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SOLID MECHANICS

INVESTIGATION OF THE SPATIAL DEFORMATIONS AND TRANSVERSE VIBRATIONS IN MAIN LINKS OF BIG BAND SAW MACHINES

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ABSTRACT: In this paper, the spatial deformations and transverse vibrations in the cutting mechanisms of big band saw machines are investigated. Functions describing the static deformations in two mutually perpendicular planes are received. Transverse vibrations and dynamic deformations are also studied. Functions describing these vibrations and deformations have been received. Diagrams and surfaces, showing the type of the static and dynamic deformations are built. The proposed theoretical expressions, diagrams and surfaces can be used to design new band saw machines, as well as to study the influence of various dynamic processes that occur in operational mode.

KEY WORDS: band saw machines, elastic line, spatial deformations, transverse vibrations.

1 INTRODUCTION

The band saw machines are a certain class of woodworking machines that are used in various technological processes of the woodworking industry. According to the diameters of the leading wheels and the width of the band saw blade, the band saw machines are divided into different groups. Of particular interest are the big band saw machines. These machines are used for sawing logs with very large diameters. As a result, very big loads occur during operation [1–3]. These loads cause large deformations in the main links of the machines.

The main purpose of this study is to investigate the influence of the static and dynamic loads on the upper shaft of big band saw machines when the leading wheel is mounted between the two bearing supports.

To achieve this purpose, the following main tasks must be solved:

- investigation of the upper shaft deformations as a result of static loads;

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- study of the transverse vibrations and deformations of the upper shaft as a result of dynamic loads;
- construction of plane and spatial diagrams showing the deformations of the upper shaft when changing different parameters.

2 Expose

To solve the problems posed, a scheme of the cutting mechanism of the band saw machine is presented. The dynamic model is also shown. This model is used to solve the main tasks of the presented research.

2.1 Scheme of the cutting mechanism

The scheme of the cutting mechanism is shown in Fig. 1 [4–6].

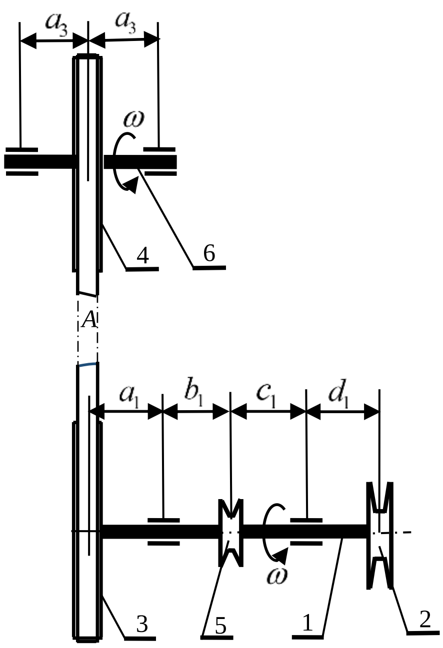

Fig. 1: Cutting mechanism. The following symbols are defined: 1 – main shaft, 2 and 5 – belt pulleys, 3 and 4 – leading wheels, A – band saw blade, and 6 – upper shaft.

2.2 Dynamic model

Figure 2 shows the dynamic model. This model is presented by the author in his previous work [6] and it is built for the case where the leading wheel is mounted symmetrically between the two bearing supports.

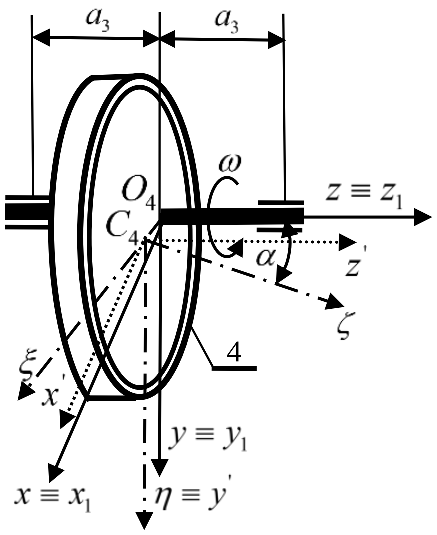

Fig. 2: Dynamic model.

The following coordinate systems are used to obtain expressions for calculating the deformations of the upper shaft [6]:

- fixed coordinate system O_4xyz and moving coordinate system $O_4x_1y_1z_1$. In the initial moment the axes of the two coordinate systems coincide;
- coordinate system $C_4x'y'z'$, whose axes are parallel to the axes of the moving coordinate system;
- coordinate system $C_4\xi\eta\varsigma$. The axes of this coordinate system are principal axes of inertia.

The linear and angular deviations e and α are also shown in the figure as $e = O_4C_4$ and α is the angle between the axes ζ and z.

2.3 Static and dynamic loads on the upper shaft

The static and dynamic loads, as well as the full dynamic reactions have been determined by the author in his previous research [6]. They can be calculated from the expressions below.

$$
\begin{aligned}
S_x &= S_x^{\mathrm{st}} + S_x^{\mathrm{d}}\,, \qquad & S_y &= S_y^{\mathrm{st}} + S_y^{\mathrm{d}}\,,\\
T_x &= T_x^{\mathrm{st}} + T_x^{\mathrm{d}}\,, \qquad & T_y &= T_y^{\mathrm{st}} + T_y^{\mathrm{d}}\,.
\end{aligned} \tag{1}
$$

3 Deformations of the Upper Shaft as a Result of Static Loads

We consider the static loads of the upper shaft in two mutually perpendicular planes, O_4yz and O_4xz. These loads are shown in Fig. 3.

