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THERMAL STABILITY ANALYSIS OF FUNCTIONALLY GRADED NANO BEAMS BASED ON HIGHER ORDER SHEAR DEFORMATION THEORY WITH TWO VARIABLES

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ABSTRACT: In this work, the thermal buckling of nano functionally graded beams is investigated using nonlocal higher order deformation shear theory with two variables. The variation of properties of FG material is made through the thickness using a power law and exponential form. Governing equations are derived using the principle of virtual work. The temperature variation is assumed to be uniform. The critical buckling temperature is calculated for various boundary conditions of the beam: simply supported and clamped-clamped beams. The nonlocal parameter, power law index, slenderness radio and boundary conditions on the critical buckling temperature of the FG nano beams are analyzed.

KEY WORDS: functionally graded materials, beam theory, thermal buckling, nonlocal.

1 INTRODUCTION

In recent years, several research projects have been carried out with particular attention to functional gradient materials (FGM). This new material, which was created in Japan in the 1990s, is a composite material that has variable mechanical properties following a function in one or two given directions. It is made by mixing the particles of two or more different materials, for example, metal and ceramic. The idea behind the creation of FGM is to build a material that can resist well in very high temperature environments (up to 1700°C). From this point, one can appreciate the importance of research on the thermal behavior of FGM structures. Temperature variation is one of the major factors influencing the structural stability of engineering structures and in specific of FG materials. This problem is analyzed in various research works.

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Song [1] investigate the thermal buckling and post-buckling behaviors of functionally graded graphene nano platelet (GPL)-strengthened composite multi-layered beams containing an open edge crack and resting on a Pasternak-type elastic foundation, based on the theory of first-order shear deformation beams, including the geometric non-linearity of von Krmn. The obtained non-linear partial differential equations of equilibrium are discretized by the differential quadrature method, and then an iterative method is used to obtain the thermal buckling loads and post-buckling load-deflection curves. Sahmani [2] presented prediction of buckling behavior of size-dependent micro beams made of functionally graded materials including thermal environment effect using strain gradient elasticity theory. It was revealed temperature change plays more important role in the buckling behavior of FGM micro beams with higher values of dimensionless length scale parameter. Rahimnami [3] investigated thermal buckling analysis of functionally graded rectangular nano plates. Nonlocal elasticity theory and third-order shear deformation theory are used and two types of uniform and nonlinear temperature distributions are considered. They found that increasing the aspect ratio or voltage will decrease the critical temperatures. Yimingfu [4] studies the thermal buckling and post-buckling of functionally graded tubes whose material properties are temperature-dependent based on a refined beam model. The nonlinear governing equations of the tubes are obtained by the generalized variational principle and the two-step perturbation technique is used to solve these equations. Ansari [5] The buckling and vibration responses of nano plates made of functionally graded materials subjected to thermal loading are studied in pre-buckling domain with considering the effect of surface stress. Gurtin–Murdoch elasticity theory is incorporated into the classical plate theory to develop a non-classical plate model including the surface effects. They revealed that in the presence of surface stress effect, the influence of material property gradient index on the critical thermal buckling load is more prominent for FGM nano plates with lower length-to-thickness ratios. Trinh [6] proposed an analytical method for the vibration and buckling behavior of functional gradient (FG) beams with different boundary conditions under mechanical and thermal loads with three cases of the temperature rise through the thickness, which are uniform, linear and nonlinear. The state space approach is used to find exact solutions. Ebrahimi [7] examined the thermal effects on the buckling of functionally graded (FG) nano beams subjected to various types of thermal loading including uniform, linear and non-linear temperature changes basing on the nonlocal third-order shear deformation beam theory. Ziane [8] developed an analytical method based on the Galerkin technique for predicting the thermal buckling of simply supported and clamped-clamped porous FGM box beams. Uniform temperature rise and temperature dependency of the constituents is also taken into account. Trung-Kiennguyen [9] examined the hygro-thermal effects on vibration and buckling analysis of functionally graded beams. Hyperbolic higher-order shear deformation theory is used and Ritz solution method to solve problems with different boundary conditions. Wu [10] investigated the imperfection sensitivity of thermal post-buckling behavior of functionally graded carbon nano tube-reinforced composite (FG-CNTRC) beams subjected to in-plane temperature variation. The differential quadrature method in conjunction with modified Newton–Raphson technique is employed to determine the thermal post-buckling equilibrium path of imperfect FG-CNTRC beams. Medani [11] presented the static and dynamic behavior of Functionally Graded Carbon Nanotubes (FG-CNT)-reinforced porous sandwich (PMPV) polymer plate. The model of nanocomposite plate is investigated within the first order shear deformation theory (FSDT). Two types of porous sandwich plates are supposed (sandwich with face sheets reinforced / homogeneous core and sandwich with homogeneous face sheets / reinforced core). Song [12] investigates thermal buckling and post buckling behaviors of functionally graded graphene nano platelet (GPL)-reinforced composite multilayer beams containing an open edge crack and resting on a Pasternak-type elastic foundation based on the first-order shear deformation beam theory including von Krmn geometric nonlinearity. Thermal buckling of functionally graded (FG) porous nano composite beams subjected to a thermal gradient are studied by Yas [13] using generalized differential quadrature method (GDQM). According to their results, the performance of graphene wafers (GPLs) is strongly affected by their geometry. The buckling and lateral buckling of thin-walled functionally graded (FG) open-section beams with various types of material distributions are studied by Bouafia [14] studied the small scale impact on the vibrational properties of functionally graded" (FG) nanoplate embedded in an elastic medium. The formulation in this work is based on the four-unknown refined integral plate theory on aggregate with the nonlocal elasticity theory. Contrary to other theories, this one involves only four unknown variables [15]. The flexural-torsional buckling and lateral buckling of thin-walled functionally graded open-section beams are discussed by using finite element method to solve Governing buckling equations. Kumar [16] examined a refined trigonometric higher-order shear deformation theory with the conjunction of nonlocal theory for the vibrational response of functionally graded porous nanoplate. The displacement field is chosen based on assumptions that the out of the plane displacement consists of bending and shear components whereas the transverse shear-strain has nonlinear variation along the thickness direction [17]. The role of the spatial variation of the nonlocal parameter on the free vibration of functionally graded sandwich nanoplates is investigated in this study. The key achievement of this work is that the classical nonlocal elasticity theory is modified to take into account the dependence of nonlocal parameters on the varying of materials through the thickness of the functionally graded sandwich nanoplates. Vinh [18] studied the effect of the variation of nonlocal parameter throw the thickness on the free vibration of functionally graded doubly curved nanoscale shells using the first order shear deformation theory.

