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FREQUENCIES OF TRIANGULAR PLATE WITH TWO-DIMENSIONAL PARABOLIC THICKNESS UNDER TEMPERATURE FIELD

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ABSTRACT: The effect of two-dimensional parabolic thickness on frequencies of triangular plate (isosceles, right angled, and scalene triangle) is investigated for clamped edge condition under one dimensional temperature field. First three modes of vibration are computed on different variation of plate parameters using Rayleigh-Ritz method. The objective of this study is to illustrate a numerical data in the form of mode frequencies and showing how variation in mode frequencies can be control by choosing an appropriate variation in plate parameters. A comparative study of the frequencies with other available results is also shown here.

KEY WORDS: parabolic thickness, triangle plate, temperature, frequencies.

1 INTRODUCTION

Plates are used for many structural applications in the fields of engineering. To develop an authentic model, we must know the vibration characteristics of plates because the excess of vibration causes system failure or its less efficiency. In order to optimize vibration or to get desired frequencies, we must have to be very careful in choosing the variations in plate parameters. The following literature study shows how plate parameters effect the vibration of plates.

One-dimensional temperature and two-dimensional thickness impact on frequencies of triangle plate [1] has been computed using Rayleigh-Ritz method with clamped edge conditions. The frequencies of isosceles triangle plates [2] with different edge conditions has been computed using pb-2 Rayleigh-Ritz method, on the basis of

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Mindlin plate theory. Time period analysis of vibration of skew plate [3] and square plate [4] made up of nonhomogeneous material under temperature environment has been studied and effect of various plate parameters have been discussed. Rayleigh-Ritz technique has been employed to examine the behaviour of vibrational frequencies of non-homogeneous rectangle plates [5] with parabolic thickness under exponentially temperature distribution. Lower and upper bounds for vibrational frequencies demonstrating the effect of plate parameters of clamped plates [6] with general thickness variations has been computed. Vibration of cantilevered triangular plates [7] with variable thickness and arbitrary plan form has been solved using the finite element technique and natural frequencies and mode shapes of the plate have also been computed. Frequencies of homogeneous viscoelastic clamped rectangular plate [8] using Rayleigh-Ritz method has been discussed and time period, logarithmic decrement and deflection function on various values of plate parameters have been computed. Natural thermal vibration of rectangle plate [9] with circular variation in density and exponential variation in Poisson's ratio has been studied and first two modes have been computed. Free vibration of right-angled triangle plates [10] with all possible combinations of clamped and simply supported edge conditions has been analysed and an accurate analytical solution have been obtained. Circular variation in both thickness and Poisson's ratio impact on thermal induced natural vibration of skew plate [11] has been analysed using Rayleigh-Ritz method. Vibrations of a triangular plate [12] on various edge conditions has been studied by using Rayleigh-Ritz method. A comprehensive study of the free vibration of completely free triangular plates [13] has been presented by using Rayleigh-Ritz method and frequencies and nodal patterns have been computed. Vibration of laminated triangular plate [14] with free clamped free edge condition for different materials has been studied. A solution method has been proposed for the vibration analysis of arbitrarily-shaped triangular plates [15] with elastically restrained edges. A mathematical model has been presented to analyse the free vibration of clamped rectangle plate [16] with linear thickness and circular Poisson's ratio using classical plate theory.

In this study an effect of two-dimensional parabolic thickness and one-dimensional linear temperature on mode frequencies of triangular plate is presented in tabular form. A tabular comparison of mode frequencies with available published results is also provided in order to support the findings of present study.

2 Analysis

Consider a viscoelastic triangle plate having aspect ratio $\theta = b/c$ and $\mu = c/a$, two dimensional thickness l as shown in Fig. 1. Now transform the given triangle into right angled triangle using the transformation $x = a\zeta + b\psi$ and $y = c\psi$ as shown in Fig. 2.