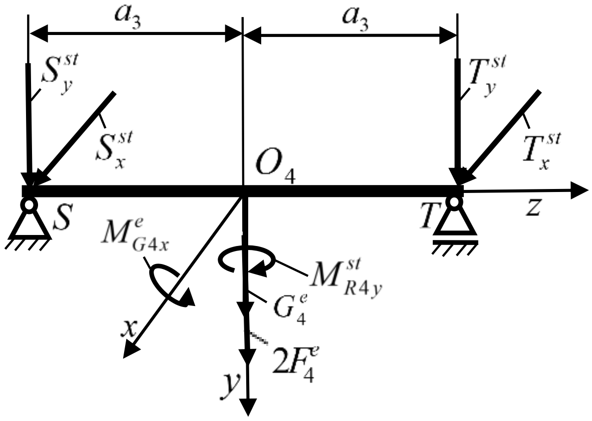

Fig. 3: Static load of the upper shaft.

The static loads can be calculated from the following dependencies:

$$M^{e}_{G4x} = m_4 g e \sin\alpha\,, \qquad M^{\rm st}_{R4y} = \frac{1}{2}\Big(\frac{mK_{\Delta(\lambda)}bHu}{V} + R_\Sigma\Big) r_4 \cos\alpha\,,$$
$$G^{\rm e}_4 = m_4 g\,, \qquad F^{\rm e}_4 = 1.5N_{\rm c}/V\,, \tag{2}$$

where m_4 and r_4 are the mass and the radius of the leading wheel 4 and g is the acceleration of gravity. $K_{\Delta(\lambda)}$ is the specific work of the cutting [6, 7]. R_Σ is the total resistance force. H is the thickness of the workpiece, u is the feeding speed, b is the width of the cutter. m is a coefficient $(0 \le m \le 1)$. $N_{\rm c}$ is the cutting power and V is the cutting speed. The linear deviation (eccentricity) e and angular deviation α are shown in Fig. 2 as $e = O_4C_4$ and α is the angle between the axes ζ and z. Expressions for calculating the static reactions are written below.

$$\begin{aligned} S^{\rm st}_x &= -\frac{r_4}{4a_3}\Big(\frac{mK_{\Delta(\lambda)}bHu}{V} + R_\Sigma\Big)\cos\alpha\,,\\ T^{\rm st}_x &= \frac{r_4}{4a_3}\Big(\frac{mK_{\Delta(\lambda)}bHu}{V} + R_\Sigma\Big)\cos\alpha\,,\\ S^{\rm st}_y &= -\frac{m_4 g}{2a_3}(e\sin\alpha + a_3) - \frac{1.5N_{\rm c}}{V}\,,\\ T^{\rm st}_y &= \frac{m_4 g}{2a_3}(e\sin\alpha - a_3) - \frac{1.5N_{\rm c}}{V}\,. \end{aligned} \tag{3}$$

3.1 STATIC DEFORMATIONS IN A VERTICAL PLANE

To determine the static deformations, the following algorithm is used:

1. obtaining equations of the bending moments for each part of the shaft;

2. compilation of the differential equations of elastic lines;

3. obtaining expressions for determining the static deformations in a vertical plane for each part of the shaft.

In this case, we use Fig. 4 and apply the described algorithm.

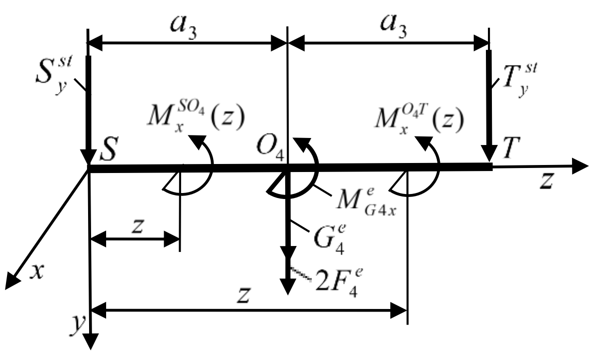

Fig. 4: Static loads of the upper shaft in a vertical plane Syz.

I. PART SO_4 $0 \leq z \leq a_3$

$$\begin{aligned} &M_x^{SO_4}(z) = -S_y^{\mathrm{st}} z\,, \\ &EJ_x y'' = S_y^{\mathrm{st}} z\,, \\ &EJ_x y = \frac{S_y^{\mathrm{st}}}{6} z^3 + S_1 z + T_1\,, \end{aligned} \tag{4}$$

where E is the modulus of elasticity, J_x is the axial moment of inertia.

II. PART O_4T $a_3 \leq z \leq 2a_3$

$$\begin{aligned} &M_x^{O_4T}(z) = -(G_4^e + 2F_4^e + S_y^{\mathrm{st}})z - M_{G4x}^e + (G_4^e + 2F_4^e)a_3\,, \\ &EJ_x y'' = (G_4^e + 2F_4^e + S_y^{\mathrm{st}})z + M_{G4x}^e - (G_4^e + 2F_4^e)a_3\,, \\ &EJ_x y = \frac{(G_4^e + 2F_4^e + S_y^{\mathrm{st}})}{6} z^3 + \frac{[M_{G4x}^e - (G_4^e + 2F_4^e)a_3]}{2} z^2 + S_2 z + T_2\,. \end{aligned} \tag{5}$$

