Journal of Theoretical and Applied Mechanics, Sofia, Vol.53 (2023) pp. 19-33
INTERDISCIPLINARY TOPICS

MINIMIZATION OF THE DRIVE TORQUE OF THE TROLLEY MOVEMENT MECHANISM DURING TOWER CRANE STEADY SLEWING

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[Received: 06 August 2021. Accepted: 17 October 2022]

doi: https://doi.org/10.55787/jtams.23.53.1.19

ABSTRACT: In the article, the optimization of the trolley movement along the boom during steady slewing of the tower crane which provides minimization of the driving force is considered. The optimization was performed using a mathematical model of the mechanism of the trolley movement, which is presented by the system of differential equations. This system is reduced to the one differential equation of the sixth order, which describes the moving torque of the mechanism of the trolley movement.

The variational problem of minimization of the root mean square value of the moving torque of the trolley movement mechanism is stated and solved. In the process of optimization, low- (pendulum) and high-frequency (caused by the change in moving torque of the mechanism) oscillations of the elements of the mechanism during its start are revealed. These oscillations are eliminated at the beginning of the steady movement due to the optimal start mode.

In the process of solving the variational problem, the minimum condition of the integral functional (root mean square value of the moving torque of the trolley movement mechanism), represented by a linear differential equation of the twelfth order, is solved by analytical methods.

The obtained result – trolley optimal movement law – was analyzed based on kinematic, dynamic, and energy features. The smoothness of the movement, elimination of the low- and high-frequency oscillations, and minimizations of

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root mean square value of drive torque during start are the main advantages of the calculated optimal control.

KEY WORDS: crane, trolley, torque, optimization, criterion, motion, oscillation, load.

1 INTRODUCTION

1.1 PROBLEM STATEMENT

It is often possible to combine operations when operating tower cranes to increase their capacity. An example of such a combination is the simultaneous operation of the mechanisms of the trolley movement and the slewing of the crane. As previous studies have shown, when combining the operation of these mechanisms in the elements of the drive mechanisms and the structure of the crane there are significant dynamic loads caused by the oscillations of the load on the flexible suspension. One of the reasons for these oscillations is connected with the changes in the moving torque of the drive mechanisms. In addition, the magnitude of the driving torque depends on energy consumption, because it is proportional to the current in the motor windings, which heats them and leads to irreversible energy consumption. Moreover, this reduces the reliability of the electric motors due to overheating of the winding insulation.

Therefore, the problem of reducing the moving torque of the drive mechanisms when combining their operation is relevant and meets modern requirements for the use of tower cranes in energy-saving operational modes.

1.2 ANALYSIS OF RECENT STUDIES AND PUBLICATIONS

A significant number of papers have been devoted to the study of dynamic loads in cranes (in particular, tower cranes) and oscillations of the load on the flexible suspension [1–7].

In the papers [4–7] the dynamics of the trolley movement mechanism and hoisting of the load for different types of cranes were studied. The oscillations of the load on a flexible suspension have been described in these works. In [8] the joint movement of the trolley movement the crane slewing is considered. Here, the drive of the trolley movement mechanism was controlled during the slewing of the crane with the load on the flexible suspension. The research aims to reduce the load oscillations.

In the article [9] the joint movement of both - crane slewing and trolley moving mechanisms is considered. The optimal control of joint movement was stated and solved via the metaheuristic optimization method [10]. Its application is caused by constraints of the problem, which are reflected with electric drives limitations. A similar goal (elimination of the load pendulum oscillations) was under investigation in a vast number of optimization approaches [11–13]. In the paper [11] the optimization problem of reducing load oscillations on a flexible suspension during the operation of the crane slewing mechanism by using a complex dynamic integral criterion was solved. In the paper [12] it is proposed to optimize the process of starting the electric motor of the mechanism of horizontal movement of the crane by controlling the torque of the drive, to ensure the minimum start duration and eliminate oscillations of the load on the flexible suspension. An important issue in optimization of the crane mechanisms modes is a criterion to minimize. One of the most common criteria to minimize is the duration of a mechanism operation. The solution of optimization problems according to this criterion has the form of an "on-off" function [13], which causes additional loads on the elements of the drive mechanisms and the metal structure of the crane.

In addition, works [14, 15] should be mentioned as they contain some interesting and promising results. For instance, in the article [14] the double-pendulum load oscillations were considered. For the design of a controller, the authors applied Lyapunov and LaSalle's theorems to provide the stability of the controlled motion. Numerical results show the reduction of the load oscillation. In the study [15] a shaper was developed to control the trolley movement during a jib slewing. Here an artificial neural network was trained to cope with the varying of the cable length. Lab experiments supported the theoretical results. However, load oscillations were not completely eliminated.

Analysis of the scientific approaches in this area shows a wide range of possible solutions for load oscillations elimination during the joint operation of the mechanisms of the tower crane. It supports the statement that this direction is relevant and requires further studies. Among them, we may stress optimal-based ones. In the current work, we focus on the calculation of optimal control of the trolley during slewing of the crane.