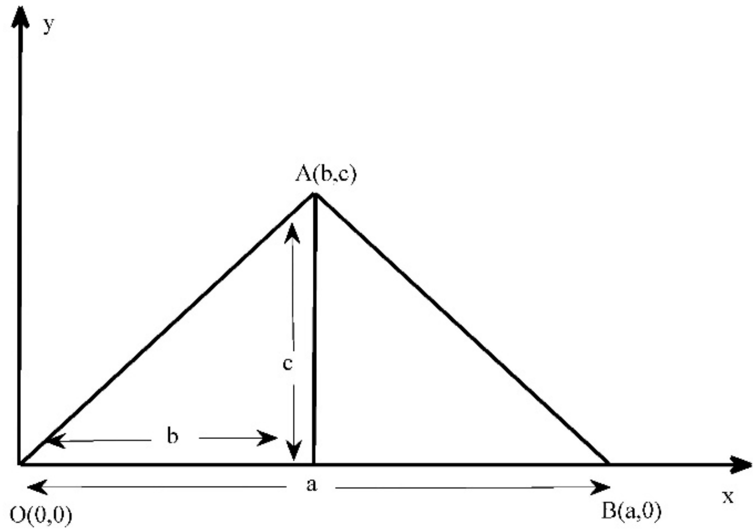

Fig. 1: Triangle plate.

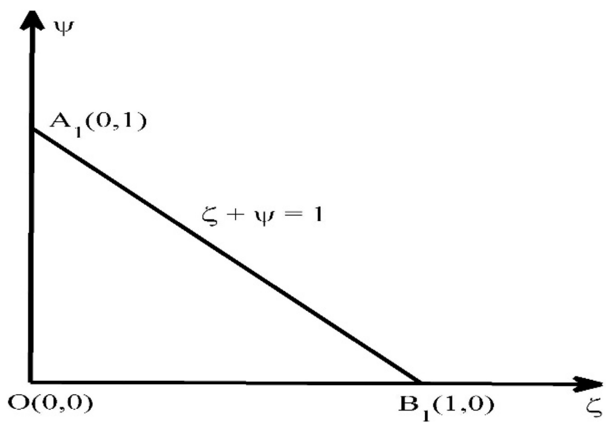

Fig. 2: Transformed triangle plate.

The kinetic energy and strain energy for vibration of triangle plate are [17]:

(1) $$T_s = \frac{1}{2}\rho\omega^2 \int_0^1 \int_0^{1-\zeta} l\Phi^2 acd\psi d\zeta \,,$$

$$V_s = \frac{1}{2}\int_0^1\int_0^{1-\zeta} D_1\Bigg[\frac{1}{a^4}\Big(\frac{\partial^2\Phi}{\partial\zeta^2}\Big)^2 + \Big(\frac{b^2}{a^2c^2}\frac{\partial^2\Phi}{\partial\zeta^2} + \frac{1}{c^2}\frac{\partial^2\Phi}{\partial\psi^2} - \frac{2b}{ac^2}\frac{\partial^2\Phi}{\partial\zeta\partial\psi}\Big)^2 + 2\nu\Big(\frac{1}{a^2}\frac{\partial^2\Phi}{\partial\zeta^2}\Big)\Big(\frac{b^2}{a^2c^2}\frac{\partial^2\Phi}{\partial\zeta^2} + \frac{1}{c^2}\frac{\partial^2\Phi}{\partial\psi^2} - \frac{2b}{ac^2}\frac{\partial^2\Phi}{\partial\zeta\partial\psi}\Big) + 2(1-\nu)\Big(-\frac{b}{a^2c}\frac{\partial^2\Phi}{\partial\zeta^2} + \frac{1}{ac}\frac{\partial^2\Phi}{\partial\zeta\partial\psi}\Big)^2\Bigg](ac)d\psi d\zeta\,, \tag{2}$$

where Φ is deflection function, ω is natural frequency and $D_1 = El^3/12\left(1-\nu^2\right)$ is flexural rigidity, here E and ν is Young's modulus and Poisson's ratio of the plate.

Applying Rayleigh Ritz method, we have:

$$L = \delta(V_s - T_s) = 0\,. \tag{3}$$

Using Eqs. (1) and (2), we have

$$L = \frac{1}{2}\int_0^1\int_0^{1-\zeta} D_1\Bigg[\frac{1}{a^4}\Big(\frac{\partial^2\Phi}{\partial\zeta^2}\Big)^2 + \Big(\frac{b^2}{a^2c^2}\frac{\partial^2\Phi}{\partial\zeta^2} + \frac{1}{c^2}\frac{\partial^2\Phi}{\partial\psi^2} - \frac{2b}{ac^2}\frac{\partial^2\Phi}{\partial\zeta\partial\psi}\Big)^2 + 2\nu\Big(\frac{1}{a^2}\frac{\partial^2\Phi}{\partial\zeta^2}\Big)\Big(\frac{b^2}{a^2c^2}\frac{\partial^2\Phi}{\partial\zeta^2} + \frac{1}{c^2}\frac{\partial^2\Phi}{\partial\psi^2} - \frac{2b}{ac^2}\frac{\partial^2\Phi}{\partial\zeta\partial\psi}\Big) + 2(1-\nu)\Big(-\frac{b}{a^2c}\frac{\partial^2\Phi}{\partial\zeta^2} + \frac{1}{ac}\frac{\partial^2\Phi}{\partial\zeta\partial\psi}\Big)^2\Bigg](ac)d\psi d\zeta - \frac{1}{2}\rho\omega^2\int_0^1\int_0^{1-\zeta} l\Phi^2(ac)d\psi d\zeta\,. \tag{4}$$