The constants of integration S_i, T_i $(i = 1, 2)$ are determined by the boundary conditions. From these conditions, we obtain the following non-homogeneous algebraic system:

$$
\begin{aligned}
&\frac{S_y^{\mathrm{st}}}{6} O^3 + S_1 O + T_1 = 0\,, \\
&\frac{(G_4^e + 2F_4^e + S_y^{\mathrm{st}})}{6}(2a_3)^3 + \frac{[M_{G4x}^e - (G_4^e + 2F_4^e)a_3]}{2}(2a_3)^2 \\
&\qquad\qquad\qquad\qquad + S_2 2a_3 + T_2 = 0\,, \\
&\frac{S_y^{\mathrm{st}}}{6} a_3^3 + S_1 a_3 + T_1 = \frac{(G_4^e + 2F_4^e + S_y^{\mathrm{st}})}{6} a_3^3 \\
&\qquad\qquad + \frac{[M_{G4x}^e - (G_4^e + 2F_4^e)a_3]}{2} a_3^2 + S_2 a_3 + T_2\,, \\
&\frac{S_y^{\mathrm{st}}}{2} a_3^2 + S_1 = \frac{(G_4^e + 2F_4^e + S_y^{\mathrm{st}})}{2} a_3^2 + [M_{G4x}^e - (G_4^e + 2F_4^e)a_3]a_3 + S_2
\end{aligned}
\tag{6}
$$

From the solution of this system, we get expressions for calculating the constants of integration S_i, T_i $(i = 1,\ 2)$ in the final form

$$
\begin{aligned}
S_1 &= -\frac{a_3[(G_4^e + 2F_4^e + 8S_y^{\mathrm{st}})a_3 + 3M_{G4x}^e]}{12}\,, \qquad T_1 = 0\,, \\
S_2 &= \frac{a_3[(5G_4^e + 10F_4^e - 8S_y^{\mathrm{st}})a_3 - 15M_{G4x}^e]}{12}\,, \\
T_2 &= \frac{a_3^2[3M_{G4x}^e - (G_4^e + 2F_4^e)a_3]}{6}\,.
\end{aligned}
\tag{7}
$$

3.2 STATIC DEFORMATIONS IN A HORIZONTAL PLANE

We use Fig. 5 and apply the algorithm described above. In this case, the horizontal plane is rotated about the axis of rotation of an angle $\pi/2$.

Expressions for determining the static deformations in the horizontal plane
I. PART SO_4 $0 \le z \le a_3$

$$
\begin{aligned}
M_y^{SO_4}(z) &= S_x^{\mathrm{st}} z\,, \\
EJ_y x'' &= S_x^{\mathrm{st}} z\,, \\
EJ_y x &= \frac{S_x^{\mathrm{st}}}{6} z^3 + S_3 z + T_3\,.
\end{aligned}
\tag{8}
$$

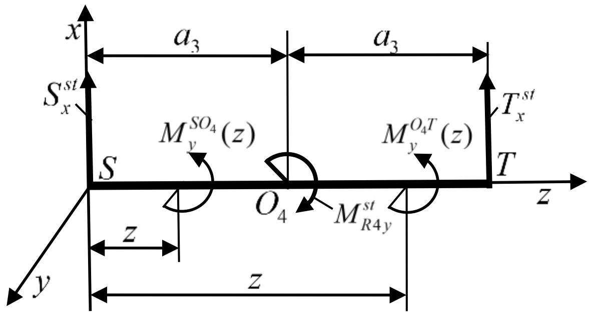

Fig. 5: Static loads of the upper shaft in a horizontal plane Sxz.

II. PART O_4T $a_3 \leq z \leq 2a_3$

(9)
$$\begin{aligned} &M_y^{O_4T}(z) = S_x^{\text{st}} z + M_{R4y}^{\text{st}}\,, \\ &EJ_y x'' = S_x^{\text{st}} z + M_{R4y}^{\text{st}}\,, \\ &EJ_y x = \frac{S_x^{\text{st}}}{6} z^3 + \frac{M_{R4y}^{\text{st}}}{2} z^2 + S_4 z + T_4\,. \end{aligned}$$

Non-homogeneous algebraic system

(10)
$$\begin{aligned} &\frac{S_x^{\text{st}}}{6} 0^3 + S_3 0 + T_3 = 0\,, \\ &\frac{S_x^{\text{st}}}{6}(2a_3)^3 + \frac{M_{R4y}^{\text{st}}}{2}(2a_3)^2 + S_4 2a_3 + T_4 = 0\,, \\ &\frac{S_x^{\text{st}}}{6} a_3^3 + S_3 a_3 + T_3 = \frac{S_x^{\text{st}}}{6} a_3^3 + \frac{M_{R4y}^{\text{st}}}{2} a_3^2 + S_4 a_3 + T_4\,, \\ &\frac{S_x^{\text{st}}}{2} a_3^2 + S_3 = \frac{S_x^{\text{st}}}{2} a_3^2 + M_{R4y}^{\text{st}} a_3 + S_4\,. \end{aligned}$$

Expressions for calculating the constants of integration S_j, T_j ($j = 3, 4$) in final form

(11)
$$\begin{aligned} &S_3 = -\frac{a_3(3M_{R4y}^{\text{st}} + 8a_3 S_x^{\text{st}})}{12}\,, \qquad T_3 = 0\,, \\ &S_4 = -\frac{a_3(15M_{R4y}^{\text{st}} + 8a_3 S_x^{\text{st}})}{12}\,, \qquad T_4 = \frac{M_{R4y}^{\text{st}} a_3^2}{2}\,. \end{aligned}$$

4 DYNAMIC DEFORMATIONS OF THE UPPER SHAFT

In this part, the dynamic deformations of the upper shaft when the leading wheel is mounted between the two bearing supports are analyzed. These deformations occur

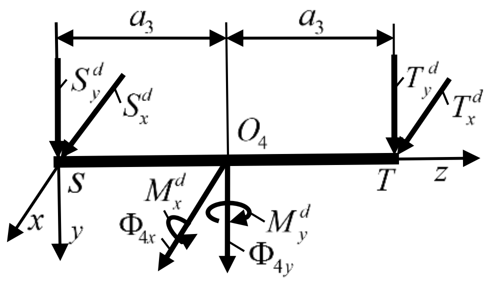

Fig. 6: Upper shaft-dynamic loads and reactions.

as a result of the inaccuracies with which the leading wheel is made. For this purpose, it is necessary to know the dynamic reactions in the bearing supports, as well as the components of the centrifugal force and the dynamic moment that load the shaft. These forces and moments are shown in Fig. 6.