In this paper, an analysis of thermal buckling of functionally graded beam is investigated for different types of volume fraction distribution of constituents materials. The small scale effect is taken into account by using Eringen theory. The higher order shear deformation theory is utilized with neglecting of the membrane effect. A parametric study is made to understand the stability behavior of FG nano beam.

2 Theoretical Formulation

In this work, a beam of length L, thickness h and width b is considered in the conventional Cartesian coordinate system (x, y, z), as shown in Fig. 1. In this system, x refers to the longitudinal direction, while z represents the transverse direction.

The beam studied is an FGM beam whose constituents are varied through thickness:

$$P(z,T) = P_c V_c + P_m V_m \,, \tag{1}$$

with P denotes a mechanical property of the material (Young's modulus, thermal expansion coefficients, etc.), V is the volume fraction of each material, the subscripts

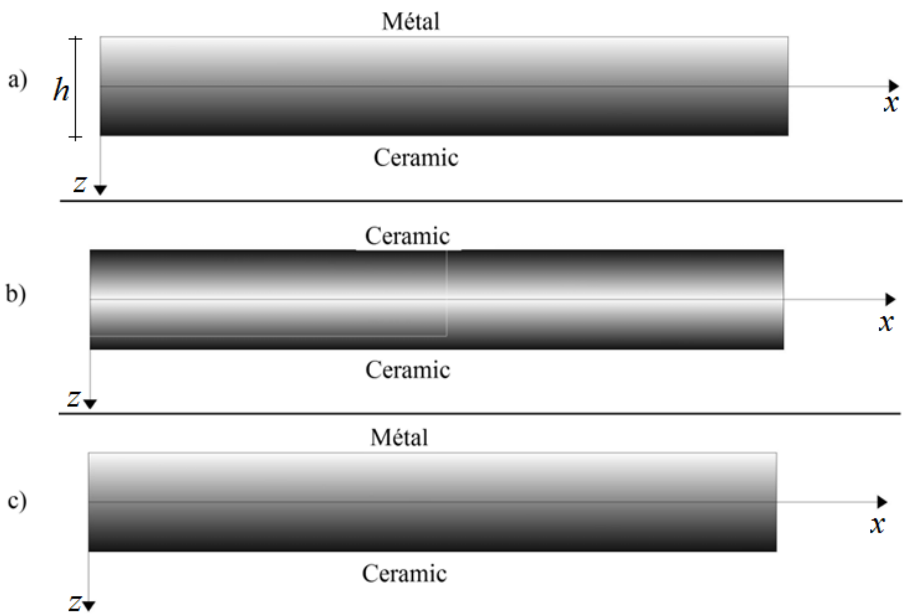

Fig. 1: Geometry of FGM beam: a) type 1; b) type 2; c) type 3.

c and m denote ceramic and metal, respectively. The relationship between the metal and ceramic fractions can be written as follows:

$$V_c + V_m = 1 \Rightarrow V_m = 1 - V_c \,. \tag{2}$$

The mixture law of Young's modulus and coefficient of thermal expansion can be written as follows [19]:

$$E(z,T) = (E_c - E_m)\, V_c + E_m \,, \tag{3}$$

$$\alpha(z,T) = (\alpha_c - \alpha_m)\, V_c + \alpha_m \,. \tag{4}$$

Three FGM beam's models are used in this study:

$$V_c = \Big(\frac{1}{2} + \frac{z}{h}\Big)^k \qquad \text{FGM type 1 (P-FGM)} \tag{5}$$

