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ANALYSIS OF REFLECTION AND TRANSMISSION OF AXIALLY-SYMMETRIC WAVES FOR SELF-REINFORCED HALF SPACE OVER A POROELASTIC HALF SPACE

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ABSTRACT: The objective of this paper is to study the reflection and transmission phenomena in axially symmetric waves in two half spaces, in which the lower one is poroelastic half space, and the upper one is self-reinforced half space. Employing Biot's theory of wave propagation, reflection and transmission coefficients of two dilatational waves, and shear wave are computed against angle of incidence. Numerical results are presented graphically for two types of poroelastic solids, namely, for upper half space, self-reinforced material with the longitudinal, shear modulus and reinforcement parameters respectively and for lower half space the sandstone saturated with water is used. From the numerical results, it is clear that the reflection and transmission coefficients are strongly affected as a function of angle of incidence.

KEY WORDS: self-reinforcement half space, poroelastic half space, reflection, transmission, angle of incidence.

1 INTRODUCTION

During the past half century in many scientific fields, wave reflection and transmission phenomena have received much attention in the domains such as Marine Seismology, Geotechnical Engineering, Acoustics, and Geophysics. Analysis of reflection and transmission phenomena can be used to understand various materials. Sandstone is a great source for quartz which is very useful in our day to day life. Sandstone deposits may be cylindrical in shape, and may be surrounded by poroelastic medium such as soil or rock. The study of reflection and transmission of waves incidented at the said interface gives the information pertaining to sandstone deposits. Since

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the reflection and transmission coefficients are functional dependent on the medium consisting of two non-homogeneous layers separated by a horizontal interface. In acoustics, the said coefficients are used to understand the effect of various materials on their acoustic environments. In other domains such as Geophysics and Medicine, the pertinent analysis can be used as Non Destructive Evaluation (NDE) tool. Employing transfer matrix method, Bogy and Gracewski [1] derived the reflection coefficient for plane waves in a fluid incident on a layered elastic half-space. Sinha and Elsibai [2] investigated the reflection of thermoelastic waves from the free surface of solid half space, and at the welded interface of two semi-infinite media. In the paper [3], the reflection coefficient is calculated for shear wave that incidents from within the solid on its boundary. Singh [4] exploits the reflection of $P$and SV waves from the free surface of an elastic solid with generalized thermo-diffusion. Employing Biot's theory of poroelasticity [5], reflection of plane waves at boundaries of a poroelastic half space is discussed by Tajuddin and Hussaini [6]. In the paper [6], it is clear that the overlapping parameter between solid and fluid plays a significant role in generating the reflected slow dilatational wave. Dai and Kaung [7] calculated the reflection and transmission of elastic waves at the interface between water and double porosity solids. The analytical expressions of all the three phase velocities of qP, qSV and qSH waves are derived by Chattopadhyay *et al.* [8]. Reuven Gordon [9] explained the reflection of cylindrical surface waves. The reflection and transmission of plane waves between two different fluids saturated with porous half spaces are given in the paper [10] wherein variations of amplitude ratios with angle of incidence are calculated. The closed form analytical expressions for reflection and transmission coefficients of waves are derived [11]. Samal and Chattaraj [12] discussed the surface wave propagation in fiber-reinforced anisotropic elastic layer between liquid saturated porous half space and uniform liquid layer. Rajneesh *et al.* [13] investigated reflection and transmission of plane waves at the loosely bonded interface of an elastic solid half space and a micro polar thermo elastic diffusion half-space. Semi relativistic reflection and transmission coefficients for two spinless particles separated by exponential and rectangular shaped potential barriers are investigated by Ethyl *et al.* [14]. Malla *et al.* [15] determined reflection and transmission coefficients of the axially symmetric body waves incident on a base of a semi-infinite solid cylinder using Whittaker's function. Sindhuja *et al.* [16] discussed shear wave propagation in magneto poroelastic medium sandwiched between self-reinforced poroelastic medium and poroelastic half space. To the best of author's knowledge, reflection and transmission phenomenon of axially symmetric waves is not yet investigated for two half spaces problem. Hence, in the present work, using the model [15], the same is investigated in the framework of Biot's theory of poroelasticity.

This paper is organized as follows. In Section 2, formulation and solution of the problem is given. Boundary conditions are given in Section 3. In Section 4, numerical results are described. Finally, conclusion is given in Section 5.

2 Formulation and Solution of the Problem

Consider the solid of two half spaces, in which upper half space is self-reinforced (say M_1), and the lower one is poroelastic half space (say M_2). If a wave is incidents in the lower half space, two dilatational waves, and a shear wave either reflect in the lower half space or transmit into the upper half space as shown in Figure 1. The r-axis is taken as interface of the half spaces, and z-axis is taken along the thickness of the two half spaces. The equations of motion of a homogeneous, isotropic poroelastic solid in absence of dissipation are [5]:

$$\begin{aligned} &N\nabla^2\vec{u} + \nabla(Ae + Q\varepsilon) = \frac{\partial^2}{\partial t^2}(\rho_{11}\vec{u} + \rho_{12}\vec{U}), \\ &\nabla(Qe + R\varepsilon) = \frac{\partial^2}{\partial t^2}(\rho_{12}\vec{u} + \rho_{22}\vec{U}) \,. \end{aligned} \tag{1}$$

where $\vec{u} = (u, v, w)$ and $\vec{U} = (U, V, W)$ are displacement vectors of solid and fluid, respectively, e and ε are the dilatations of solid and fluid, $P = A + 2N$, Q and R are all poroelastic constants, e and ε are the dilatations of solid and fluid. The mass coefficients ρ_{11}, ρ_{12} and ρ_{22} satisfy the relations $\rho_1 = \rho_{11} + \rho_{12} = (1 - \beta)\rho_s$, $\rho_2 = \rho_{12} + \rho_{22} = \beta\rho_f$, $\rho = \rho_1 + \rho_2 = \rho_s + \beta(\rho_f - \rho_s)$. Here ρ_1 and ρ_2 are masses of