The expressions for calculating the dynamic reactions are presented below

$$S_x^{\mathrm{d}} = S_{\mathrm{bm}} \cos \omega t\,, \qquad S_y^{\mathrm{d}} = S_{\mathrm{bm}} \sin \omega t\,,$$
$$T_x^{\mathrm{d}} = T_{\mathrm{bm}} \cos \omega t\,, \qquad T_y^{\mathrm{d}} = T_{\mathrm{bm}} \sin \omega t\,, \tag{12}$$

where the amplitudes S_{bm} and T_{bm} are calculated from the following dependencies:

$$\begin{aligned} S_{\mathrm{bm}} &= -\frac{1}{4a_3}\Big(\frac{mK_{\Delta(\lambda)}bHu}{V} + R_\Sigma\Big) e \cos\alpha \\ &\quad + \frac{\omega^2}{2}\Big[\frac{1}{a_3}(J_\xi - J_\zeta - m_4 e^2)\sin\alpha - m_4 e\Big]\cos\alpha, \\ T_{\mathrm{bm}} &= \frac{1}{4a_3}\Big(\frac{mK_{\Delta(\lambda)}bHu}{V} + R_\Sigma\Big) e \cos\alpha \\ &\quad - \frac{\omega^2}{2}\Big[\frac{1}{a_3}\left(J_\xi - J_\zeta - m_4 e^2\right)\sin\alpha + m_4 e\Big]\cos\alpha\,. \end{aligned} \tag{13}$$

The components of the centrifugal force Φ_{4x} and Φ_{4y} can be calculated from the expressions (14) written below

$$\Phi_{4x} = m_4\omega^2 e \cos\alpha \cos\omega t\,, \qquad \Phi_{4y} = m_4\omega^2 e \cos\alpha \sin\omega t\,. \tag{14}$$

The components of the dynamic moment M_x^{d} and M_y^{d} are shown in Fig. 6. These moments can be calculated from the dependencies (15).

$$M_x^{\mathrm{d}} = M_{\mathrm{bm}} \sin\omega t\,, \qquad M_y^{\mathrm{d}} = M_{\mathrm{bm}} \cos\omega t\,, \tag{15}$$

where $M_{\mathrm{bm}} = \frac{1}{2}\Big[\Big(\frac{mK_{\Delta(\lambda)}bHu}{V} + R_\Sigma\Big) e\cos\alpha - \omega^2(J_\xi - J_\varsigma - m_4 e^2)\sin 2\alpha\Big]$.

4.1 Dynamic deformations in a vertical plane

In this part, we study the dynamic deformations of the upper shaft in the vertical plane Syz. The inertial force Φ_{4y} and the moment M_x^{d} act in this plane. The differential equation of the transverse vibrations is written in the following form [8]:

$$\frac{EJ}{A_{\mathrm{ms}}\rho}\frac{\partial^4 y}{\partial z^4}+\frac{\partial^2 y}{\partial t^2}=0\,, \tag{16}$$

where E is the modulus of elasticity, $J = J_x = J_y$ is the axial moment of inertia, A_{ms} is the cross-sectional area of the shaft, ρ is the density of the material.

The solution of the differential equation for each part of the shaft is written below

$$y_j(z,\,t)=Z_{4i}(z)\sin\omega t\,,\qquad (i=1,\,2),\ (j=i+2)\,. \tag{17}$$

$Z_{4i}(z)$ is a function that depends on the z coordinate and has the following form:

$$Z_{4i}(z)=\overline{S}_i\cos\omega_0 z+\overline{T}_i\sin\omega_0 z+\overline{V}_i\cosh\omega_0 z+\overline{W}_i\sinh\omega_0 z,\quad (i=1,\,2), \tag{18}$$

where $\omega_0^4 = A_{\mathrm{ms}}\rho\omega^2/EJ$. The constants of integration $\overline{S}_i$, $\overline{T}_i$, $\overline{V}_i$, $\overline{W}_i$, $(i = 1,\ 2)$ are determined by the boundary conditions for each part of the shaft. In this case, there are two parts that change in the following intervals:

$$\begin{array}{ll}\text{I. Part } SO_4 & 0\le z\le a_3\\ \text{II. Part } O_4T & a_3\le z\le 2a_3\end{array} \tag{19}$$