$$V_c = \Big[1 - a\Big(\frac{1}{2} - \frac{z}{h}\Big) + b\Big(\frac{1}{2} - \frac{z}{h}\Big)^c\Big]^k \qquad \text{FGM type 2 (P-FGM)} \tag{6}$$

$$\begin{cases} E(z) = E_0 e^{\omega z},\ E_0 = \sqrt{E_c E_m},\ \omega = \dfrac{1}{h}\ln\Big(\dfrac{E_c}{E_m}\Big) \\ \alpha(z) = \alpha_0 e^{\omega z},\ \alpha_0 = \sqrt{\alpha_c \alpha_m},\ \omega = \dfrac{1}{h}\ln\Big(\dfrac{\alpha_c}{\alpha_m}\Big) \end{cases} \qquad \text{FGM type 3 (E-FGM)} \tag{7}$$

The displacement field has only two variables [20]. The axial displacement of a point $M(x,y)$ is composed of two parts: a displacement due to bending $(-z\frac{\partial w_b}{\partial x})$; and another due to shearing $(-f(z)\frac{\partial w_s}{\partial x})$. The displacement component of the membrane is neglected. While the transverse displacement consists of two parts: one part due to bending and the other part due to shear. This is expressed in the following relationship:

$$\begin{aligned} u(x,z) &= -z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x}\,, \\ w(x,z) &= w_b + w_s \,. \end{aligned} \tag{8}$$

$f(z)$ is the shape function defining the distribution of strain and transverse shear stress across the thickness [21]

$$f(z) = \frac{h}{\pi}\sin(\frac{\pi z}{h}) \,.$$

The elastic strain at a point is written as follows:

$$\begin{aligned} \varepsilon_x &= -z\frac{\partial^2 w_b}{\partial x^2} - f(z)\frac{\partial^2 w_s}{\partial x^2}\,, \\ \gamma_{xz} &= g(z)\frac{\partial w_s}{\partial x} \quad \text{with} \quad g(z) = 1 - \frac{\partial f(z)}{\partial z}\,. \end{aligned} \tag{9}$$

The virtual work principle states that the variation of the total energy is zero at equilibrium. This energy consists of the difference between the strain energy and the work of the external forces

$$\delta U - \delta V = 0\,. \tag{10}$$

The elastic strain energy depends on the stress, normal and tangential, and the strain. This can be expressed as follows:

$$\begin{aligned}\delta U &= \int \sigma_x \delta\varepsilon_x + \tau_{xz}\delta\gamma_{xz} dV \\ &= \int -M_b \frac{\partial^2 \delta w_b}{\partial x^2} - M_s \frac{\partial^2 \delta w_s}{\partial x^2} + Q\frac{\partial \delta w_s}{\partial x}\,.\end{aligned} \tag{11}$$

Here, N, M_b, M_s and Q represent the internal forces which are defined by

$$\{N, M_b, M_s\} = \int \sigma_x \{1, z, f(z)\} dz \quad \text{and} \quad Q = \int G(z)g(z)dz\,. \tag{12}$$

In this study, the beam is subjected to a thermal axial force N^T due to uniform temperature variation ΔT:

$$N^T = \int E(z,T)\alpha(z,T)\Delta T \cdot dz \tag{13}$$

with $\Delta T = T - T_0$. T_0 is the reference temperature, $T_0 = 25°\text{C}$ and T is the final temperature.

The variation of the work of the external forces is expressed as follows:

$$\delta V = \int N^T \left(\frac{\partial w(x,z)}{\partial x}\frac{\partial \delta w(x,z)}{\partial x}\right) dx\,. \tag{14}$$

Equation (10) leads to the following thermal equilibrium equations:

$$\delta w_b : \frac{\partial^2 M_b}{\partial x^2} - N^T \frac{\partial^2 (w_b + w_s)}{\partial x^2} = 0\,, \tag{15}$$

$$\delta w_s : \frac{\partial^2 M_s}{\partial x^2} + \frac{\partial Q}{\partial x} - N^T \frac{\partial^2 (w_b + w_s)}{\partial x^2} = 0\,. \tag{16}$$

The relationship between non-local stresses and strains can be written using Eringen's theory [22, 25] which assumes that the stress at one point depends on the strain at all points of the body

$$\sigma_x - \mu \frac{\partial^2 \sigma_x}{\partial x^2} = E(z)\varepsilon_x\,, \tag{17}$$

$$\tau_{xz} - \mu \frac{\partial^2 \tau_{xz}}{\partial x^2} = G(z)\gamma_{xz}\,. \tag{18}$$

Combining the expressions (17) and (18) and the equations (12), the internal forces can be written as follows:

$$M_b - \mu \frac{\partial^2 M_b}{\partial x^2} = -B\frac{\partial^2 M_b}{\partial x^2} - C\frac{\partial^2 M_s}{\partial x^2}\,, \tag{19}$$

$$M_s - \mu \frac{\partial^2 M_s}{\partial x^2} = -D_s\frac{\partial^2 M_b}{\partial x^2} - H\frac{\partial^2 M_s}{\partial x^2}\,, \tag{20}$$

$$Q - \mu \frac{\partial^2 Q}{\partial x^2} = A_s\frac{\partial w_s}{\partial x}\,. \tag{21}$$