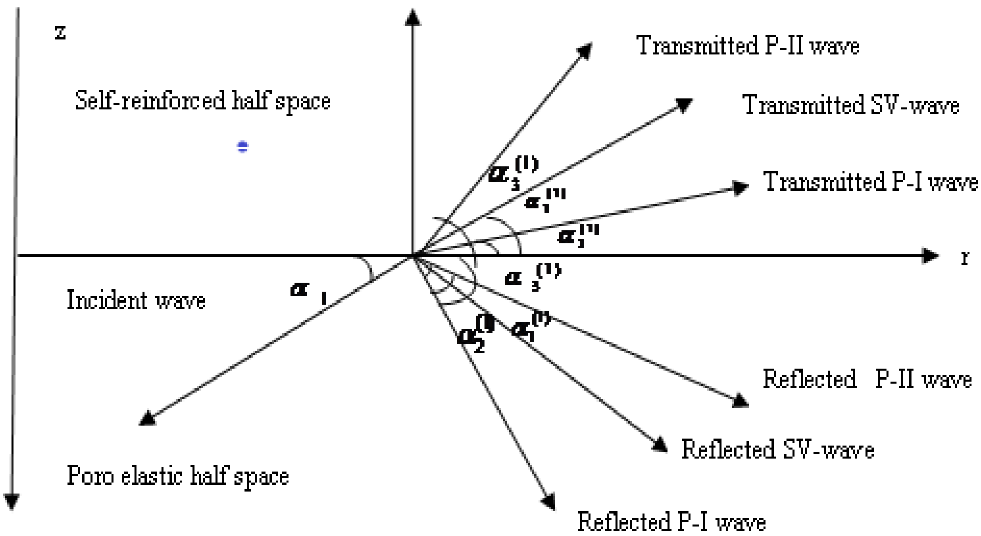

Fig. 1: Geometry of the problem.

solid and fluid per unit volume of aggregate, ρ is total mass of solid-fluid aggregate per unit volume. ρ_s and ρ_f are mass densities of the solid and fluid, respectively. β is the porosity of aggregate. The solid stresses σ_{ij} and fluid pressure s are given by

$$\sigma_{ij} = 2Ne_{ij} + (Ae + Q\varepsilon)\delta_{ij}, \qquad (i, j = 1, 2, 3), \qquad s = Qe + R\varepsilon\,. \tag{2}$$

In eq. (2), δ_{ij} is the well-known Kronecker delta function. The axially symmetric waves in the lower half space, and upper half space are considered in the following subsections.

2.1 Upper self-reinforced half space

If a wave incidents in the lower poroelastic half space, and if it transmits, then from the theory of isotropic poroelasticity, there exist two dilatational and a shear transmitted waves in the upper self-reinforced poroelastic half space. The stress tensor σ_{ij} for a self-reinforced layer along preferred direction $\vec{a}$ is [16]

$$\begin{aligned}\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_l e_{kl}\delta_{ij} + a_i a_j e_{kk}) \\ + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) + \beta a_k a_m e_{km} a_i a_j\,,\end{aligned} \tag{3}$$

Here $i, j, k,\ l,\ m = 1, 2, 3$, e_{ij}'s are the strain components given by

$$e_{ij} = \frac{1}{2}\Big(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\Big) \quad \text{and} \quad \vec{a} = (a_1, a_2, a_3)^T$$

is the preferred unit direction of reinforcement. For axisymmetric waves, which are under consideration, let $\vec{u} = (u_1,\ 0,\ w_1)$ and $\vec{U} = (U_1,\ 0,\ W_1)$ be the displacement components. Consider the potential decomposition of the displacements as given below:

$$\begin{aligned} u_1 &= \frac{\partial \varphi_{11}}{\partial r} - \frac{\partial \psi_{11}}{\partial z}\,, \qquad & w_1 &= \frac{\partial \varphi_{11}}{\partial z} + \frac{\partial \psi_{11}}{\partial r} + \frac{\psi_{11}}{r}\,, \\ U_1 &= \frac{\partial \varphi_{12}}{\partial r} - \frac{\partial \psi_{12}}{\partial z}\,, \qquad & W_1 &= \frac{\partial \varphi_{12}}{\partial z} + \frac{\partial \psi_{12}}{\partial r} + \frac{\psi_{12}}{r}\,. \end{aligned} \tag{4}$$

In the above φ_{ij} and ψ_{ij} are displacement potentials pertaining to the upper half space, and are functions of r, z and t. Following the paper [15], the displacement potentials φ_{ij} and ψ_{ij} are assumed as given below:

$$\varphi_{11} = \begin{cases} \varphi_{(m)} + \varphi_n^{(m)} = A_m e^{i(\omega t - \delta_m(r\sin\alpha_m - z\cos\alpha_m))} \\ \qquad\qquad + A_n^{(m)} e^{i(\omega t - \delta_n(r\sin\alpha_n^{(m)} - z\cos\alpha_n^{(m)}))}, \quad m = 1, 2 \\ \varphi_{(3)} + \varphi_3^{(3)} = \ A_3 e^{i(\omega t - \delta_3(r\cos\alpha_3 - z\sin\alpha_3))} \\ \qquad\qquad + A_3^{(3)} e^{i(\omega t - \delta_3(r\sin\alpha_3^{(3)} - z\cos\alpha_3^{(3)}))}, \end{cases}$$

(5)
$$\varphi_{12} = \begin{cases} \varphi_{(m)} + \varphi_n^{(m)} = A_m e^{i(\omega t - \delta_m\mu_n(r\sin\alpha_m - z\cos\alpha_m))} \\ \qquad\qquad + A_n^{(m)} e^{i(\omega t - \delta_n\mu_n(r\sin\alpha_n^{(m)} - z\cos\alpha_n^{(m)}))}, \quad m = 1, 2, \\ \varphi_{(3)} + \varphi_3^{(3)} = \ A_3 e^{i(\omega t - \delta_3\mu_3(r\cos\alpha_3 - z\sin\alpha_3))} \\ \qquad\qquad + A_3^{(3)} e^{i(\omega t - \delta_3\mu_3(r\sin\alpha_3^{(3)} - z\cos\alpha_3^{(3)}))}, \end{cases}$$

$$\psi_{11} = A_3^{(m)} e^{i(\omega t - \delta_3(r\sin\alpha_3^{(m)} - z\cos\alpha_3^{(m)}))}, \quad m = 1, 2, 3,$$
$$\psi_{12} = A_3^{(m)} e^{i(\omega t - \delta_3\mu_3(r\sin\alpha_3^{(m)} - z\cos\alpha_3^{(m)}))}, \quad m = 1, 2, 3.$$