The boundary conditions for two parts of the shaft are eight in number. From these conditions, we calculate the constants of integration. They are also eight in numer. Thus, we obtain a non-homogeneous algebraic system. This algebraic system is written below

$$\begin{aligned}
&\overline{S}_1+\overline{V}_1=0\,,\\
&-\overline{T}_1+\overline{W}_1=\frac{S_{\mathrm{bm}}}{EJ\omega_0^3}\,,\\
&-\overline{S}_1+\overline{V}_1=0\,,\\
&\overline{S}_1\cos\omega_0a_3+\overline{T}_1\sin\omega_0a_3+\overline{V}_1\cosh\omega_0a_3+\overline{W}_1\sinh\omega_0a_3\\
&\qquad-\overline{S}_2\cos\omega_0a_3-\overline{T}_2\sin\omega_0a_3-\overline{V}_2\cosh\omega_0a_3-\overline{W}_2\sinh\omega_0a_3=0,\\
&-\overline{S}_1\sin\omega_0a_3+\overline{T}_1\cos\omega_0a_3+\overline{V}_1\sinh\omega_0a_3+\overline{W}_1\cosh\omega_0a_3\\
&\qquad+\overline{S}_2\sin\omega_0a_3-\overline{T}_2\cos\omega_0a_3-\overline{V}_2\sinh\omega_0a_3-\overline{W}_2\cosh\omega_0a_3=0\,,\\
&\overline{S}_2\cos 2\omega_0a_3+\overline{T}_2\sin 2\omega_0a_3+\overline{V}_2\cosh 2\omega_0a_3+\overline{W}_2\sinh 2\omega_0a_3=0\,,
\end{aligned} \tag{20}$$

$$-\overline{S}_2 \cos 2\omega_0 a_3 - \overline{T}_2 \sin 2\omega_0 a_3 + \overline{V}_2 \cosh 2\omega_0 a_3 + \overline{W}_2 \sinh 2\omega_0 a_3 = 0\,,$$
$$-\overline{S}_2 \sin 2\omega_0 a_3 + \overline{T}_2 \cos 2\omega_0 a_3 - \overline{V}_2 \sinh 2\omega_0 a_3 - \overline{W}_2 \cosh 2\omega_0 a_3 = \frac{T_{\text{bm}}}{EJ\omega_0^3}\,.$$

This system is written below in matrix-vector form:

$$\mathbf{P}\boldsymbol{y}_1 = \boldsymbol{p}\,, \tag{21}$$

where $\mathbf{P}$ is the basic matrix of the linear system. This matrix is obtained from the coefficients of the constants $\overline{S}_i$, $\overline{T}_i$, $\overline{V}_i$, $\overline{W}_i$, $(i = 1,\ 2)$ and has the following form:

$$\mathbf{P} = \tag{22}$$
$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \cos\omega_0 a_3 & \sin\omega_0 a_3 & \cosh\omega_0 a_3 & \sinh\omega_0 a_3 & -\cos\omega_0 a_3 & -\sin\omega_0 a_3 & -\cosh\omega_0 a_3 & -\sinh\omega_0 a_3 \\ -\sin\omega_0 a_3 & \cos\omega_0 a_3 & \sinh\omega_0 a_3 & \cosh\omega_0 a_3 & \sin\omega_0 a_3 & -\cos\omega_0 a_3 & -\sinh\omega_0 a_3 & -\cosh\omega_0 a_3 \\ 0 & 0 & 0 & 0 & \cos 2\omega_0 a_3 & \sin 2\omega_0 a_3 & \cosh 2\omega_0 a_3 & \sinh 2\omega_0 a_3 \\ 0 & 0 & 0 & 0 & -\cos 2\omega_0 a_3 & -\sin 2\omega_0 a_3 & \cosh 2\omega_0 a_3 & \sinh 2\omega_0 a_3 \\ 0 & 0 & 0 & 0 & -\sin 2\omega_0 a_3 & \cos 2\omega_0 a_3 & -\sinh 2\omega_0 a_3 & -\cosh 2\omega_0 a_3 \end{bmatrix}$$

$\boldsymbol{y}_1$ is a column vector. This vector is formed from the unknown constants of integration

$$\boldsymbol{y}_1 = \left[\overline{S}_1\ \overline{T}_1\ \overline{V}_1\ \overline{W}_1\ \overline{S}_2\ \overline{T}_2\ \overline{V}_2\ \overline{W}_2\right]^T\,. \tag{23}$$

$\boldsymbol{p}$ is a column vector. It is formed from the free members of the system

$$\boldsymbol{p} = \left[0\ \frac{S_{\text{bm}}}{EJ\omega_0^3}\ 0\ 0\ 0\ 0\ 0\ \frac{T_{\text{bm}}}{EJ\omega_0^3}\right]^T\,. \tag{24}$$

The solutions of the system (21) can be written as follows:

$$\overline{S}_i = \frac{\Delta_{\overline{S}_i}}{\Delta_P}\,,\quad \overline{T}_i = \frac{\Delta_{\overline{T}_i}}{\Delta_P}\,,\quad \overline{V}_i = \frac{\Delta_{\overline{V}_i}}{\Delta_P}\,,\quad \overline{W}_i = \frac{\Delta_{\overline{W}_i}}{\Delta_P}\,,\quad (i = 1,\ 2)\,, \tag{25}$$

where $\Delta_P = \det \mathbf{P}$ is the determinant of the matrix $\mathbf{P}$. This determinant is non-zero, because otherwise the mechanical system will operate in resonance mode. The determinants $\Delta_{\overline{S}_i}$, $\Delta_{\overline{T}_i}$, $\Delta_{\overline{V}_i}$ and $\Delta_{\overline{W}_i}$ $(i = 1,\ 2)$ are determinants of the matrices formed by the matrix $\mathbf{P}$ in which the corresponding columns are replaced by the column vector $\boldsymbol{p}$ as follows:

$\Delta_{\overline{S}_i}$ $(i = 1,\ 2)$ −the first or fifth column of the matrix **P**,
$\Delta_{\overline{T}_i}$ $(i = 1,\ 2)$ −the second or sixth column of the matrix **P**,
$\Delta_{\overline{V}_i}$ $(i = 1,\ 2)$ −the third or seventh column of the matrix **P**,
$\Delta_{\overline{W}_i}$ $(i = 1,\ 2)$ −the fourth or eighth column of the matrix **P**.