By replacing the expressions (19), (20) and (21) in the system of equations (15) and (16), the non-local differential equations in terms of displacement for the thermal buckling of FGM nano beams are obtained as follows:

$$-\mu\Big(N^T\frac{\partial^4(w_b+w_s)}{\partial x^4}\Big) + D\frac{\partial^4 w_b}{\partial x^4} + D_s\frac{\partial^4 w_s}{\partial x^3 4} + N^T\frac{\partial^2(w_b+w_s)}{\partial x^2} = 0\,, \tag{22}$$

$$-\mu\Big(N^T\frac{\partial^4(w_b+w_s)}{\partial x^4}\Big) + D_s\frac{\partial^4 w_b}{\partial x^4} + H\frac{\partial^4 w_s}{\partial x^3 4} - A_s\frac{\partial^2 w_s}{\partial x^2} + N^T\frac{\partial^2(w_b+w_s)}{\partial x^2} = 0\,, \tag{23}$$

where B, C, D, D_s, H and A_s are the stiffnesses of the FGM beam

$$\{B, C, D, D_s, H\} = \int E\,(z,T)\,\{z, f(z), z^2, zf(z), f(z)^2\}dz\,,$$
$$A_s = \int G\,(z,T)\,g(z)dz.$$

3 Solution Methodology

Here, an exact solution of the differential equations for thermal buckling of functionally graded nano beams is presented:

$$\begin{aligned} w_b &= \sum_{i=1}^{N} Wb_i F(x)\,,\\ w_s &= \sum_{i=1}^{N} Ws_i F(x)\,. \end{aligned} \tag{24}$$

The function $F(x)$ depends on the boundary conditions [23]:

$$\text{Simply supported (SS)}: \quad F(x) = \sin(\lambda x)$$
$$\text{Clamped-Clamped (CC)}: \quad F(x) = \sin^2(\lambda x)$$

$\lambda = (n\pi)/L$, n is an integer.

Replacing the displacement expression (24) in the system of differential equations gives the following matrix system:

$$([K] - N^T [K^T]) \{u\} = 0 \tag{25}$$

with

$$[K] = \begin{bmatrix} D\dfrac{\partial^4 F(x)}{\partial x^4} & D_s\dfrac{\partial^4 F(x)}{\partial x^4} \\ D_s\dfrac{\partial^4 F(x)}{\partial x^4} & H\dfrac{\partial^4 F(x)}{\partial x^4} - A_s\dfrac{\partial^2 F(x)}{\partial x^2} \end{bmatrix}, \quad [K^T] = \begin{bmatrix} \dfrac{\partial^2 F(x)}{\partial x^2} & \dfrac{\partial^2 F(x)}{\partial x^2} \\ \dfrac{\partial^2 F(x)}{\partial x^2} & \dfrac{\partial^2 F(x)}{\partial x^2} \end{bmatrix}.$$

The critical buckling temperature of the FGM nano beam is obtained by solving the eigenvalue problem (25). T_{cr} has the following expression:

$$T_{\text{cr}} = \frac{-DH\left(\frac{\partial^4 F(x)}{\partial x^4}\right)^2 + DA_s\frac{\partial^2 F(x)}{\partial x^2} + D_s^2\frac{\partial^4 F(x)}{\partial x^4}}{A_s\left(\frac{\partial^2 F(x)}{\partial x^2}\right)^2 + \frac{\partial^4 F(x)}{\partial x^4}(-D - H + 2D_s)} \frac{1}{\int_{-h/2}^{h/2} E(z)\alpha(z)dz} + T_0\,. \tag{26}$$

4 Results and Discussion

To understand the thermal buckling of FGM nano beams, a detailed parametric study is carried out. For this purpose, an FGM material consisting of Si3N4 ceramic and SUS304 steel will be used. Their material characteristics are presented in Table 1. The parameters taken into account in the parametric study are: the slenderness ratio of the beam, the power law index "k", the non-local parameter "μ" and the boundary conditions.

Firstly, some comparative studies are carried out to validate the accuracy and effectiveness of the present method. Table 2 gives the critical buckling temperature