In the above, α_m is the angle made by the m-th incident wave with the normal, $\alpha_n^{(m)}$ is the angle made by the n-th transmitted wave with the normal when m-th wave incidents, $\varphi_{(m)}$ are incident wave potentials, $\varphi_n^{(m)}$ are transmitted wave potentials, ω is frequency, $\delta_m$are wavenumbers of incident waves, δ_t are wavenumbers of transmitted waves, $A_n^{(m)}$ are coefficients of transmitted wave potentials, A_m are coefficients of potentials of incident waves, $T_n^{(m)} = (A_n^{(m)}/A_m)$ are transmission coefficients of n-th transmitted wave when m-th wave is incident. In all the above, incident and transmitted waves are either dilatational or shear. Using eq. (5) in eq. (4), after a long calculation, the solid displacements are obtained as

(6)
$$u_1 = \begin{cases} -i\delta_m\varphi_{(m)}\sin\alpha_m + \sum_{n=1}^{2} \delta_n\varphi_n^{(m)}\sin\alpha_n^{(m)} - \delta_3\varphi_3^{(m)}\cos\alpha_3^{(m)},\ m=1,2. \\ i\delta_m\varphi_{(m)}\cos\alpha_m + \sum_{n=1}^{2} \delta_n\varphi_n^{(m)}\sin\alpha_n^{(m)} - \delta_3\varphi_3^{(3)}\cos\alpha_3^{(3)},\ m=3. \end{cases}$$
$$w_1 = \begin{cases} -i\delta_m\varphi_{(m)}\cos\alpha_m + \sum_{n=1}^{2} \delta_n\varphi_n^{(m)}\cos\alpha_n^{(m)} + (\varphi_n^{(m)}/r) + \delta_3\varphi_3^{(m)}\sin\alpha_3^{(m)},\ m=1,2. \\ i\delta_m\varphi_{(m)}\sin\alpha_m + \sum_{n=1}^{2} \delta_n\varphi_n^{(m)}\cos\alpha_n^{(m)} + (\varphi_n^{(m)}/r) + \delta_3\phi_3^{(3)}\sin\alpha_3^{(3)},\ m=3. \end{cases}$$

Using the displacement components of eq. (6) and stress displacement relations (3), the relevant stress components are obtained, and are given by

$$(\sigma_{rz})_1 = \begin{cases} A_1(B_1+B_4)+A_2(B_2+B_3) \\ A_1(B_5+B_6)+A_2(B_7+B_8) \end{cases}, \tag{7}$$
$$(\sigma_{zz})_1 = \begin{cases} C_1D_1+C_2D_2+C_3D_3 \\ C_1D_4+C_2D_5+C_3D_6 \end{cases}$$

where

$$A_1 = (\alpha a_1 a_3 + 2a_1 a_3(\mu_L - \mu_T) + \beta a_1^3 a_3)\,,$$
$$A_2 = (\mu_T + \mu_L - \mu_T(a_1^2 + a_3^2) + \beta a_1^2 a_3^2)\,,$$
$$B_1 = \delta_m^2 \varphi_{(m)}(\sin\alpha_m)^2 - \sum_{n=1}^{2} \delta_n \varphi_n^{(m)}(\sin\alpha_n^{(m)})^2 - \delta_3\varphi_3^{(m)}\cos 2\alpha_3^{(m)},\ m=1,2,$$
$$B_2 = \delta_m^2 \varphi_{(m)}\sin 2\alpha_m - \sum_{n=1}^{2} \delta_n^2 \varphi_n^{(m)}\sin 2\alpha_n^{(m)} - (\varphi_n^{(m)}/r)$$
$$+ \delta_3^2\varphi_3^{(m)}\cos 2\alpha_3^{(m)} + (i/r)\delta_n\varphi_n^{(m)}\sin\alpha_n^{(m)}\,,$$
$$B_3 = \delta_m^2 \varphi_{(m)}\sin 2\alpha_m - \sum_{n=1}^{2} \delta_n \varphi_n^{(m)}\sin 2\alpha_n^{(m)} - \delta_3\varphi_3^{(m)}\cos 2\alpha_3^{(m)},$$
$$B_4 = \delta_m^2 \varphi_m(\sin\alpha_m)^2 - \sum_{n=1}^{2} \delta_n \varphi_n^{(m)}(\sin\alpha_n^{(m)})^2 - \delta_3\varphi_3^{(m)}\cos 2\alpha_3^{(m)},$$
$$B_5 = \delta_m \varphi_{(m)}\cos\alpha_m + \sum_{n=1}^{2} \delta_n^2 \varphi_n^{(m)}\sin\alpha_n^{(m)} - \delta_3\varphi_3^{(3)}\cos\alpha_3^{(3)},$$
$$B_6 = \delta_m^2 \varphi_{(m)}\cos 2\alpha_m - \sum_{n=1}^{2} \delta_n \varphi_n^{(m)}\cos\alpha_n^{(m)} + (\varphi_n^{(m)}/r)$$
$$+ \delta_3\varphi_3^{(3)}\sin\alpha_3^{(k)} + (i/r)\delta_n\varphi_n^{(m)}\cos\alpha_n^{(m)},$$
$$B_7 = \delta_m \varphi_{(m)}\cos 2\alpha_m - \sum_{n=1}^{2} \delta_n \varphi_n^{(m)}\sin 2\alpha_n^{(m)} - \delta_3\varphi_3^{(3)}\cos 2\alpha_3^{(3)},$$
$$B_8 = \delta_m \varphi_{(m)}\cos 2\alpha_m - \sum_{n=1}^{2} \delta_n \varphi_n^{(m)}\sin 2\ \alpha_n^{(m)} - \delta_3\varphi_3^{(3)}\cos 2\alpha_3^{(3)},$$
$$C_1 = (\lambda + \alpha(a_1^2 + a_3^2) + \beta a_1^2 a_3^2),$$
$$C_2 = (\alpha a_1 a_3 + 2a_1 a_3(\mu_L - \mu_T) + \beta a_1 a_3^3),$$
$$C_3 = \lambda + 2\mu_T + 2\alpha a_3^2 + 4a_3^2(\mu_L - \mu_T) + \beta a_3^4,$$