For example, the determinant $\Delta_{\overline{S}_1}$ is written in the following way:

(26)
$$\Delta_{\overline{S}_1} =$$
$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{S_{\text{bm}}}{EJ\omega_0^3} & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sin\omega_0 a_3 & \cosh\omega_0 a_3 & \sinh\omega_0 a_3 & -\cos\omega_0 a_3 & -\sin\omega_0 a_3 & -\cosh\omega_0 a_3 & -\sinh\omega_0 a_3 \\ 0 & \cos\omega_0 a_3 & \sinh\omega_0 a_3 & \cosh\omega_0 a_3 & \sin\omega_0 a_3 & -\cos\omega_0 a_3 & -\sinh\omega_0 a_3 & -\cosh\omega_0 a_3 \\ 0 & 0 & 0 & 0 & \cos 2\omega_0 a_3 & \sin 2\omega_0 a_3 & \cosh 2\omega_0 a_3 & \sinh 2\omega_0 a_3 \\ 0 & 0 & 0 & 0 & -\cos 2\omega_0 a_3 & -\sin 2\omega_0 a_3 & \cosh 2\omega_0 a_3 & \sinh 2\omega_0 a_3 \\ \frac{T_{\text{bm}}}{EJ\omega_0^3} & 0 & 0 & 0 & -\sin 2\omega_0 a_3 & \cos 2\omega_0 a_3 & -\sinh 2\omega_0 a_3 & -\cosh 2\omega_0 a_3 \end{bmatrix}$$

4.2 Dynamic deformations in a horizontal plane

The inertial force Φ_{4x} and the moment M_y^{d} act in the horizontal plane Sxz.

The differential equation of the transverse vibrations is written below [8]:

(27)
$$\frac{EJ}{A_{\text{ms}}\rho}\frac{\partial^4 x}{\partial z^4} + \frac{\partial^2 x}{\partial t^2} = 0\,.$$

The symbols in this equation are explained in differential equation (16). The solution of the above equation for each part of the shaft is written below:

(28)
$$x_j(z,t) = Z_{3i}(z)\cos\omega t\,, \qquad (i = 1,\, 2),\ (j = i+2)\,,$$

where $Z_{3i}(z)$ is a function that depends on the z coordinate

(29)
$$Z_{3i}(z) = \overline{S}_j\cos\omega_0 z + \overline{T}_j\sin\omega_0 z + \overline{V}_j\cosh\omega_0 z + \overline{W}_j\sinh\omega_0 z\,, \quad (i = 1,\, 2),\ (j = i+2)\,.$$

The parameter ω_0 is explained in expression (18). $\overline{S}_j$, $\overline{T}_j$, $\overline{V}_j$, $\overline{W}_j$, $(j = i+2)$ are constants of integration that are determined by the boundary conditions for each part of shaft. These parts are described in (19). And in this case, we obtain a non-homogeneous algebraic system. This system is written below:

(30)
$$\begin{aligned} &\overline{S}_3 + \overline{V}_3 = 0\,,\\ &-\overline{S}_3 + \overline{V}_3 = 0\,,\\ &\overline{S}_3\cos\omega_0 a_3 + \overline{T}_3\sin\omega_0 a_3 + \overline{V}_3\cosh\omega_0 a_3 + \overline{W}_3 sh\omega_0 a_3\\ &\qquad - \overline{S}_4\cos\omega_0 a_3 - \overline{T}_4\sin\omega_0 a_3 - \overline{V}_4\cosh\omega_0 a_3 - \overline{W}_4\sinh\omega_0 a_3 = 0\,,\\ &-\overline{S}_3\sin\omega_0 a_3 + \overline{T}_3\cos\omega_0 a_3 + \overline{V}_3\sinh\omega_0 a_3 + \overline{W}_3 ch\omega_0 a_3\\ &\qquad + \overline{S}_4\sin\omega_0 a_3 - \overline{T}_4\cos\omega_0 a_3 - \overline{V}_4\sinh\omega_0 a_3 - \overline{W}_4\cosh\omega_0 a_3 = 0\,, \end{aligned}$$

$$
\begin{aligned}
-\overline{S}_3 \sin\omega_0 a_3 + \overline{T}_3 \cos\omega_0 a_3 - \overline{V}_3 \sinh\omega_0 a_3 - \overline{W}_3 \cosh\omega_0 a_3 \\
+ \overline{S}_4 \sin\omega_0 a_3 - \overline{T}_4 \cos\omega_0 a_3 + \overline{V}_4 sh\omega_0 a_3 \\
+ \overline{W}_4 \cosh\omega_0 a_3 = \frac{m_4\omega^2 e\cos\alpha}{EJ\omega_0^3}\,, \\
\overline{S}_3 \cos\omega_0 a_3 + \overline{T}_3 \sin\omega_0 a_3 - \overline{V}_3 \cosh\omega_0 a_3 - \overline{W}_3 \sinh\omega_0 a_3 \\
- \overline{S}_4 \cos\omega_0 a_3 - \overline{T}_4 \sin\omega_0 a_3 + \overline{V}_4 \cosh\omega_0 a_3 \\
+ \overline{W}_4 \sinh\omega_0 a_3 = \frac{M_{bm}}{EJ\omega_0^2}\,, \\
\overline{S}_4 \cos 2\omega_0 a_3 + \overline{T}_4 \sin 2\omega_0 a_3 + \overline{V}_4 \cosh 2\omega_0 a_3 + \overline{W}_4 \sinh 2\omega_0 a_3 = 0\,, \\
-\overline{S}_4 \cos 2\omega_0 a_3 - \overline{T}_4 \sin 2\omega_0 a_3 + \overline{V}_4 \cosh 2\omega_0 a_3 + \overline{W}_4 \sinh 2\omega_0 a_3 = 0\,.
\end{aligned}
$$

This system can be written in matrix-vector form.