1.3 The purpose of the study

The purpose of this work is to minimize the moving torque of the drive mechanism of the trolley movement of the tower crane in steady slewing mode. The obtained optimal control law must meet the requirements of load oscillations elimination. The additional requirement to meet is smoothness of the optimal control, which positively influences on the overall dynamical loads of the mechanisms. To achieve the purpose, the following tasks must be solved: 1) to develop the mathematical model of the dynamical system "tower crane - load"; 2) to state the optimal control problem and find its solution; 3) to analyze the obtained results.

2 Main Material

A dynamic model of a tower crane has been developed to optimize the movement modes of the trolley movement mechanism during the steady movement of the slewing mechanism (Fig.1). In this case, the boom system of the crane is represented as a mechanical system with absolutely rigid bodies, except for the traction rope of the trolley, which is an elastic body, and a flexible suspension, which oscillates in the plane of the trolley movement. The moment of inertia I and the moving torque M of the drive of the trolley movement mechanism are reduced to the axis of the drum rotation. The body with a moment of inertia I is connected to the trolley with mass m_1 by an elastic rope with the coefficient of rigidity C or C' depending on the trolley movement direction. The load with the mass m is connected to the center of the trolley with the flexible suspension.

The given dynamic model of the boom crane system is presented as a holonomic mechanical system with three degrees of freedom, where the generalized coordinates are the linear coordinates of the centers of mass of the trolley z and the load x, as well as the angular coordinate of rotation of the trolley movement drive β. The angular deviation of the flexible suspension of the load from the vertical is determined by the angular coordinate υ.

Moving torque of the drive M causes the change of generalized coordinates. The resistance force to the movement of the trolley W, which is always directed opposite

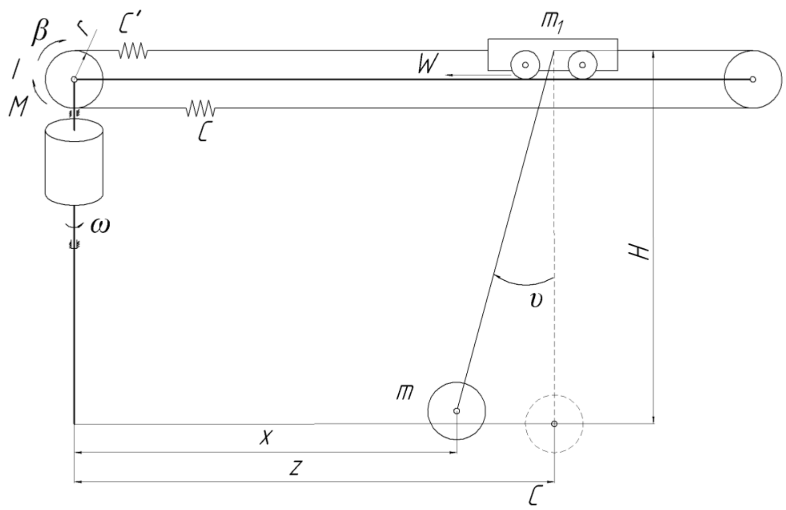

Fig. 1: The dynamic model of the mechanism of the trolley movement during steady crane slewing.

to the movement of the trolley, causes the system as well. The length of the flexible suspension of the load H is a constant value. The angular velocity of the crane slewing ω is a constant value as well.

The angular coordinate of the deviation υ of the flexible suspension from the vertical varies in a small range, which is supported by the practical observations [5]. That is the way it may be determined by the following dependence:

$$\upsilon = \frac{z - x}{H} \ . \tag{1}$$

For a dynamic model (Fig. 1), we may obtain differential equations of motion by using well-known Lagrange equations [16]

$$\begin{aligned} &\frac{d}{dt}\frac{\partial T}{\partial \dot\beta} - \frac{\partial T}{\partial \beta} = M - \frac{\partial \Pi}{\partial \beta}; \\ &\frac{d}{dt}\frac{\partial T}{\partial \dot z} - \frac{\partial T}{\partial z} = -\frac{\partial \Pi}{\partial z}; \\ &\frac{d}{dt}\frac{\partial T}{\partial \dot x} - \frac{\partial T}{\partial x} = -\frac{\partial \Pi}{\partial x}, \end{aligned} \tag{2}$$

where T and Π – the kinematic and potential energy of the dynamic model of the boom system of the crane respectively.

Now we may write down the functions of the kinetic and potential energy of the system

$$T = \frac{1}{2} I \dot\beta^2 + \frac{1}{2} m_1 \left(\omega^2 z^2 + \dot z^2\right) + \frac{1}{2} m \left(\omega^2 x^2 + \dot x^2\right) \ , \tag{3}$$

$$\Pi = \frac{1}{2} \left(\beta r - z\right)^2 + mgH \left(1 - \cos\upsilon\right) \ , \tag{4}$$

where r is the radius of the drum; g is the acceleration of free fall.