$$
\begin{aligned}
D_1 &= \delta_m^2 \varphi_{(m)} \sin^2 \alpha_m - \sum_{n=1}^{2} \delta_n \varphi_n^{(m)} \sin 2\alpha_n^{(m)} + \delta_3^2 \varphi_3^{(m)} \cos 2\alpha_3^{(m)}),\ m = 1, 2,\\
D_2 &= \delta_m^2 \varphi_{(m)} \sin 2\alpha_k - \sum_{n=1}^{2} \delta_n \varphi_n^{(m)} \sin 2\alpha_n^{(m)} + \delta_3^2 \varphi_3^{(m)} \cos 2\alpha_3^{(m)}),\ m = 3,\\
D_3 &= \delta_m^2 \varphi_{(m)} \sin 2\alpha_m + \sum_{n=1}^{2} \delta_n \varphi_n^{(m)} (\sin 2\alpha_{nt}^{(m)} - \cos^2 \alpha_n^{(m)}) - \delta_3^2 \varphi_3^{(m)} \sin \alpha_3^{(m)},\ m = 1, 2,\\
D_4 &= \delta_m^2 \varphi_{(m)} \cos^2 \alpha_k + \sum_{n=1}^{2} \delta_n \varphi_n^{(m)} (\sin 2\alpha_n^{(m)} - \cos^2 \alpha_n^{(m)}) + \delta_3^2 \varphi_3^{(m)} \sin \alpha_3^{(m)},\ m = 3,\\
D_5 &= -\delta_m^2 \varphi_{(m)} \sin 2\alpha_m - \sum_{n=1}^{2} \delta_n \varphi_n^{(m)} (\cos^2 \alpha_n^{(m)} + (1/r) \cos \alpha_n^{(m)} + \sin 2\alpha_n^{(m)})\\
&\quad + \delta_3^2 \varphi_3^{(m)} \cos 2\alpha_3^{(m)}), m = 1, 2,\\
D_6 &= (\delta_m^2 \varphi_{(m)} \cos^2 \alpha_3 - \sum_{n=1}^{2} \delta_n \varphi_3^{(3)} (\cos^2 \alpha_n^{(m)} + (1/r) \cos \alpha_n^{(m)} + \sin 2\alpha_n^{(m)})\\
&\quad + \delta_3^2 \varphi_3^{(3)} \cos 2\alpha_3^{(3)}),\ m = 3,\\
\mu_n &= \frac{(\rho_{11} R - \rho_{12} Q) - (PR - Q^2) V_n^{-2}}{(\rho_{22} Q - \rho_{12} R)}, \quad n = 1, 2, 3,\\
\mu_m &= \frac{(\rho_{11} R - \rho_{12} Q) - (PR - Q^2) V_m^{-2}}{(\rho_{22} Q - \rho_{12} R)}, \quad m = 1, 2, 3.
\end{aligned}
$$

V_1 and V_2 are the velocities of dilatational waves of first and second kind, respectively, and V_3 is the velocity of shear wave.

2.2 Lower poroelastic half space

If a wave incidents in the lower half space, and if it is reflected then from the theory of isotropic poroelasticity there exists two reflected dilatational and a reflected shear wave. In order to solve the pertinent problem, it is convenient to introduce displacement potentials φ_{ij} and ψ_{ij} are given below:

$$
\begin{aligned}
u_2 &= \frac{\partial \varphi_{21}}{\partial r} - \frac{\partial \psi_{21}}{\partial z}, \qquad w_2 = \frac{\partial \varphi_{21}}{\partial z} + \frac{\partial \psi_{21}}{\partial r} + \frac{\psi_{21}}{r}\\
U_2 &= \frac{\partial \varphi_{22}}{\partial r} - \frac{\partial \psi_{22}}{\partial z}, \qquad W_2 = \frac{\partial \varphi_{22}}{\partial z} + \frac{\partial \psi_{22}}{\partial r} + \frac{\psi_{22}}{r}.
\end{aligned} \tag{8}
$$

The displacement potentials φ_{ij} and ψ_{ij} which are functions of r, z, and t are given by [6]

$$\varphi_{21} = \begin{cases} \varphi_{(m)} + \varphi_l^{(m)} = & A_m e^{i(\omega t - \delta_m(r\sin\alpha_m - z\cos\alpha_m))} \\ & + A_l^{(m)} e^{i(\omega t - \delta_t(r\sin\alpha_l^{(m)} - z\cos\alpha_l^{(m)}))},\ m = 1, 2, \\ \varphi_{(3)} + \varphi_3^{(3)} = & A_3 e^{i(\omega t - \delta_3(r\cos\alpha_3 - z\sin\alpha_3))} \\ & + A_3^{(3)} e^{i(\omega t - \delta_3(r\sin\alpha_3^{(3)} - z\cos\alpha_3^{(3)}))}, \end{cases}$$