Table 1: Mechanical properties of ceramic and metal

Material	E (GPa)	α (1/°C)
Si3N4	322.20	7.48E-06
SUS304	207.79	1.53E-05

Table 2: Critical temperature of FGM beam type 1

k	L/h	SS					CC				
		10	25	30	35	40	10	25	30	35	40
0	Ref. [24]	1080.50	176.31	122.58	90.12	69.03	4067.80	698.95	485.29	358.17	275.79
	$\mu = 0$	1084.10	175.63	122.05	89.71	68.70	4157.76	697.65	485.86	357.57	274.07
	$\mu = 1$	987.01	159.86	111.09	81.65	62.53	2980.93	500.19	348.34	256.36	196.50
	$\mu = 2$	905.65	146.68	101.93	74.92	57.38	2323.33	389.84	271.49	199.81	153.15
1	Ref. [24]	905.65	154.10	107.14	78.77	30.34	3524.50	605.59	426.14	314.06	240.90
	$\mu = 0$	947.62	153.52	106.69	78.42	60.06	3629.55	609.72	424.65	312.54	239.56
	$\mu = 1$	862.49	139.73	97.11	71.38	54.66	2602.23	437.14	304.46	224.08	171.75
	$\mu = 2$	791.40	128.22	89.10	65.49	50.16	2028.17	340.71	237.29	174.64	133.86
3	Ref. [24]	808.98	132.16	91.89	67.56	51.75	3046.90	523.53	363.83	286.19	206.54
	$\mu = 0$	812.39	131.67	91.51	67.26	51.51	3107.05	522.80	364.15	268.03	205.45
	$\mu = 1$	739.41	119.84	83.29	61.22	46.88	2227.62	374.83	261.08	192.16	147.30
	$\mu = 2$	678.46	109.97	76.42	56.17	43.02	1736.20	292.14	203.49	149.77	114.81
4	Ref. [24]	664.90	108.70	75.58	55.57	42.57	2503.70	430.20	300.51	221.55	169.98
	$\mu = 0$	667.89	108.30	75.26	55.32	42.37	2714.36	434.30	301.60	221.58	169.65
	$\mu = 1$	607.89	98.57	68.50	50.35	38.56	1946.08	311.37	216.23	158.86	121.63
	$\mu = 2$	557.79	90.44	62.86	46.20	35.38	1516.77	242.68	168.53	123.82	94.80

of FGM type 1 beams for different values of L/h, k and μ for two cases of boundary condition: SS and CC. These results obtained by the present work are compared by those found by Hosseini [24]. The results are in excellent agreement.

Figures 2 and 3 show the variation of the critical temperature as a function of the slenderness ratio L/h, for different values of μ, for the FGM type 1 beam. It can be seen that T_{cr} decreases as the slenderness ratio L/h increases, and this for both cases:$k = 1$ and $k = 10$. It can also be seen that the values of *Tcr* are significantly larger in the case of clamped-clamped beams (Fig. 3).

The thermal buckling of FGM type 2 beams as a function of the L/h ratio is shown in Fig. 4 and Fig. 5. The values of T_{cr} in this case are lower than those in the case of FGM type 1. The ceramic distribution in FGM type 1 is symmetrical. It is 100% in both faces, and decreases towards the centre of the beam. In the case of FGM type 2, the ceramic reaches 100% in the upper side of the beam and 0% in the lower side. This variation in material characteristics explains the flexibility of the beam in the case of FGM type 2 compared to the case of FGM type 1.

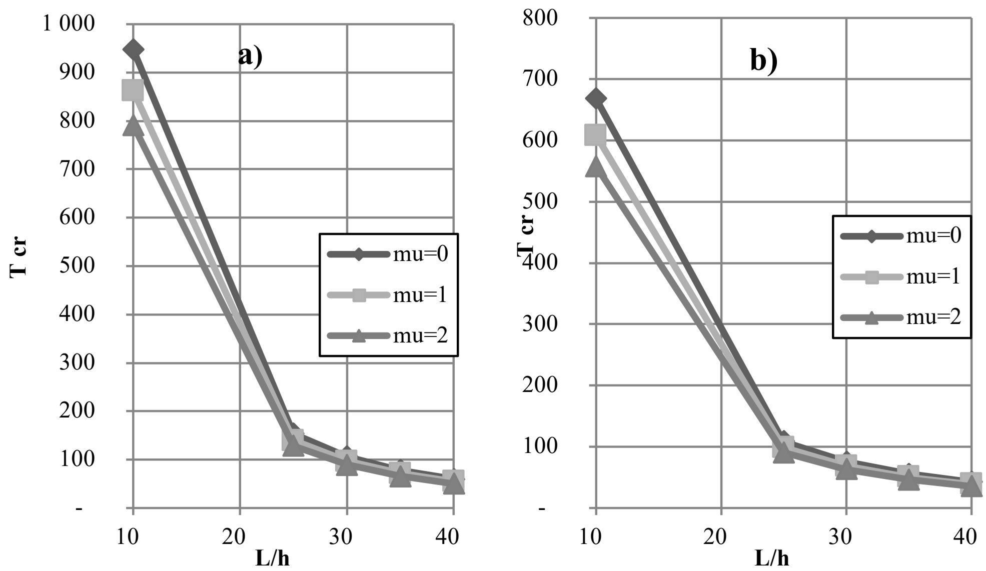

Fig. 2: Variation of T_{cr} as a function of L/h for FGM type 1, SS: a) $k = 1$; b) $k = 10$.