Introducing two-dimensional parabolic thickness as

$$l = l_0(1-\beta_1\zeta^2)(1-\beta_2\psi^2)\,, \tag{5}$$

where l_0 are the thickness at origin. Also β_1, β_2 are tapering parameters.

One-dimensional temperature on the plate is assumed to be linear as [1]

$$\tau = \tau_0(1-\zeta)\,, \tag{6}$$

where τ and τ_0 denote the temperature on and at the origin respectively. The modulus of elasticity is takes as in [1]

$$E = E_0(1-\gamma\tau)\,, \tag{7}$$

where E_0 is the Young's modulus at $\tau = 0$ and γ is called slope of variation

$$E = E_0\Big(1 - \alpha\{1 - \zeta\}\Big)\,, \tag{8}$$

where $\alpha = \gamma\tau_0$, $(0 \le \alpha < 1)$ is called thermal gradient.

Using Eqs. (5) and (8), the functional in Eq. (4) become

$$\begin{aligned} L = \frac{D_0}{2}\int_0^1\int_0^{1-\zeta}\Bigg\{&[1-\alpha(1-\zeta)](1-\beta_1\zeta^2)^3(1-\beta_2\psi^2)^3 \\ &\times\Bigg[(1+\theta^2)^2\Big(\frac{\partial^2\Phi}{\partial\zeta^2}\Big)^2 + \Big(\frac{\partial^2\Phi}{\partial\psi^2}\Big)^2 + \frac{2(2\theta^2+1-\nu)}{\mu^2}\Big(\frac{\partial^2\Phi}{\partial\zeta\partial\psi}\Big)^2 \\ &+ \frac{2(\nu+\theta^2)}{\mu^2}\Big(\frac{\partial^2\Phi}{\partial\zeta^2}\Big)\Big(\frac{\partial^2\Phi}{\partial\psi^2}\Big) - \frac{4\theta(1+\theta^2)}{\mu}\Big(\frac{\partial^2\Phi}{\partial\zeta^2}\Big)\Big(\frac{\partial^2\Phi}{\partial\zeta\partial\psi}\Big) \\ &- \frac{4\theta}{\mu^3}\Big(\frac{\partial^2\Phi}{\partial\psi^2}\Big)\Big(\frac{\partial^2\Phi}{\partial\zeta\partial\psi}\Big)\Bigg]\Bigg\}(ac)d\psi d\zeta \\ &- \frac{1}{2}\rho\omega^2\int_0^1\int_0^{1-\zeta}(1-\beta_1\zeta^2)(1-\beta_2\psi^2)\Phi^2 ac\,d\psi d\zeta\,, \end{aligned} \tag{9}$$

where $D_0 = E_0 l_0^3/12(1-\nu^2)$ and $\lambda^2 = \rho\omega^2 l_0 a^2/D_0$.

Deflection function is taken as

$$\Phi(\zeta,\psi) = \Big[(\zeta)^e(\psi)^f(1-\zeta-\psi)^g\Big]\left[\sum_{i=0}^{n}\Psi_i\left\{(\zeta)\,(\psi)\,(1-\zeta-\psi)\right\}^i\right], \tag{10}$$

where Ψ_i, $i = 0, 1, 2, \dots, n$ are unknowns and the value of e, f, g can be 0, 1 and 2, corresponding to given edge condition.

To minimize Eq. (10), we have

$$\frac{\partial L}{\partial \Psi_i} = 0, \quad i = 0, 1, \dots, n. \tag{11}$$

Solving Eq. (11), we have frequency equation

$$\left|P - \lambda^2 Q\right| = 0\,, \tag{12}$$

where $P = [p_{ij}]_{i,j=0,1,\dots,n}$ and $Q = [q_{ij}]_{i,j=0,1,\dots,n}$ are square matrix of order $(n+1)$.