(9)
$$\varphi_{22} = \begin{cases} \varphi_{(m)} + \varphi_l^{(m)} = & A_m e^{i(\omega t - \delta_m\mu_m(r\sin\alpha_m - z\cos\alpha_m))} \\ & + A_l^{(m)} e^{i(\omega t - \delta_l\mu_l(r\sin\alpha_l^{(m)} - z\cos\alpha_l^{(m)}))},\ m = 1, 2, \\ \varphi_{(3)} + \varphi_3^{(3)} = & A_3 e^{i(\omega t - \delta_3\mu_3(r\cos\alpha_3 - z\sin\alpha_3))} \\ & + A_3^{(3)} e^{i(\omega t - \delta_3\mu_3(r\sin\alpha_3^{(3)} - z\cos\alpha_3^{(3)}))}, \end{cases}$$

$$\psi_{21} = A_3^{(m)} e^{i(\omega t - \delta_3(r\sin\alpha_3^{(m)} - z\cos\alpha_3^{(m)}))},\ m = 1, 2, 3,$$
$$\psi_{22} = A_3^{(m)} e^{i(\omega t - \delta_3\mu_3(r\sin\alpha_3^{(m)} - z\cos\alpha_3^{(m)}))},\ m = 1, 2, 3.$$

Using eq. (9) in eq. (8), one obtains
(10)
$$u_2 = \begin{cases} -i\{\delta_m\varphi_{(m)}\sin\alpha_m + \sum_{l=1}^{2}\delta_l\varphi_l^{(m)}\sin\alpha_l^{(m)} - \delta_3\varphi_3^{(m)}\cos\alpha_3^{(m)},\ m=1,2. \\ i\delta_3\varphi_{(m)}\cos\alpha_m + \sum_{l=1}^{2}\delta_3\varphi_l^{(m)}\sin\alpha_{3l}^{(m)} - \delta_3\phi_3^{(3)}\cos\alpha_3^{(3)},\ m=3. \end{cases}$$

Using these displacement components in eq. (2), the non-zero stress components are derived, which are given by:

$$(11)\quad (\sigma_{rz})_2 = \begin{cases} N\delta_m^2\varphi_{(m)}\sin 2\alpha_m - \sum_{l=1}^{2}\delta_l\varphi_l^{(m)}\sin 2\alpha_l^{(m)} - (\varphi_l^{(m)}/r^2) \\ \quad + \delta_3^2\varphi_3^{(m)}\cos 2\alpha_3^{(m)} + (i/r)\delta_l\varphi_l^{(m)}\sin\alpha_l^{(m)},\ m = 1, 2 \\ N\delta_m^2\varphi_{(m)}\cos 2\alpha_m - \sum_{l=1}^{2}\delta_l\varphi_l^{(m)}\sin 2\alpha_l^{(m)} - (\varphi_l^{(m)}/r^2) \\ \quad + \delta_3^2\varphi_3^{(m)}\cos 2\alpha_3^{(m)} + (i/r)\delta_l\varphi_l^{(m)}\sin\alpha_l^{(m)},\ m = 3. \end{cases}$$

$$(\sigma_{zz})_2 = \begin{cases} -N\delta_m^2\varphi_{(m)}\cos^2\alpha_m + \sum_{l=1}^{2}\delta_i\varphi_l^{(m)}\cos^2\alpha_l^{(m)} + N\delta_3^2\varphi_3^{(m)}\sin 2\alpha_3^{(m)} \\ \quad +(-i/r)(A+Q\mu_m)\delta_m\varphi_{(m)}\sin\alpha_m + \sum_{l=1}^{2}\delta_i\varphi_l^{(m)}(A+Q\mu_l)\sin\alpha_l^{(m)} \\ \quad -(A+Q\mu_3)\delta_3\varphi_3^{(m)}\cos\alpha_3^{(m)} - N\delta_l\varphi_l^{(m)}\cos\alpha_l^{(m)},\ m=1,2, \\ N\delta_m^2\varphi_{(m)}\sin 2\alpha_m + \sum_{i=1}^{2}\delta_l^2\varphi_l^{(m)}\cos^2\alpha_l^{(m)} + N\delta_l^2\varphi_l^{(m)}\sin 2\alpha_l^{(m)} \\ +(-i/r)(A+Q\mu_3)\delta_m\varphi_{(m)}\cos\alpha_m + \sum_{l=1}^{2}(A+Q\mu_3)\delta_l\varphi_l^{(m)}\sin\alpha_l^{(m)} \\ \quad -(A+Q\mu_l)\delta_l\varphi_l^{(m)}\cos\alpha_l^{(m)} - N\delta_3\varphi_3^{(3)}\cos\alpha_3^{(3)},\ m=3. \end{cases}$$

In eq. (11),

$$\mu_l = \frac{(\rho_{11}R - \rho_{12}Q) - (PR - Q^2)V_l^{-2}}{(\rho_{22}Q - \rho_{12}R)}, \quad l = 1,2,3.$$

3 Boundary Conditions

Assume that at $z = 0$, that is, at the interface, the stress components and displacement components must be continuous, i.e.,

$$(\sigma_{rz})_1 = (\sigma_{rz})_2, \quad (\sigma_{zz})_1 = (\sigma_{zz})_2, \quad (u)_1 = (u)_2, \quad (w)_1 = (w)_2. \tag{12}$$

From eqs. (11) and (12) after long calculation, the following relations are obtained:

$$\sum_{n=1}^{3} A_{jn}^{(m)} T_n^{(m)} = A_{j0}^{(m)}, \qquad j,m,n = 1,2,3, \tag{13}$$