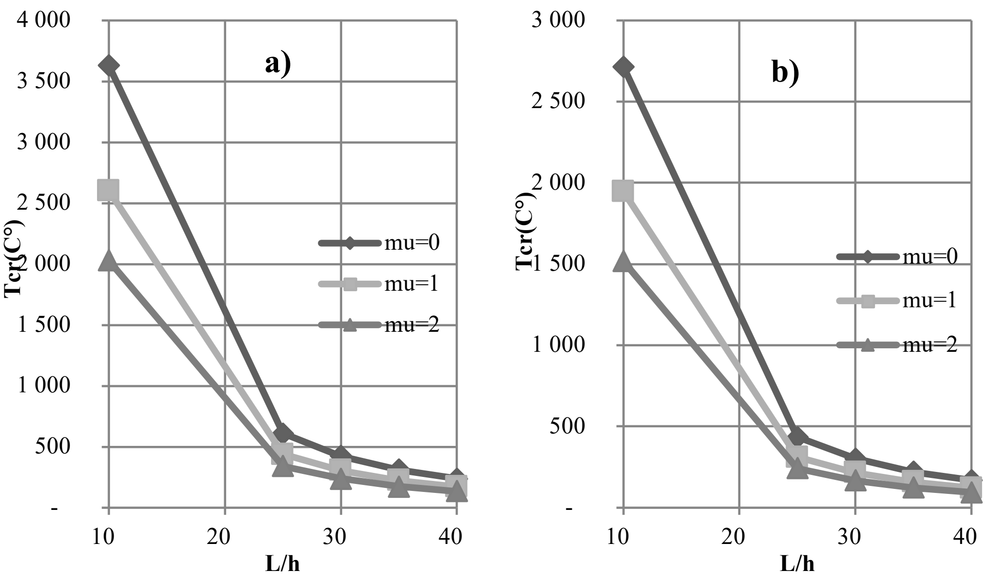

Fig. 3: Variation of T_{cr} as a function of L/h for FGM type 1, CC: a) $k = 1$; b) $k = 10$.

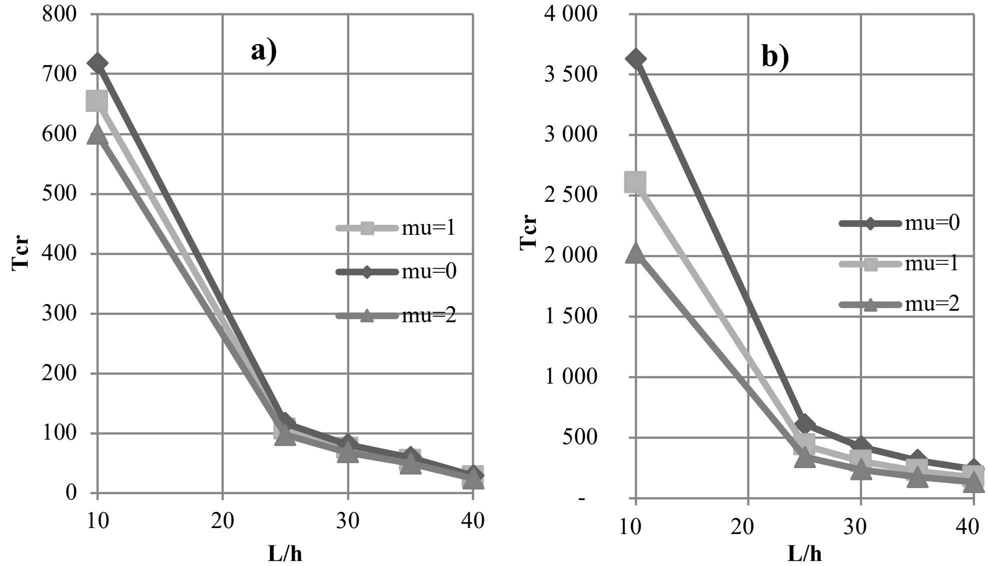

Fig. 4: Variation of T_{cr} as a function of L/h for FGM type 2, SS: a) $k = 1$; b) $k = 10$.

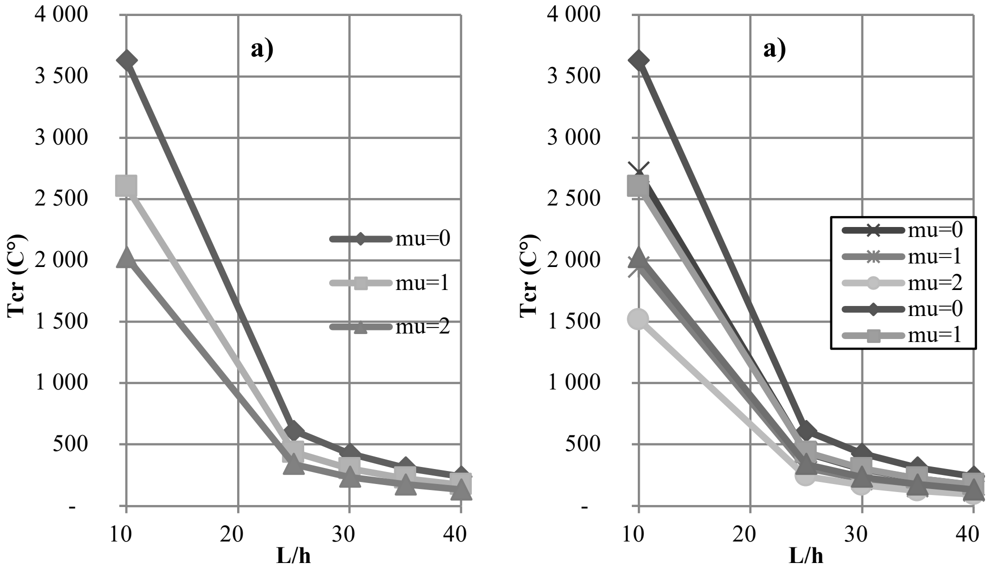

Fig. 5: Variation of T_{cr} as a function of L/h for FGM type 2, CC: a) $k=1$; b) $k=10$.