(31) $$\mathbf{Q}\boldsymbol{x}_1 = \boldsymbol{q}\,,$$

where $\mathbf{Q}$ is the basic matrix of the linear system. This matrix is obtained from the coefficients of the constants $\overline{S}_j, \overline{T}_j, \overline{V}_j, \overline{W}_j$, $(j = 3,\ 4)$ and has the following form:

(32) $$\mathbf{Q} =$$

$$
\begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\cos\omega_0 a_3 & \sin\omega_0 a_3 & \cosh\omega_0 a_3 & \sinh\omega_0 a_3 & -\cos\omega_0 a_3 & -\sin\omega_0 a_3 & -\cosh\omega_0 a_3 & -\sinh\omega_0 a_3 \\
-\sin\omega_0 a_3 & \cos\omega_0 a_3 & \sinh\omega_0 a_3 & \cosh\omega_0 a_3 & \sin\omega_0 a_3 & -\cos\omega_0 a_3 & -\sinh\omega_0 a_3 & -\cosh\omega_0 a_3 \\
-\sin\omega_0 a_3 & \cos\omega_0 a_3 & -\sinh\omega_0 a_3 & -\cosh\omega_0 a_3 & \sin\omega_0 a_3 & -\cos\omega_0 a_3 & \sinh\omega_0 a_3 & \cosh\omega_0 a_3 \\
\cos\omega_0 a_3 & \sin\omega_0 a_3 & -\cosh\omega_0 a_3 & -\sinh\omega_0 a_3 & -\cos\omega_0 a_3 & -\sin\omega_0 a_3 & \cosh\omega_0 a_3 & \sinh\omega_0 a_3 \\
0 & 0 & 0 & 0 & \cos 2\omega_0 a_3 & \sin 2\omega_0 a_3 & \cosh 2\omega_0 a_3 & \sinh 2\omega_0 a_3 \\
0 & 0 & 0 & 0 & -\cos 2\omega_0 a_3 & -\sin 2\omega_0 a_3 & \cosh 2\omega_0 a_3 & \sinh 2\omega_0 a_3
\end{bmatrix}
$$

$\boldsymbol{x}_1$ is a column vector. This vector is formed from the unknown constants of integration

(33) $$\boldsymbol{x}_1 = \left[\overline{S}_3\ \overline{T}_3\ \overline{V_3}\ \overline{W}_3\ \overline{S}_4\ \overline{T}_4\ \overline{V}_4\ \overline{W}_4\right]^T\,.$$

$\boldsymbol{q}$ is a column vector. It is formed from the free members of the system

(34) $$\boldsymbol{q} = \left[0\ 0\ 0\ 0\ \frac{m_4\omega^2 e\cos\alpha}{EJ\omega_0^3}\ \frac{M_{\mathrm{bm}}}{EJ\omega_0^2}\ 0\ 0\right]^T\,.$$

The solutions of this system (31) is written below:

(35) $$\overline{S}_j = \frac{\Delta_{\overline{S}_j}}{\Delta_Q},\quad \overline{T}_j = \frac{\Delta_{\overline{T}_j}}{\Delta_Q},\quad \overline{V}_j = \frac{\Delta_{\overline{V}_j}}{\Delta_Q},\quad \overline{W}_j = \frac{\Delta_{\overline{W}_j}}{\Delta_Q},\quad (j = 3,\ 4)\,,$$

where $\Delta_Q = \det \mathbf{Q}$ is the determinant of the matrix **Q**. The determinants $\Delta_{\overline{S}_j}$, $\Delta_{\overline{T}_j}$, $\Delta_{\overline{V}_j}$ and $\Delta_{\overline{W}_j}$ $(j = 3,\ 4)$ are determinants of the matrices formed by the matrix **Q** in the same way as the determinants in expressions (25). For example, the determinant $\Delta_{\overline{S}_3}$ is written in the following way:

(36)
$$\Delta_{\overline{S}_3} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sin\omega_0 a_3 & \cosh\omega_0 a_3 & \sinh\omega_0 a_3 & -\cos\omega_0 a_3 & -\sin\omega_0 a_3 & -\cosh\omega_0 a_3 & -\sinh\omega_0 a_3 \\ 0 & \cos\omega_0 a_3 & \sinh\omega_0 a_3 & \cosh\omega_0 a_3 & \sin\omega_0 a_3 & -\cos\omega_0 a_3 & -\sinh\omega_0 a_3 & -\cosh\omega_0 a_3 \\ \frac{m_4\omega^2 e\cos\alpha}{EJ\omega_0^3} & \cos\omega_0 a_3 & -\sinh\omega_0 a_3 & -\cosh\omega_0 a_3 & \sin\omega_0 a_3 & -\cos\omega_0 a_3 & \sinh\omega_0 a_3 & \cosh\omega_0 a_3 \\ \frac{M_{\text{bm}}}{EJ\omega_0^2} & \sin\omega_0 a_3 & -\cosh\omega_0 a_3 & -\sinh\omega_0 a_3 & -\cos\omega_0 a_3 & -\sin\omega_0 a_3 & \cosh\omega_0 a_3 & \sinh\omega_0 a_3 \\ 0 & 0 & 0 & 0 & \cos 2\omega_0 a_3 & \sin 2\omega_0 a_3 & \cosh 2\omega_0 a_3 & \sinh 2\omega_0 a_3 \\ 0 & 0 & 0 & 0 & -\cos 2\omega_0 a_3 & -\sin 2\omega_0 a_3 & \cosh 2\omega_0 a_3 & \sinh 2\omega_0 a_3 \end{bmatrix}$$