3 Numerical Illustration and Discussion

The effect of two-dimensional thickness with one-dimensional temperature impact on vibrational frequencies (first three modes) of different triangle plate is computed with clamped edges and presented with the help of tables.

Table 1 presents the vibrational frequencies of right-angled isosceles triangle corresponding to both the tapering parameters β_1, β_2 for variable value of thermal gradient α, i.e., $\alpha = 0.2, 0.4, 0.6$. From Table 1, we found that vibrational mode frequencies decrease with the increasing value of both tapering parametersβ_1, β_2. The vibrational frequency mode also decreases with the increasing value of thermal gradient α. The rate of decrement in vibrational mode frequencies corresponding to tapering β_2 is less in comparison to rate of decrement in vibrational mode frequencies corresponding to tapering β_1. The rate of decrement is also increases with the increasing value of thermal gradient α.

Table 1: Frequencies of right-angled isosceles triangle plate vs. tapering parameters β_1, β_2 for fixed aspect ratios $\theta = 0$, $\mu = 1$

$\alpha = 0.2$									
	$\beta_2 = 0.0$			$\beta_2 = 0.4$			$\beta_2 = 0.8$		
β_1	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3
0.0	100.13	366.92	832.38	97.21	358.71	810.53	95.84	355.37	800.22
0.2	98.28	361.74	818.57	95.09	353.04	795.06	93.44	349.21	784.86
0.4	96.74	357.54	807.35	93.28	348.32	783.99	91.34	344.02	772.81
0.6	95.59	354.52	798.97	91.84	344.80	775.76	89.62	340.06	763.65
0.8	94.90	352.96	794.22	90.86	342.78	769.59	88.35	337.65	761.23
$\alpha = 0.4$									
0.0	92.69	339.29	770.29	90.17	332.12	750.99	89.09	329.45	742.26
0.2	90.79	334.00	756.25	88.03	326.35	735.49	86.68	323.23	726.77
0.4	89.18	329.56	744.76	86.15	321.46	723.81	84.53	317.86	714.21
0.6	87.91	326.21	735.64	84.61	317.62	714.85	82.71	313.59	704.54
0.8	87.06	324.21	729.63	83.49	315.17	707.52	81.31	310.76	697.43
$\alpha = 0.6$									
0.0	84.60	309.17	702.92	82.54	303.17	686.40	81.77	301.25	679.55
0.2	82.63	303.70	688.57	80.34	297.28	670.56	79.33	294.92	663.64
0.4	80.91	299.00	676.23	78.37	292.12	658.23	77.11	289.35	650.21
0.6	79.49	295.25	665.91	76.70	287.90	648.08	75.81	284.69	639.71
0.8	78.44	292.66	658.64	75.39	284.89	639.60	73.61	281.28	631.52

Tables 2, 3 and 4 show the mode frequencies of right-angled scalene triangle plate corresponding to both the tapering parameters β_1, β_2 for variable value of thermal gradient α i.e., $\alpha = 0.2, 0.4, 0.6$. Here value of aspect ratio μ is taken 1.5, 2.0 and 2.5 respectively. The vibrational mode frequencies of right-angled scalene triangle plate decrease with the increasing value of both tapering parameters β_1, β_2 as well as for variable value thermal gradient α like in Table 1. But the frequency reported in Tables 2, 3 and 4 are less when compared with the frequency tabulated in Table 1. Also, the frequency reported in Table 1 is more than the frequency reported in Table 3 which is more than the frequency reported in Table 4.

Table 5 presents the mode frequencies of scalene triangle plate corresponding to thermal gradient α for variable value of both the tapering parameters β_1, β_2 i.e., $\beta_1 = \beta_2 = 0.0, 0.4, 0.8$. The increasing the thermal gradient α results the decrease in mode frequencies for variable values of both the tapering parameters β_1, β_2. Variation in both the tapering parameters β_1, β_2 provides less variation in mode frequencies in comparison to variation in mode frequencies corresponding to thermal gradient α.