where $A_{jn}^{(m)}$ are

$$\begin{aligned} A_{1n}^{(m)} &= \alpha a_1 a_3 + 2a_1 a_3(\mu_L - \mu_T) + \beta a_1^3 a_3(\sin\alpha_n^{(m)})^2 + \mu_T + \mu_L - \mu_T(a_1^2 + a_3^2) \\ &+ \beta a_1^2 a_3^2(\sin 2\ \alpha_n^{(m)} - (1/r)) + (\alpha a_1 a_3 + 2a_1 a_3(\mu_L - \mu_T) \\ &+ \beta a_1^3 a_3(\sin\alpha_t^{(m)})^2 + (\lambda + \alpha(a_1^2 + a_3^2) \\ &+ \beta a_1^2 a_3^2)\sin 2\alpha_n^{(m)} + (\alpha a_1 a_3 + 2a_1 a_3(\mu_L - \mu_T) \\ &+ \beta a_1^3 a_3(\sin 2\alpha_n^{(m)} - \cos^2\alpha_n^{(m)}) + (\lambda + 2\mu_T + 2\alpha a_3^2 + 4a_3^2(\mu_L - \mu_T) \\ &+ \beta a_3^4(\cos\alpha_n^{(m)})^2 + (1/r)\cos\alpha_n^{(m)} + \sin 2\alpha_n^{(m)}))\delta_n^2,\ n = 1,2, \\ A_{13}^{(m)} &= (\alpha a_1 a_3 + 2a_1 a_3(\mu_L - \mu_T) + \beta a_1^3 a_3\cos 2\alpha_4^{(m)} - \mu_T \\ &+ (\mu_L - \mu_T)(a_1^2 + a_3^2)\cos 2\alpha_3^{(m)})\delta_3^2, \end{aligned}$$

$$
\begin{aligned}
A_{2n}^{(m)} &= (\sin\alpha_n^{(m)} - \cos\alpha_n^{(m)})\delta_n^2,\ n = 1, 2,\\
A_{23}^{(m)} &= (\sin\alpha_3^{(3)} - \cos\alpha_3^{(3)})\delta_3^2,\\
A_{3n}^{(m)} &= \alpha a_1 a_3 + 2a_1 a_3(\mu_L - \mu_T) + \beta a_1^3 a_3((\sin 2\alpha_n^{(m)} - (\cos\alpha_n^{(m)})^2)\\
&\quad - \sin\alpha_n^{(m)}))\delta_n^2,\ n = 1, 2,\\
A_{33}^{(m)} &= \alpha a_1 a_3 + 2a_1 a_3(\mu_L - \mu_T) + \beta a_1^3 a_3(\cos\alpha_3^{(3)} - \sin\alpha_3^{(3)})\delta_3^2,\\
A_{10}^{(m)} &= (\alpha a_1 a_3 + 2a_1 a_3(\mu_L - \mu_T) + \beta a_1^3 a_3(\sin\alpha_k)^2 + \mu_T\\
&\quad + (\mu_L - \mu_T)(a_1^2 + a_3^2) + \beta a_1^2 a_3^2 \sin 2\alpha_m)\delta_m^2,\ m = 1, 2,\\
A_{20}^{(m)} &= (\lambda + \alpha(a_1^2 + a_3^2)(\sin\alpha_k)^2 + \alpha a_1 a_3 + 2a_1 a_3(\mu_L - \mu_T)\\
&\quad + \beta a_1 a_3^3(\cos\alpha_m)^2\delta_m^2,\ m = 1, 2,\\
A_{30}^{(m)} &= (\lambda + 2\mu_T + 2\alpha a_3^2 + 4a_3^2(\mu_L - \mu_T) + \beta a_3^4(\sin 2\alpha_m)\delta_m^2,\ m = 1, 2,\\
A_{10}^{(3)} &= (\alpha a_1 a_3 + 2a_1 a_3(\mu_L - \mu_T) + \beta a_1^3 a_3)(\sin 2\alpha_m)\delta_3^2,\\
A_{20}^{(3)} &= (\mu_T + \mu_L - \mu_T(a_1^2 + a_3^2) + \beta a_1^2 a_3^2)(\cos 2\alpha_m)\,)\delta_3^2,\\
A_{30}^{(3)} &= (\alpha a_1 a_3 + 2a_1 a_3(\mu_L - \mu_T) + \beta a_1^3 a_3 \sin 2\ \alpha_m)\delta_3^2.
\end{aligned}
$$

The transmission coefficients are:

$$
\begin{aligned}
T_n^{(m)} = \Big[& A_{10}^{(m)} + A_{20}^{(m)} + A_{30}^{(m)} - (A_{1n}^{(m)} + A_{2n}^{(m)} + A_{3n}^{(m)})(A_n^{(2)}/A_m)\\
& - (A_{1n}^{(m)} + A_{2n}^{(m)} + A_{3n}^{(m)})(A_n^{(3)}/A_m)\Big] \Big[A_{1n}^{(m)} + A_{2n}^{(m)} + A_{3n}^{(m)}\Big]^{-1}.
\end{aligned}
$$

4 Numerical Results

For the numerical process, polypropylene material is considered for self-reinforced upper half space [12], whose material parameters are

$$
\begin{aligned}
&\mu_T = 2.46 \times 10^9\ \text{N/m}^2, && \mu_L = 5.66 \times 10^9\ \text{N/m}^2,\\
&\alpha = 1.28 \times 10^9\ \text{N/m}^2, && \beta = 220.90 \times 10^9\ \text{kg/m}^3,\\
&\rho_{11} = 1.926137 \times 10^3\ \text{kg/m}^3, && \rho_{12} = -0.002137 \times 10^3\ \text{kg/m}^3,\\
&\rho_{22} = 0.215337 \times 10^3\ \text{kg/m}^3,
\end{aligned}
$$

whereas sandstone saturated with water is considered for poroelastic lower half space whose material parameter values are [17]

$$
\begin{aligned}
&A = 0.922 \times 10^{10}\ \text{N/m}^2, && N = 0.92 \times 10^{10}\ \text{N/m}^2,\\
&Q = 0.13 \times 10^{10}\ \text{N/m}^2, && R = 0.63 \times 10^{10}\ \text{N/m}^2,
\end{aligned}
$$

$$\rho_{11} = 1.90302 \times 10^3 \text{ kg/m}^3, \qquad \rho_{12} = 0,$$
$$\rho_{22} = 0.2268 \times 10^3 \text{ kg/m}^3.$$

Employing these values in eq. (13), reflection and transmission coefficients are computed as a function of angle of incidence, and the results are depicted in Figs. 2 to 10. The notations Rcyl-I, Rcyl-II, Tcyl-I, Tcyl-II, are used in the figures representing the reflection coefficients of upper half space, reflection coefficient of lower half space, transmission coefficient of upper half space, transmission coefficients of lower half space, respectively. When fast dilatational wave is incidents, the reflection coeffi-