Taking the necessary derivatives of functions (3) and (4) and substituting the found results in the system (2), we will have

$$\begin{aligned} &\ddot\beta = M - Cr\left(\beta r - z\right), \\ &m_1 \ddot z - m_1 \omega^2 z = C\left(\beta r - z\right) - \frac{mg}{H}\left(z - x\right) - W, \\ &m \ddot x - m\omega^2 x = \frac{mg}{H}\left(z - x\right). \end{aligned} \tag{5}$$

From the last equation of system (5) we express the coordinate z through x, as a result, we receive

$$
z = \Big(1 - \frac{H}{g}\omega^2\Big)x + \frac{H}{g}\ddot{x},
$$
(6)
$$
\dot{z} = \Big(1 - \frac{H}{g}\omega^2\Big)\dot{x} + \frac{H}{g}\dddot{x},
$$
$$
\ddot{z} = \Big(1 - \frac{H}{g}\omega^2\Big)\ddot{x} + \frac{H}{g}\overset{\mathrm{IV}}{x}.
$$

From the second equation of system (5) we may find the expression

(7)
$$
C\,(\beta r - z) = m_1\ddot{z} - m_1\omega^2 z + m\ddot{x} - m\omega^2 x + W
$$

and express from it the angular coordinate of the drum

(8)
$$
\beta = \frac{1}{r}\left[\left(C - m_1\omega^2\right)z + m_1\ddot{z} + m\ddot{x} - m\omega^2 x + W\right].
$$

Now we substitute expressions z and $\ddot{z}$ from (6) into dependence (8). As a result, we have

(9)
$$
\begin{aligned}
\beta =& \frac{1}{r}\Big\{\Big[(C - m_1\omega^2)\Big(1 - \frac{H}{g}\omega^2\Big) - m\omega^2\Big]x \\
&+ \Big[(C - 2m_1\omega^2)\frac{H}{g} + m_1 + m\Big]\ddot{x} + m_1\frac{H}{g}\overset{\mathrm{IV}}{x} + W\Big\}.
\end{aligned}
$$

Taking the time derivatives of expression (9) will lead to the following:

(10)
$$
\begin{aligned}
\dot{\beta} =& \frac{1}{r}\Big\{\Big[(C - m_1\omega^2)\Big(1 - \frac{H}{g}\omega^2\Big) - m\omega^2\Big]\dot{x} \\
&+ \Big[(C - 2m_1\omega^2)\frac{H}{g} + m_1 + m\Big]\dddot{x} + m_1\frac{H}{g}\overset{\mathrm{V}}{x}\Big\},
\end{aligned}
$$

(11)
$$
\begin{aligned}
\ddot{\beta} =& \frac{1}{r}\Big\{\Big[(C - m_1\omega^2)\Big(1 - \frac{H}{g}\omega^2\Big) - m\omega^2\Big]\ddot{x} \\
&+ \Big[(C - 2m_1\omega^2)\frac{H}{g} + m_1 + m\Big]\overset{\mathrm{IV}}{x} + m_1\frac{H}{g}\overset{\mathrm{VI}}{x}\Big\}.
\end{aligned}
$$

Substitute the expressions z and $\ddot{z}$ in the dependence (7) gives the next result:

$$
\begin{aligned}
C(\beta r - z) =& m_1\frac{H}{g}\overset{\mathrm{IV}}{x} + \Big[m + m_1\Big(1 - 2\frac{H}{g}\omega^2\Big)\Big]\ddot{x} \\
&- \Big[m + m_1\Big(1 - \frac{H}{g}\omega^2\Big)\Big]\omega^2 x + W\,.
\end{aligned}
$$

Now we substitute the obtained expression and dependence (11) in the first equation of system (5), as a result, we obtain the expression of the moving torque

$$
\begin{aligned}
M = {} & \frac{Im_1}{Cr}\frac{H}{g}\overset{\text{VI}}{x} \\
& + \Big\{\frac{I}{Cr}\Big[(C - m_1\omega^2)\frac{H}{g} + m_1\Big(1 - \frac{H}{g}\omega^2\Big) + m\Big] + m_1\frac{H}{g}r\Big\}\overset{\text{IV}}{x} \\
& + \Big\{\frac{I}{Cr}\Big[(C - m_1\omega^2)\Big(1 - \frac{H}{g}\omega^2\Big) - m\omega^2\Big] + \Big[m + m_1\Big(1 - 2\frac{H}{g}\omega^2\Big)\Big]r\Big\}\ddot{x} \\
& - \Big[m + m_1\Big(1 - \frac{H}{g}\omega^2\Big)\Big]\omega^2 r x + Wr.
\end{aligned} \tag{12}
$$