Table 2: Frequencies of right-angled scalene triangle plate vs. tapering parameters β_1, β_2 for fixed aspect ratios $\theta = 0$, $\mu = 1.5$

$\alpha = 0.2$									
	$\beta_2 = 0.0$			$\beta_2 = 0.4$			$\beta_2 = 0.8$		
β_1	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3
0.0	90.31	330.94	750.27	87.90	324.46	733.30	86.94	322.54	726.94
0.2	88.74	326.67	739.30	86.09	319.73	720.84	84.87	317.38	714.26
0.4	87.47	323.34	730.49	84.56	315.92	712.17	83.08	313.14	704.81
0.6	86.55	321.10	724.45	83.38	313.27	705.75	81.63	310.09	697.72
0.8	86.06	320.25	721.49	82.63	312.00	701.55	80.62	308.48	693.92
$\alpha = 0.4$									
0.0	83.52	305.67	693.45	81.46	300.06	678.77	80.74	298.68	673.73
0.2	81.89	301.26	681.84	79.60	295.23	665.70	78.64	293.43	660.55
0.4	80.53	297.67	672.61	78.00	291.19	656.43	76.79	289.00	650.30
0.6	79.49	295.08	665.75	76.72	288.19	649.49	75.26	285.61	642.53
0.8	78.84	293.75	661.62	75.82	286.46	644.13	74.11	283.53	637.71
$\alpha = 0.6$									
0.0	76.12	278.10	631.35	74.46	273.51	619.10	74.02	272.73	615.60
0.2	74.41	273.47	619.47	72.54	268.48	605.82	71.88	267.33	602.33
0.4	72.93	269.57	609.28	70.85	264.16	595.57	69.95	262.62	591.18
0.6	71.74	266.53	601.21	69.42	260.73	587.42	68.29	258.82	582.34
0.8	70.88	264.60	595.88	68.34	258.41	581.01	66.97	256.17	576.05

Table 3: Frequencies of right-angled scalene triangle plate vs. tapering parameters β_1, β_2 for fixed aspect ratios $\theta = 0$, $\mu = 2.0$

$\alpha = 0.2$									
	$\beta_2 = 0.0$			$\beta_2 = 0.4$			$\beta_2 = 0.8$		
β_1	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3
0.0	86.61	317.37	719.29	84.40	311.57	704.40	83.59	310.19	699.76
0.2	85.15	313.47	709.47	82.70	307.22	692.83	81.65	305.43	688.13
0.4	83.98	310.48	701.75	81.28	303.77	685.09	79.98	301.59	679.24
0.6	83.15	308.56	696.43	80.21	301.43	679.77	78.64	298.97	673.07
0.8	82.74	307.95	694.50	79.54	300.45	676.29	77.73	297.57	669.97
$\alpha = 0.4$									
0.0	80.06	292.99	664.25	78.18	288.01	651.44	77.61	287.12	648.13
0.2	78.53	288.93	653.85	76.44	283.53	639.38	75.63	282.24	635.97
0.4	77.27	285.65	645.69	74.94	279.84	631.01	73.89	278.18	626.43
0.6	76.32	283.37	639.49	73.76	277.16	624.59	72.47	275.12	619.53
0.8	75.75	282.29	636.29	72.94	275.69	620.45	71.41	273.36	615.21
$\alpha = 0.6$									
0.0	72.92	266.37	604.27	71.42	262.34	593.77	71.11	262.02	591.66
0.2	71.31	262.07	593.37	69.61	257.64	581.48	69.08	256.96	579.05
0.4	69.92	258.46	584.32	68.01	253.66	571.91	67.26	252.59	569.02
0.6	68.82	255.72	576.87	66.69	250.52	564.68	65.71	249.11	561.12
0.8	68.04	254.04	572.40	65.69	248.49	558.82	64.49	246.76	555.48

4 Results, Comparison

In order to authenticate the finding of the present study a comparison of mode frequencies of present study with [1] is also given corresponding to tapering parameters β_1, β_2 and presented in tabular form.

Table 6 shows the comparison of mode frequencies of right angled isosceles scalene triangle plate (present study) and obtained in [1] corresponding to tapering parameters β_1, β_2 for fixed value of aspect ratios $\theta = 0$, $\mu = 1.0$ and thermal gradient $\alpha = 0.0$. From Table 6, we conclude that the mode frequencies of present study is higher than the mode frequencies obtained in [1] but the variation in mode frequencies (rate of decrement) is less in comparison to the variation in mode frequencies obtained in [1] for both the increasing value of tapering parameters β_1, β_2. A comparison of mode frequencies of right-angled scalene triangle plate (present study) and obtained in [1] corresponding to tapering parameters β_1, β_2 for fixed value of aspect ratios $\theta = 0$, $\mu = 1.5$ and thermal gradient $\alpha = 0.0$ is presented in Table 7.