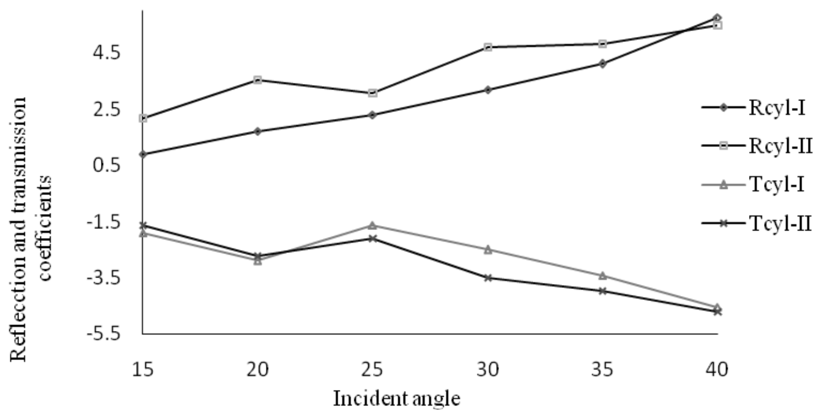

Fig. 2: Reflection and transmission coefficients as a function of angle of incidence (in degrees) for $m = 1$, $n = 1$.

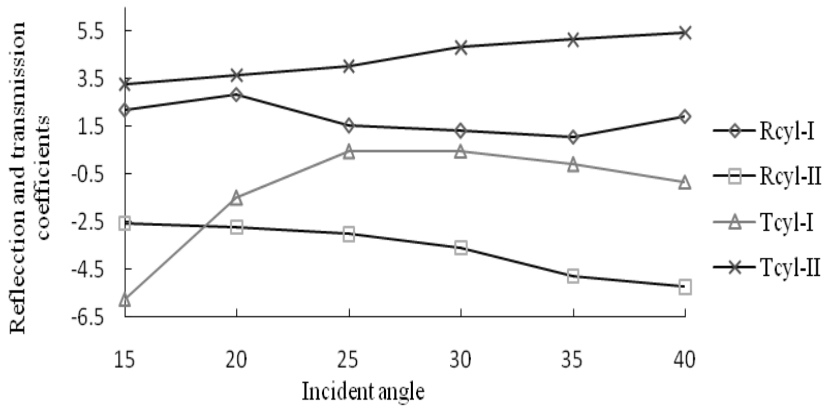

Fig. 3: Reflection and transmission coefficients as a function of angle of incidence (in degrees) for $m = 1$, $n = 2$.

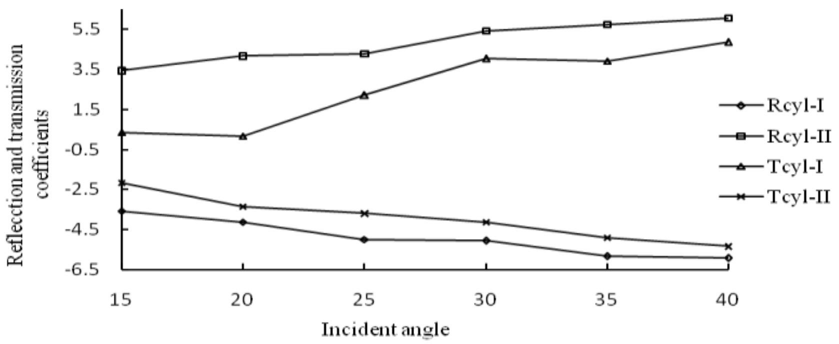

Fig. 4: Reflection and transmission coefficients as a function of angle of incidence (in degrees) for $m = 1, n = 3$.

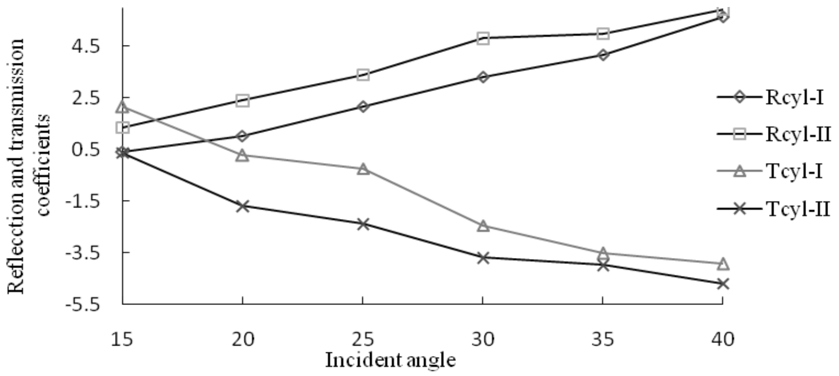

Fig. 5: Reflection and transmission coefficients as a function of angle of incidence (in degrees) for $m = 2, n = 1$.

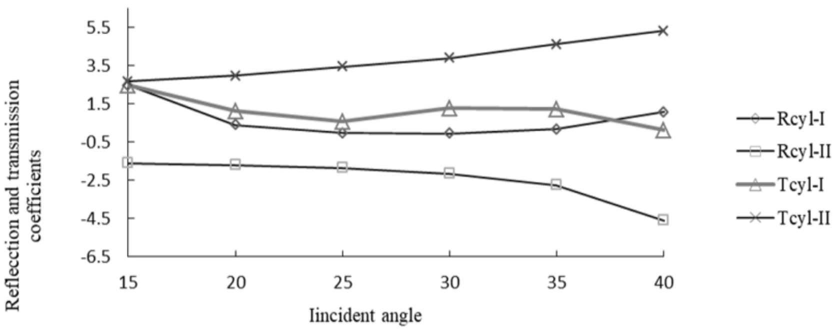

Fig. 6: Reflection and transmission coefficients as a function of angle of incidence (in degrees) for $m = 2, n = 2$.