We may represent expression (12) as follows:

$$
M = a_0 + a_1 x + a_2\ddot{x} + a_3\overset{\text{IV}}{x} + a_4\overset{\text{VI}}{x}, \tag{13}
$$

where

$$
\begin{aligned}
a_0 = {} & Wr, \qquad a_1 = -\Big[m + m_1\Big(1 - \frac{H}{g}\omega^2\Big)\Big]\omega^2 r\,, \\
a_2 = {} & \frac{I}{Cr}\Big[(C - m_1\omega^2)\Big(1 - \frac{H}{g}\omega^2\Big) - m\omega^2\Big] + \Big[m + m_1\Big(1 - 2\frac{H}{g}\omega^2\Big)\Big]r, \\
a_3 = {} & \frac{I}{Cr}\Big[(C - m_1\omega^2)\frac{H}{g} + m_1\Big(1 - \frac{H}{g}\omega^2\Big) + m\Big] + m_1\frac{H}{g}r. \\
a_4 = {} & \frac{m_1 IH}{Crg}, \qquad a_{0,1,2,3,4} = \text{const.}
\end{aligned} \tag{14}
$$

As the criterion to minimize during the controlled mode of the trolley movement mechanism we have chosen the RMS value of the driving torque, which is reduced to the axis of a drum. Thus, we have a variational problem:

$$
M_{\text{RMS}} = \Big[\frac{1}{t_1}\int_0^{t_1} M^2 dt\Big]^{1/2} \to \min\,. \tag{15}
$$

The importance of the criterion (15) may be explained by two reasons. The first of them is connected with energy efficiency: the RMS value of drive torque is proportional to the energy losses in the electric motor. The second reason reflects the need for increase of the reliability of the mechanism, which, in turn, requires minimization of the equivalent (RMS value) torque of the drive.

The solution to problem (15) $x = x(t)$, $0 \le t \le t_1$ must satisfy the boundary conditions

$$
\begin{aligned}
&t = 0 : \begin{cases} x = x_0, \ \dot{x} = 0, \ \ddot{x} = x_0\omega^2, \\ \dddot{x} = 0, \ \overset{\mathrm{IV}}{x} = x_0\omega^4, \ \overset{\mathrm{V}}{x} = 0, \end{cases} \\
&t = t_1 : \begin{cases} x = x_0 + \dfrac{Vt_1}{2}, \ \dot{x} = V, \ \ddot{x} = \Big(x_0 + \dfrac{Vt_1}{2}\Big)\omega^2, \\ \dddot{x} = V\omega^2, \ \overset{\mathrm{IV}}{x} = \Big(x_0 + \dfrac{Vt_1}{2}\Big)\omega^4, \ \overset{\mathrm{V}}{x} = V\omega^4, \end{cases}
\end{aligned} \tag{16}
$$

where x_0 is the initial position of the trolley and the load; V is the steady speed of the trolley and the load.

Note, that the variational problem (15) can be rewritten in the equivalent form

$$\int_0^{t_1} M^2 dt \to \min . \tag{17}$$

To simplify the notation of the integrand (17), we introduce the following notation of the solution x:

$$y(t) = x(t) + \frac{a_0}{a_1}, \quad 0 \le t \le t_1 \Leftrightarrow x(t) = y(t) - \frac{a_0}{a_1}, \quad 0 \le t \le t_1, \tag{18}$$

where $y(t)$ is the new unknown function to be found.

Considering the notation (18), as well as the fact that $\ddot{\ } = \ddot{y}$, $\overset{\mathrm{IV}}{x} = \overset{\mathrm{IV}}{y}$, $\overset{\mathrm{VI}}{x} = \overset{\mathrm{VI}}{y}$, we can write

$$
\begin{aligned}
M &= a_0 + a_1 x + a_2\ddot{x} + a_3\overset{\mathrm{IV}}{x} + a_4\overset{\mathrm{VI}}{x} \\
&= a_1\Big(\frac{a_0}{a_1} + x\Big) + a_2\ddot{x} + a_3\overset{\mathrm{IV}}{x} + a_4\overset{\mathrm{VI}}{x} \\
&= a_1 y + a_2\ddot{y} + a_3\overset{\mathrm{IV}}{y} + a_4\overset{\mathrm{VI}}{y} \\
&= \Big[a_1 + a_2\frac{d^2}{dt^2} + a_3\frac{d^4}{dt^4} + a_4\frac{d^6}{dt^6}\Big]y.
\end{aligned} \tag{19}
$$

The condition of the functional (17) minimum is the Euler-Poisson equation [17], which may be presented as follows:

$$\frac{\partial M^2}{\partial y} + \frac{d^2}{dt^2}\frac{\partial M^2}{\partial \ddot{y}} + \frac{d^4}{dt^4}\frac{\partial M^2}{\partial \overset{\mathrm{IV}}{y}} + \frac{d^6}{dt^6}\frac{\partial M^2}{\partial \overset{\mathrm{VI}}{y}} = 0. \tag{20}$$

Substituting the equation (19) into (20) and using a rule of differentiation for a complex function, we obtain

$$
\begin{aligned}
&2Ma_1+\frac{d^2}{dt^2}(2Ma_2)+\frac{d^4}{dt^4}(2Ma_3)+\frac{d^6}{dt^6}(2Ma_4)=0\\
&\Leftrightarrow Ma_1+a_2\frac{d^2M}{dt^2}+a_3\frac{d^4M}{dt^4}+a_4\frac{d^6M}{dt^6}=0\\
&\Leftrightarrow\Big[a_1+a_2\frac{d^2}{dt^2}+a_3\frac{d^4}{dt^4}+a_4\frac{d^6}{dt^6}\Big]^2y=0.
\end{aligned}
\tag{21}
$$