From Table 7, one can easily understand that mode frequencies of present study is

Table 4: Frequencies of right-angled scalene triangle plate vs. tapering parameters β_1, β_2 for fixed aspect ratios $\theta = 0$, $\mu = 2.5$

$\alpha = 0.2$									
	$\beta_2 = 0.0$			$\beta_2 = 0.4$			$\beta_2 = 0.8$		
β_1	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3
0.0	84.84	310.90	704.51	82.73	305.42	690.61	82.00	304.34	686.50
0.2	83.44	307.17	695.22	81.09	301.25	679.59	80.11	299.76	675.39
0.4	82.31	304.34	687.93	79.72	297.98	672.15	78.50	296.08	667.12
0.6	81.53	302.57	683.17	78.69	295.80	667.15	77.22	293.52	661.46
0.8	81.16	302.13	681.13	78.07	294.96	664.18	76.35	292.36	658.75
$\alpha = 0.4$									
0.0	78.41	286.93	650.41	76.62	282.25	638.47	76.11	281.63	635.71
0.2	76.93	283.03	640.54	74.93	277.94	626.99	74.19	276.94	623.85
0.4	75.72	279.92	632.80	73.48	274.43	618.92	72.51	273.03	615.03
0.6	74.81	277.78	627.03	72.34	271.89	612.98	71.13	270.12	608.60
0.8	74.28	276.84	624.12	71.57	270.56	609.09	70.13	268.50	604.78
$\alpha = 0.6$									
0.0	71.39	260.76	591.47	69.97	256.99	581.78	69.72	256.92	580.11
0.2	69.83	256.63	580.80	68.21	252.48	569.61	67.75	252.02	568.10
0.4	68.49	253.16	572.13	66.66	248.64	560.77	65.98	247.83	558.26
0.6	67.42	250.55	565.38	65.38	245.64	553.94	64.48	244.50	550.72
0.8	66.69	249.00	561.10	64.42	243.75	548.33	63.30	242.28	545.54

Table 5: Frequencies of scalene triangle plate vs. thermal gradient for α for fixed aspect ratios $\theta = 1/\sqrt{3}$, $\mu = \sqrt{3}/2$

$\theta = 1/\sqrt{3}$, $\mu = \sqrt{3}/2$									
	$\beta_1 = \beta_2 = 0.0$			$\beta_1 = \beta_2 = 0.4$			$\beta_1 = \beta_2 = 0.8$		
α	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3
0.2	78.67	289.04	654.74	76.28	282.33	636.64	75.82	282.60	635.09
0.4	71.86	264.65	599.27	69.49	257.93	581.22	68.85	257.52	578.31
0.6	64.34	237.76	538.08	61.95	230.93	519.97	61.09	229.67	515.35

higher than the mode frequencies obtained in [1] but the variation in mode frequencies (rate of decrement) is less in comparison to the variation in mode frequencies obtained in [1] for both the increasing value of tapering parameters β_1, β_2.

Table 6: Comparison of frequency modes of right-angled isosceles triangle with [1] vs. tapering parameters β_1, β_2 for fixed aspect ratio $\theta = 0.0$, $\mu = 1.0$ and thermal gradient $\alpha = 0.0$ (Bold font values are from [1])

$\theta = 0.0, \mu = 1.0, \alpha = 0.0$												
	$\beta_2 = 0.0$		$\beta_2 = 0.2$		$\beta_2 = 0.4$		$\beta_2 = 0.6$		$\beta_2 = 0.8$		$\beta_2 = 1.0$	
β_1	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	107.07	436.19	102.96	418.48	99.09	402.02	95.48	386.90	92.16	373.16	89.13	360.86
	107.07	**436.19**	**98.91**	**401.13**	**91.06**	**368.57**	**83.98**	**339.16**	**77.71**	**313.62**	**72.40**	**292.79**
0.2	102.96	418.18	98.84	400.90	94.96	384.56	91.35	369.56	88.01	355.95	84.98	343.76
	98.81	**401.13**	**90.94**	**368.00**	**83.57**	**337.22**	**76.82**	**309.40**	**70.85**	**285.23**	**65.83**	**265.53**
0.4	99.09	402.02	94.96	384.56	91.07	368.35	87.44	353.47	84.09	339.97	81.05	327.91
	91.06	**368.57**	**83.57**	**337.22**	**76.53**	**308.07**	**70.08**	**281.68**	**64.38**	**258.75**	**59.60**	**240.04**
0.6	95.48	386.903	91.35	369.56	87.44	353.47	83.79	338.69	80.43	325.32	77.38	313.38
	83.98	**339.16**	**76.82**	**309.40**	**70.08**	**281.68**	**63.89**	**256.56**	**58.42**	**234.70**	**53.84**	**216.84**
0.8	92.16	373.16	88.01	355.95	84.09	339.97	80.43	325.32	77.05	312.06	73.99	300.25
	77.71	**313.62**	**70.85**	**285.23**	**64.38**	**258.75**	**58.42**	**234.70**	**53.14**	**213.73**	**48.71**	**196.07**
1.0	89.13	360.86	84.98	343.76	81.05	327.91	77.38	313.38	73.99	300.25	70.93	288.58
	72.40	**292.79**	**65.83**	**265.53**	**59.60**	**240.04**	**53.84**	**216.84**	**48.71**	**196.56**	**44.38**	**179.90**