cients $Z_i^{(1)}$ $(i = 1, 2, 3)$ and transmission coefficients $T_i^{(1)}$ $(i = 1, 2, 3)$ are calculated as a function of angle of incidence and the results are depicted in Figs. 2–4. From the Figs. 2 and 3, it is observed that reflection coefficient increases, and transmission coefficient decreases as angle of incidence increases. In Fig. 4, it is seen that the reflection coefficient decreases, and the transmission coefficient, in general, increases as angle of incidence increases. When slow dilatational wave is incidents, reflection coefficient $Z_i^{(2)}$ $(i = 1, 2, 3)$ and transmission coefficient $T_i^{(2)}$ $(i = 1, 2, 3)$ are calculated as a function of angle of incidence, and the pertinent results are depicted in Figs. 5–7. From Figs. 5 and 6, it is clear that the reflection coefficient, in general, decreases and the transmission coefficient increases for both the half spaces as angle of incidence increases. From Fig. 7, it is seen that the reflection coefficient, in general, increases and the transmission coefficient decreases for both the half spaces as angle of incidence increases. When shear wave incidents, the reflection coefficient $Z_i^{(3)}$ $(i = 1, 2, 3)$ and transmission coefficient $T_i^{(3)}$ $(i = 1, 2, 3)$ are computed as a function of angle of incidence and the results are depicted in Figs. 8–10. From

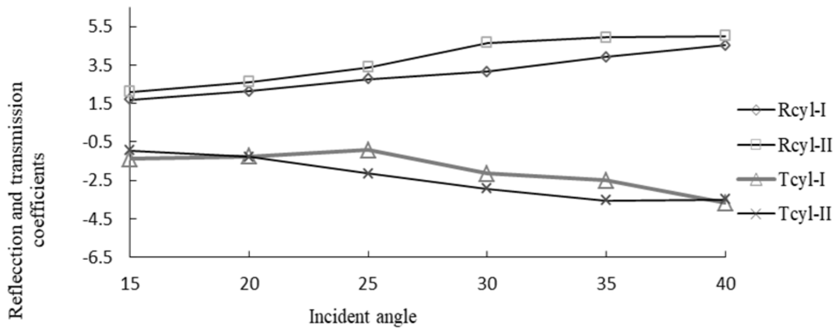

Fig. 7: Reflection and transmission coefficients as a function of angle of incidence (in degrees) for $m = 2$, $n = 3$.

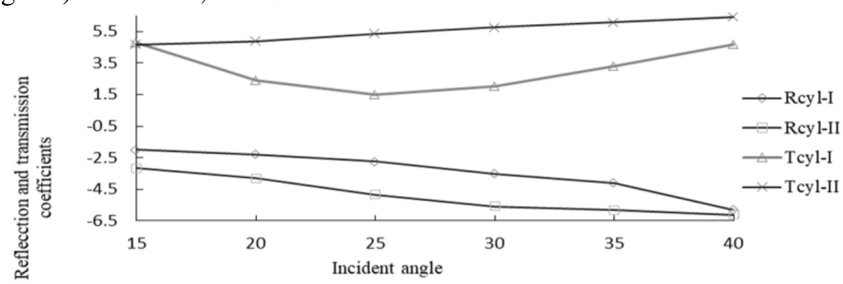

Fig. 8: Reflection and transmission coefficients as a function of angle of incidence (in degrees) for $m = 3$, $n = 1$.

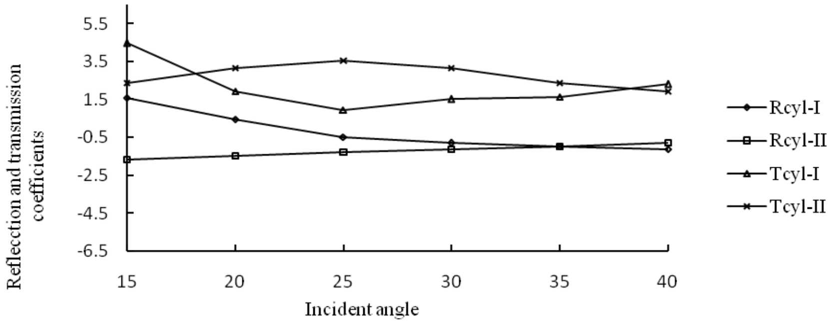

Fig. 9: Reflection and transmission coefficients as a function of angle of incidence (in degrees) for $m = 3$, $n = 2$.

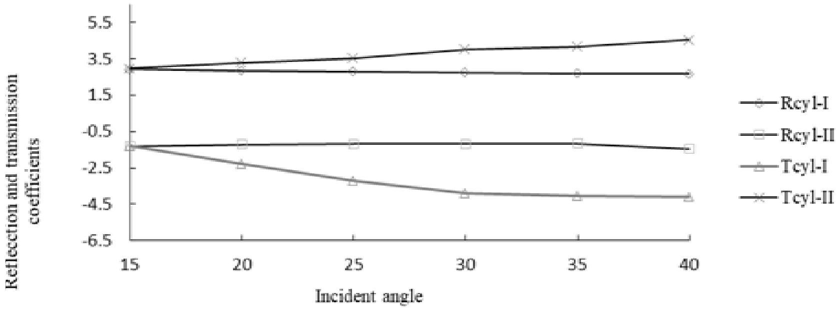

Fig. 10: Reflection and transmission coefficients as a function of angle of incidence (in degrees) for$m = 3$, $n = 3$.

Figs. 8 and 9, it is concluded that reflection coefficient increases and transmission coefficient, in general, decreases for both the half spaces. In Fig. 10, it is seen that reflection and transmission coefficient for self-reinforcement half space decrease as angle of incidence increases, whereas, in the case of poroelastic half space, the trend is reversed.

5 Conclusion

Reflection and transmission phenomena are investigated for axially symmetric waves incident at the interface of two media. Reflection and transmission coefficients are calculated as a function of angle of incidence. In the case of two dilatational waves and a shear wave, both reflection and transmission coefficients are calculated.

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