The obtained equation (21) is, in fact, a linear, homogeneous differential equation of the 12-th order. To solve it, we find the roots of a characteristic polynomial

$$Q(\lambda)=[a_1+a_2\lambda^2+a_3\lambda^4+a_4\lambda^6]^2.$$

Since the polynomial $Q(\lambda)$ is a square of a polynomial of the 6-th order, it has six roots. Each of the roots is in the 2-nd order. To find them we write the equation:

$$a_1+a_2\lambda^2+a_3\lambda^4+a_4\lambda^6=0,$$

in which to reduce the degree of the equation we enter the notation $\lambda^2=\mu$. As a result, we have an algebraic equation of the 3-rd order

$$a_1+a_2\mu+a_3\mu^2+a_4\mu^3=0. \tag{22}$$

Its roots may be found by the Cardano method [18] or approximately one of the numerical methods. If the input parameters of the problem have the following values: $m=5000$ kg, $I=30$ kgm^2, $H=10$ m, $\omega=0.075$ rad/s, $r=0.15$ m, $c=1.65\times10^5$ N/m, $V=0.85$ m/s, $x_0=7$ m, $t_1=5$ s, $W=5500$ N then the approximate solutions of equation (22) are: $\mu_1\approx-687.17$, $\mu_2\approx-3.9041$, $\mu_3\approx-0.0044954$.

Considering the equality $\lambda^2=\mu$, for the roots of the characteristic polynomial $Q(\lambda)$ we can write

$$
\begin{aligned}
\lambda_{1,2}&=\pm\sqrt{\mu_1}\approx\pm i26.214=\pm i\alpha_1,\\
\lambda_{3,4}&=\pm\sqrt{\mu_2}\approx\pm i1.91759=\pm i\alpha_2,\\
\lambda_{5,6}&=\pm\sqrt{\mu_3}\approx\pm0.067048=\pm\alpha_3,
\end{aligned}
$$

where i – imaginary unit.

Note, that all roots $\lambda_{1,2,3,4,5,6}$ of the characteristic polynomial $Q(\lambda)$ are roots of the second order. Then the general solution of the linear homogeneous differential equation (21) is written as follows:

$$
\begin{aligned}
y(t)=&(C_1+C_2t)\cos(\alpha_1t)+(C_3+C_4t)\sin(\alpha_1t)+(C_5+C_6t)\cos(\alpha_2t)\\
&+(C_7+C_8t)\sin(\alpha_2t)+(C_9+C_{10}t)e^{\alpha_3t}+(C_{11}+C_{12}t)e^{-\alpha_3t},\ 0\le t\le t_1,
\end{aligned}
$$

where $C_{1,\ldots,12} = \text{const}$.

Substituting the obtained explicit form of the function $y(t)$ in (18), we obtain the function $x(t)$

$$(23)\quad x(t) = y(t) - \frac{a_0}{a_1} = (C_1 + C_2 t)\cos(\alpha_1 t) + (C_3 + C_4 t)\sin(\alpha_1 t) + (C_5 + C_6 t)\cos(\alpha_2 t) + (C_7 + C_8 t)\sin(\alpha_2 t) + (C_9 + C_{10} t)e^{\alpha_3 t} + (C_{11} + C_{12} t)e^{-\alpha_3 t} - \frac{a_0}{a_1},\ 0 \le t \le t_1.$$

To find the coefficients $C_{1,\ldots,12}$, the expression (23) should be substituted to the boundary conditions (16) of the original problem. As a result, we have a system of linear algebraic equations of the 12-th order. Its approximate solution has the following form:

$$\begin{aligned}
&C_1 \approx -1.3540 \times 10^{-6}, && C_2 \approx 1.0536 \times 10^{-7}, && C_3 \approx 2.09208 \times 10^{-8},\\
&C_4 \approx 1.8714 \times 10^{-7}, && C_5 \approx 0.036064, && C_6 \approx -0.014894,\\
&C_7 \approx 0.020835, && C_8 \approx -0.003177, && C_9 \approx -78.196,\\
&C_{10} \approx 2.9682, && C_{11} = -99.386, && C_{12} \approx -4.41522.
\end{aligned}$$

Substituting the found $C_{1,\ldots,12}$ in (23), we obtain the final solution of the variational problem (17).

As a result of the conducted researches plots of kinematic (Figs. 2–4), dynamical (Figs. 5, 6), and power (Fig. 7) characteristics of the trolley movement mechanism during the steady slewing of the tower crane have been built.