Table 7: Comparison of frequency modes of right-angled scalene triangle with [1] vs. tapering parameters β_1, β_2 for fixed aspect ratio $\theta = 0.0$, $\mu = 1.5$ and thermal gradient $\alpha = 0.0$ (Bold font values are from [1])

$\theta = 0.0, \mu = 1.5, \alpha = 0.0$												
	$\beta_2 = 0.0$		$\beta_2 = 0.2$		$\beta_2 = 0.4$		$\beta_2 = 0.6$		$\beta_2 = 0.8$		$\beta_2 = 1.0$	
β_1	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	96.65	393.73	93.05	378.56	89.67	364.53	786.54	351.69	83.66	340.11	81.05	329.80
	79.21	**322.70**	**72.21**	**293.23**	**65.50**	**265.22**	**59.20**	**239.05**	**53.42**	**215.22**	**48.32**	**194.29**
0.2	93.05	378.56	89.44	363.44	86.04	349.46	82.89	336.66	79.99	325.12	77.38	314.87
	74.12	**301.25**	**67.41**	**273.15**	**60.98**	**246.44**	**54.93**	**221.48**	**49.38**	**198.75**	**44.48**	**178.81**
0.4	89.67	364.53	86.04	349.46	82.63	335.51	79.45	322.76	76.54	311.26	73.91	301.05
	69.45	**281.99**	**63.02**	**255.11**	**56.85**	**229.55**	**51.03**	**205.66**	**45.68**	**183.90**	**40.96**	**164.82**
0.6	86.54	351.69	82.89	336.66	79.45	322.76	76.26	310.05	73.32	298.60	70.67	288.43
	65.30	**265.39**	**59.13**	**239.57**	**53.19**	**214.99**	**47.58**	**192.01**	**42.42**	**171.08**	**37.85**	**152.72**
0.8	83.66	340.11	79.99	325.12	76.54	311.26	73.32	298.60	70.37	287.19	67.70	277.07
	61.74	**251.97**	**55.81**	**227.00**	**50.09**	**203.23**	**44.68**	**180.98**	**39.67**	**160.71**	**35.24**	**142.93**
1.0	81.05	329.80	77.38	314.87	73.91	301.05	70.67	288.43	67.70	277.07	65.02	267.01
	58.85	**242.24**	**53.13**	**217.91**	**47.61**	**194.73**	**42.38**	**173.03**	**37.52**	**153.23**	**33.20**	**135.87**

5 Conclusions

Vibrational frequencies (first three modes) of various triangle plate with clamped edges comprises the effect of two-dimensional parabolic thickness and one-dimensional temperature effect is computed and presented. Authors would like to conclude the following facts:

- Two-dimensional parabolic thickness provides higher frequencies when compared with the frequencies in case of two-dimensional linear thickness [1]. The frequencies of present study coincide with frequency mode obtained in [1] for right angled isosceles triangle plate at$\beta_1 = \beta_2 = 0.0$.
- The variation in frequencies in case of two-dimensional parabolic thickness is less when in comparison to the variation in frequencies in case two-dimensional linear thickness [1].
- Increase in both the plate parameters (tapering and temperature) decrease the frequencies triangle plates.
- A numerical data in the form of modes provides how various plate parameters effectively affect the frequency modes. Also, the variation in frequencies can be control by choosing appropriate variation in plate parameters.

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