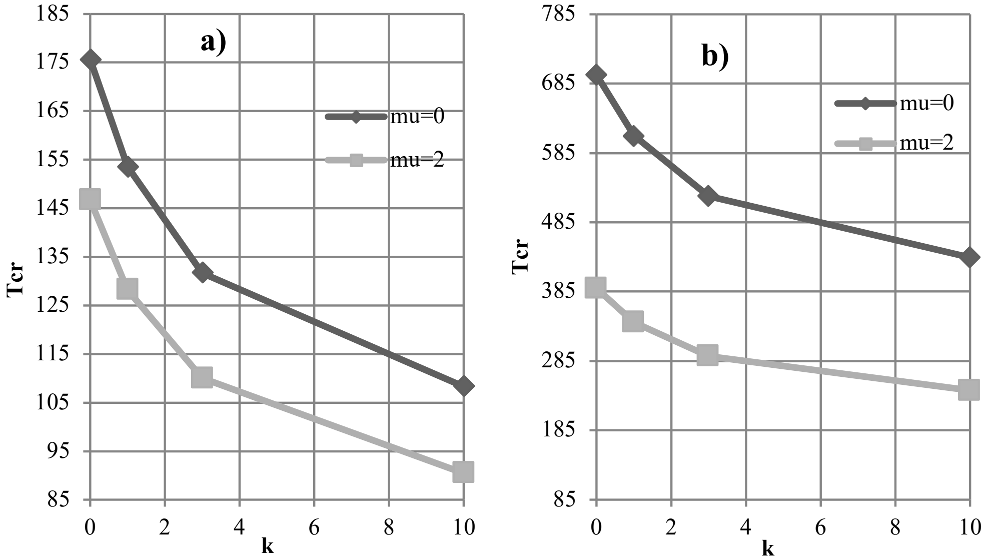

Fig. 6: Variation of T_{cr} as a function of k for FGM type 1, $L/h = 25$: a) SS; b) CC.

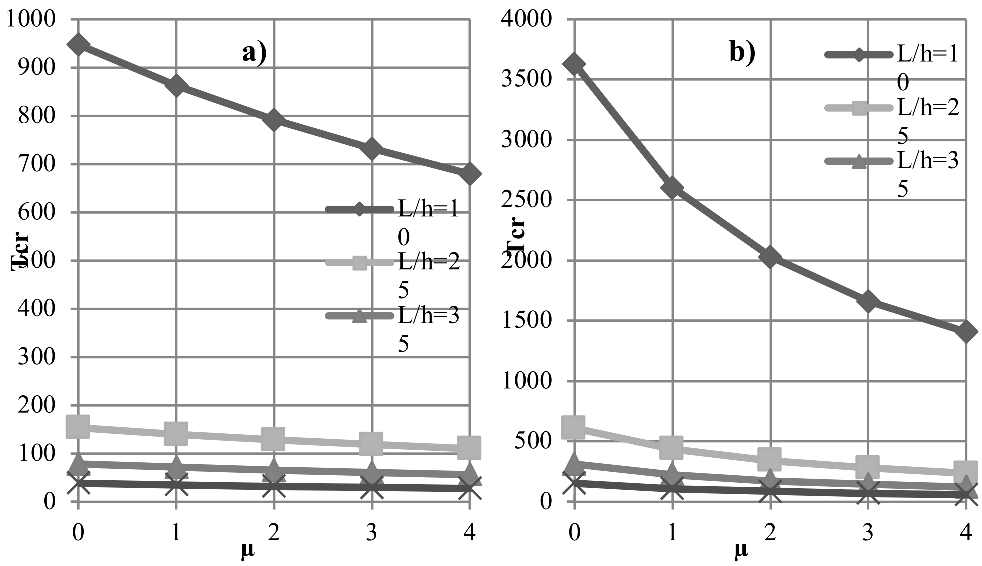

Fig. 7: Variation of T_{cr} as a function of μ for FGM type 1, $k = 1$: a) SS; b) CC.