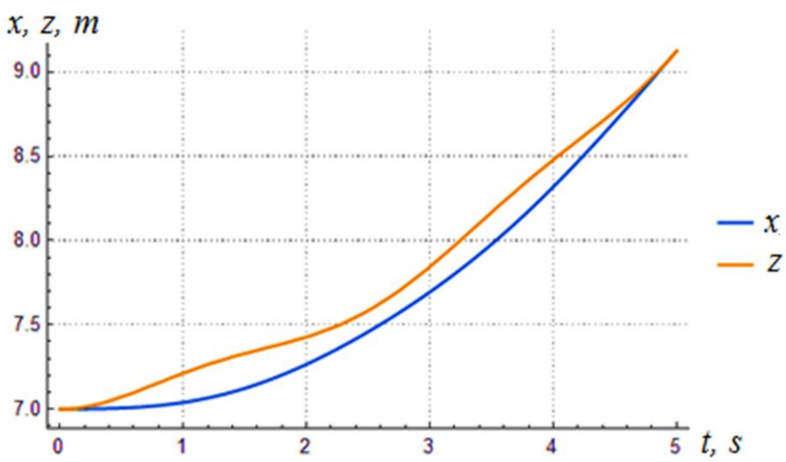

Fig. 2: Plot of the load and the trolley coordinates.

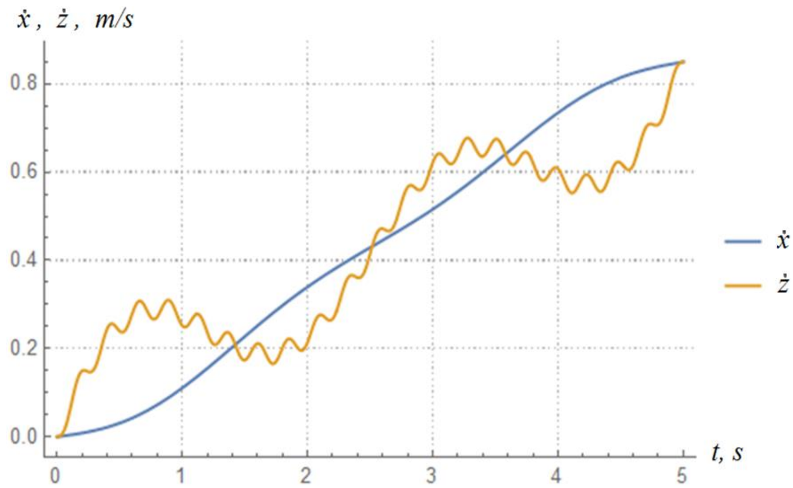

Fig. 3: Plot of the load and the trolley velocities.

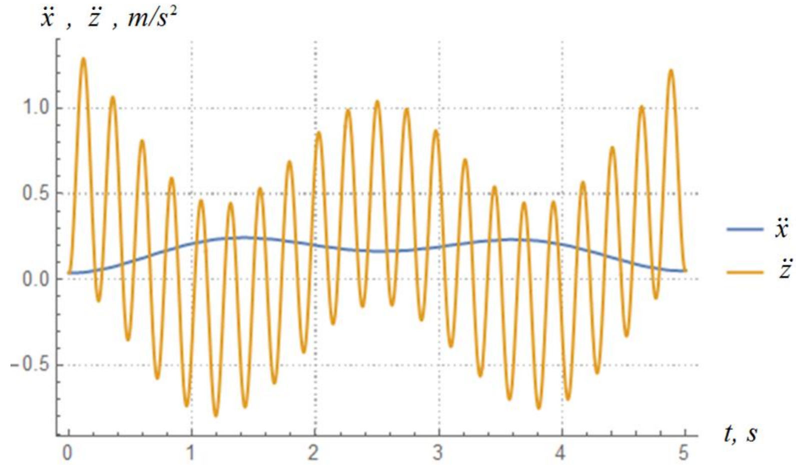

Fig. 4: Plot of the load and the trolley accelerations.

Figures 2 and 3 clearly show that at the end of the start positions and velocities of the trolley and load are the same. It indicates the absence of oscillations of the load relating to the trolley during steady movement. Low-frequency oscillations of the load relating to the trolley are observed only during the start of the system (Fig. 3). High-frequency oscillations, which are caused by the traction rope of the trolley, superimpose to the low-frequency oscillations (Figs. 4 and 5). In Fig. 6 one may observe the moving torque in the trolley movement mechanism drive. Here

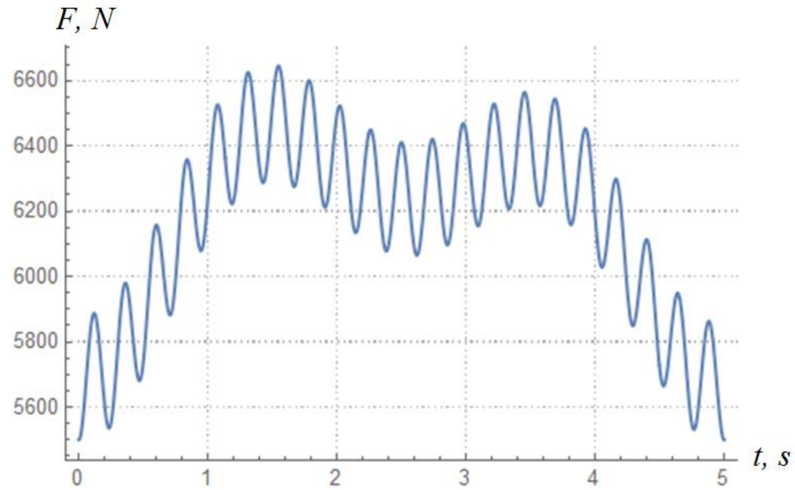

Fig. 5: Plot of the force in the traction rope of the trolley.

