and displacements into account. The following main conclusions could be drawn:

- The length of the added mass is a linear function of the acceleration and the water depth.

- Inertia related pressures are caused by a combination of horizontal and vertical acceleration of the fluid, and the dynamic pressures are constant over the depth.

- A comparison with standard formulas from Westergaard and Karman showed that the new theory results in

  - increased added mass and maximum pressures.
  - Increased pressures in the upper third of the reservoir dam where control elements are located.

- There is a defined vertical displacement of the water surface. The relative vertical displacement, normalised with the water depth, increases for decreasing water depth.

The uniform added mass theory provides a simple tool to estimate the added mass and inertia induced pressures (dynamic and hydrostatic) for vertical dams under earthquake excitation.

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