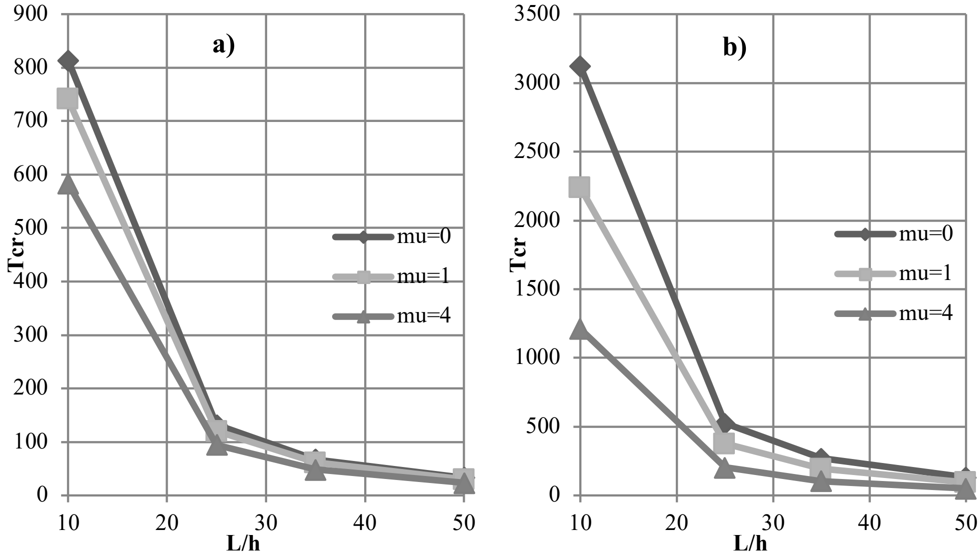

Fig. 8: Variation of T_{cr} as a function of L/h for FGM type 3: a) SS; b) CC.

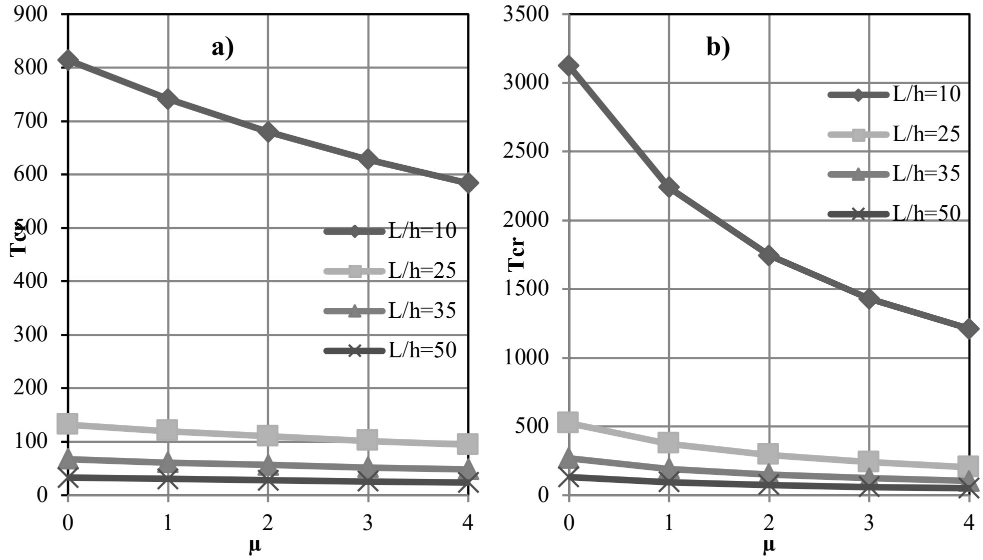

Fig. 9: Variation of T_{cr} as a function of μ for FGM type 3: a) SS; b) CC.

Figure 6 shows the variation of T_{cr} as a function of the power law index k, in the case of FGM type 1 beams. The critical buckling temperature decreases as the value of k increases. This is due to the fact that by increasing the coefficient k the volume fraction of the metal increases and therefore the Young's modulus decreases. This decrease in Young's modulus causes the beam to become more flexible, which leads to a reduced of the critical buckling temperature. The values of T_{cr} in the case of CC beams are larger.

The effect of scale factor variation on T_{cr} is shown in Fig. 7 for different values of slenderness ratio L/h. The increase of mu leads to a decrease of T_{cr}. Taking into account the scaling effect through the mu coefficient makes the beam more flexible. Therefore, the local classical theory overestimates the value of T_{cr} in the analysis of nano beams. In Fig. 8 and Fig. 9, the effect of L/h and mu on the thermal buckling behavior of E-FGM beams is shown.

5 Conclusion

This paper presents an analytical study of the buckling behaviour of a nano-functional gradient beam subjected to a uniform temperature variation. The model used is based on the higher order strain theory with two parameters and the non-local Eringen constitutive relations. The virtual work principle is used to derive the system of differential equations. The results are calculated for two types of boundary conditions (SS and CC) and three types of FG materials. It is deduced that the critical buckling temperature increases as the slenderness ratio of the beam decreases. The same effect is recorded with the power law index. In addition, it is found that the consideration of the scale effect has a significant influence in the case of nano or micro beams. Neglecting this effect overestimates the critical buckling temperature, which makes it necessary to take into account the effect of scale in the thermal analysis of the stability of nano and micro beams. Furthermore, all results for clamped beams are higher than those for simply supported boundary conditions. The use of clamped boundary conditions increases the stiffness of the beam. It was found that ignoring the displacement due to the membrane effect has no influence on the thermal buckling behavior of the nano beam. On the other hand, this ignorance simplifies the formulation of the problem and thus the solution of the differential equations.

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