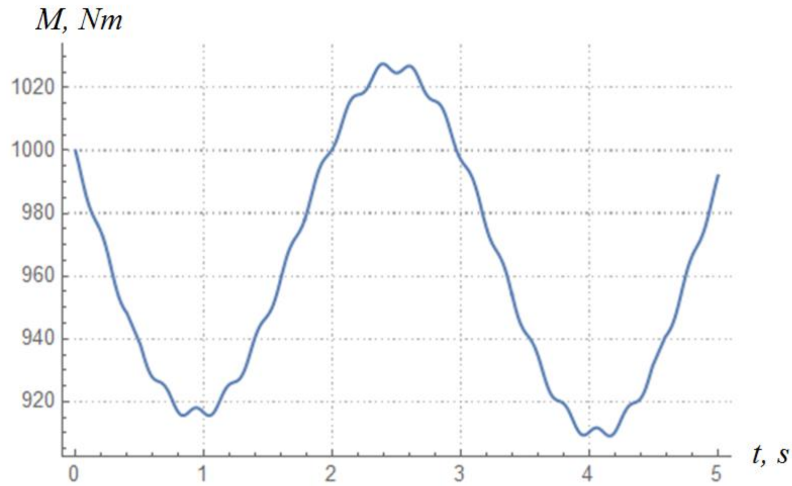

Fig. 6: Plot of the moving torque in the drive of the trolley movement mechanism.

both low- and high-frequency oscillations of the torque are presented. Moreover, low-frequency oscillations are caused by deviations of the load suspension from the vertical, and low-frequency ones are caused by the stiffness of the traction rope and by the character of the drive torque change.

Low-frequency oscillations of the force in the traction body (Fig. 5) and the moving torque (Fig. 6) are clearly expressed. At the same time, high-frequency oscilla-

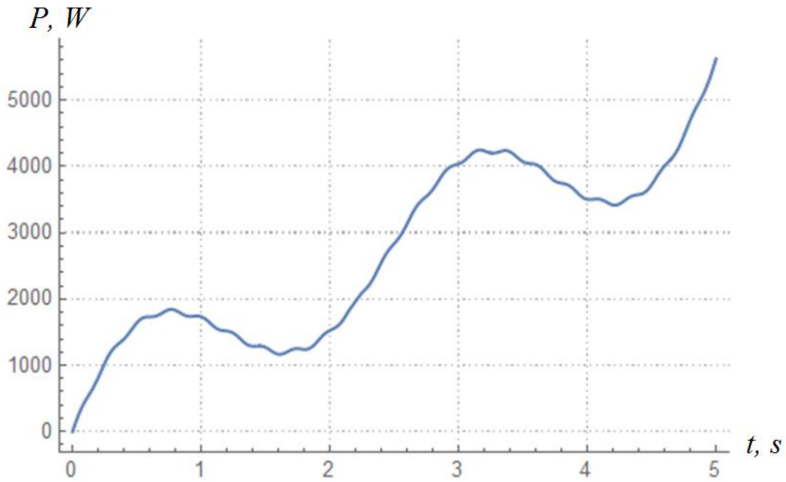

Fig. 7: Plot of the power of the trolley movement mechanism drive.

tions have a minor effect on the moving torque (Fig. 6). However, they have a major effect on the traction force.

The power of the drive of the trolley movement mechanism (Fig. 7) during its start increases with obviously expressed low-frequency oscillations and minor high-frequency oscillations. The highest value of power is reached at the end of the start process and is 5.8 kW.

Therefore, the optimization of the start mode of the trolley movement mechanism during steady movement of the tower crane slewing allowed to minimize the RMS value of the driving torque. There are insignificant low-frequency oscillations of the torque, which are eliminated at the begging of the steady movement. It is caused by the elimination of the load oscillations at this moment.

Minimization of the overall level of moving torque of the trolley movement mechanism allows to an increase the reliability of its operation, and also reduces power consumption. The latter reduces the heating of windings of the electric motor and, in turn, increases their operation life.

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3 Conclusions

1. In the presented study the dynamic model of the trolley movement mechanism during steady tower crane slewing has been developed. The system of differential equations of the second order, which is a mathematical model of the dynamical system, has been found. It was used for a statement of the varia-

tional problem. The latter involved the RMS value of the moving torque of the trolley movement mechanism (the criterion to minimize).

2. The condition of the minimum of the criterion is found, which has been represented by a homogeneous linear with constant coefficients differential equation of the twelfth order. To solve it the roots of the characteristic polynomial have been found. Based on them, an analytical solution of the differential equation was obtained, which is the optimal law of motion of the load. In the following calculations, the optimal modes of movement of other parts of the trolley motion mechanism have been found.

3. Analysis of the obtained results revealed the low-and high-frequency oscillations of kinematic, dynamic, and power characteristics. They are caused by the pendulum oscillations of the load on a flexible suspension and elastic oscillations of the traction rope of the trolley. However, at the beginning of the steady movement, these oscillations are eliminated. This effect increases the operational capacity of the crane and reduces the tension of the crane operator.

4. The undesired feature on the obtained optimal control is the non-zero value of moving torque at the moments $t = 0$ and $t = t_1$. This may increase the overall level of dynamical loads in the metal structure of the crane. Thus, further investigations in this direction are connected with the application of different criteria to minimize. They should refer to the higher derivatives of moving torque and/or consumed power, etc. Taking into account constraints, which are caused by the limitation of electric drives of the crane mechanisms, is also an important issue, that should be considered in the following studies.

REFERENCES

[1] M. STANIŠIĆ (1985) On a new theory of the dynamic behavior of the structures carrying moving masses. *Ingenieur-Archiv* **55** (3) 176-185.

[2] D.C.D. OGUAMANAM, J.S. HANSEN, G.R. HEPPLER (2001) Dynamics of a three-dimensional overhead crane system. *Journal of Sound and Vibration* **242** (3) 411-426.

[3] G. SUN, M. KLEEBERGER (2003) Dynamic responses of hydraulic mobile crane with consideration of the drive system. *Mechanism and Machine Theory* **38** (12) 1489-1508.

[4] T. CAMPARA, H. BUKVIC, D. SPRECIC (2009) Ability to Control Swinging of Payload during the Movement of the Rotary Cranes Mechanism. 4th International Conference on Intelligent Technologies in Logistics and Mechatronics Systems, Kaunas, Lithuania, 52-55.

[5] S. KAZAK (1968) “The dynamics of tower cranes”. Mechanical engineering, Moscow (in Russian).

[6] A.N. ORLOV (1992) General Cranes Dynamic Model. "Optimization of the parameters of construction and road machinery". Yaroslavl Polytechnic Institute Publishing House, Yaroslavl (in Russian).

[7] R.P. GERASIMYAK, V.A. LESHCHEV (2008) "Analysis and Synthesis of Crane Electromechanical Systems". SMIL, Odessa (in Ukrainian).

[8] R.P. GERASIMYAK, E.V. NAIDENKO (2008) Control of an asynchronous electric drive of slewing mechanisms that reduces dynamic loads. *Problems of an automated electric drive. Theory and practice* (Kharkiv) **30** 111-112 (in Russian).

[9] V. LOVEIKIN, Y. ROMASEVYCH, L. SHYMKO, D. MUSHTIN, Y. LOVEIKIN (2021) Optimal control of simultaneous tower crane slewing and trolley movement. *Journal of Theoretical and Applied Mechanics* **51**(4) 421-436.

[10] Y. ROMASEVYCH, V. LOVEIKIN (2018) A novel multi-epoch particle swarm optimization technique. *Cybernetics and information technologies* **18**(3) 62-74.

[11] V. LOVEIKIN, YU. ROMASEVYCH, I. KADYKALO, A. LIASHKO (2019) Optimization of the swinging mode of the boom crane upon a complex integral criterion. *Jornal of Theoretical and Applied Mechanics* **49**(3) 285-296.

[12] E.V. NAYDENKO (2007) Control of the electric drive of mechanisms of horizontal movement with the suspended freight. *Electrical engineering and electrical equipment* **69** 17-22 (in Russian).

[13] Y. ROMASEVYCH, V. LOVEIKIN, O. STEKHNO (2019) Closed-loop optimal control of a system Trolley – Payload. *UPB Scientific Bulletin, Series D: Mechanical Engineering* **81**(2) 3-12.

[14] Z. MENGHUA, Z. YONGFENG, J. BING, M. CHANGHUI, C. XINGONG (2020) Modeling and energy-based sway reduction control for tower crane systems with double-pendulum and spherical-pendulum effects. *Measurement and Control* **53**(1-2) 141-150.

[15] SM. FASIH, Z. MOHAMED, AR. HUSAIN, L. RAMLI, AM. ABDULLAHI, W. ANJUM (2020) Payload swing control of a tower crane using a neural network-based input shaper. *Measurement and Control* **53**(7-8) 1171-1182.

[16] R.F. GANS (2013) "Engineering Dynamics: From the Lagrangian to Simulation" (Mechanical Engineering). 2013th Edition. Springer.

[17] K.W. CASSEL (2013) "Variational methods with applications in science and engineering". Cambridge University Press.

[18] G.A. KORN, T.M. KORN (2013) "Mathematical Handbook for Scientists and Engineers: Definitions, Theorems, and Formulas for Reference and Review". Courier Corporation.