In this part of the article, we obtained the expressions $y_j(z,t)$ and $x_j(z,t)$ for research and analysis of the transverse vibrations of the upper shaft in vertical and horizontal planes. The constants of integration $\overline{S}_i$, $\overline{T}_i$, $\overline{V}_i$, $\overline{W}_i$ and $\overline{S}_j$, $\overline{T}_j$, $\overline{V}_j$, $\overline{W}_j$ are used to determine the functions $Z_{4i}(z)$ and $Z_{3i}(z)$. These functions depends on the z coordinate and take part in the expressions, describing the vertical and horizontal displacements of each part of the upper shaft.

5 Numerical Example

The results of the carried out computer experiments are presented in Figs. 7–9. We use initial data, presented in technical literature [7, 9–11]:

5.1 Static deformations

Figure 7 shows the deformations of the upper shaft in two mutually perpendicular planes. These deformations are caused by static forces and moments that load the

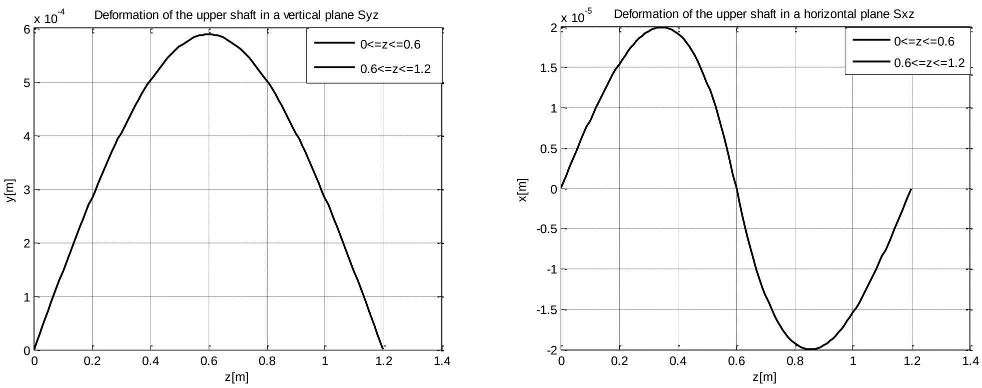

Fig. 7: Deformations of the upper shaft in vertical and horizontal planes.

shaft in operation mode. The diagrams are drawn using the equations of the elastic lines, obtained in the theoretical part of the article. The figures show that the diagrams are plane and continuous lines over the entire length of the shaft. It can also be seen that the deformations in the two bearing supports are zero, i.e. when $z = 0$ m and $z = 1.2$ m. This is because the two supports are considered as perfectly rigid bodies. We can also determine the most endangered cross section of the upper shaft. As can be seen from the figures, the values of the coordinate z for the two planes are different. For the vertical plane, the maximum value of the deformation is at $z = 0.6$ m. For the horizontal plane these values are two, $z = 0.35$ m and $z = 0.85$ m.

5.2 Dynamic deformations

Figure 8 shows the dynamic deformations of the upper shaft. We can see the deformations of the shaft in three different sections:$z = 0.25$ m, $z = 0.6$ m, $z = 0.9$ m. These graphics are functions of the time t and they change by the harmonic law. We can also determine the amplitudes of the vibrations for the three cross-sections. The greatest deviation is in the section, where $z = 0.25$ m. This section is the most dangerous cross-section.

Figure 9 shows the deformations of the upper shaft. These surfaces are functions of two arguments: the time t, which changes from 0 to 0.8 seconds, and coordinate z, which maximum value is equal to the length of the shaft, i.e. $z_{\max} = 1.2$ m.

We can draw parallel vertical planes for each value of the coordinate z and see how corresponding cross-sections vibrate, as well as to measure the amplitudes of the vibrations for these sections. All cross-sections vibrate by the harmonic law. The figures show at what value of the coordinate z deformations of the shaft are equal to zero. Obviously, these are the two supports.

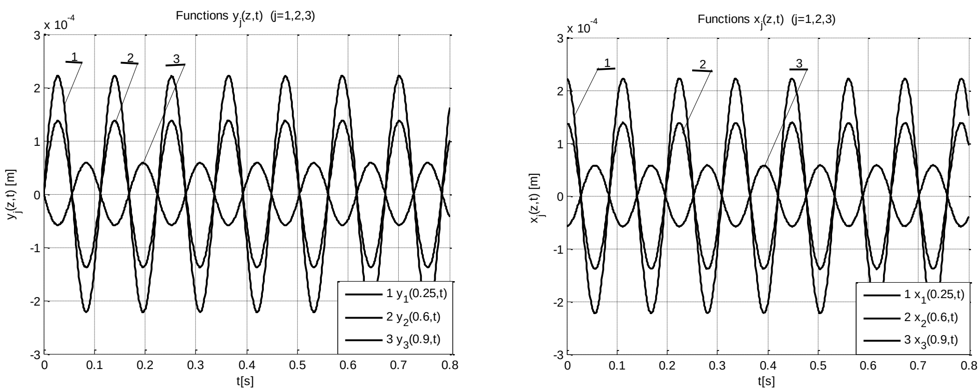

Fig. 8: Dynamic deformations of the upper shaft in vertical and horizontal planes.

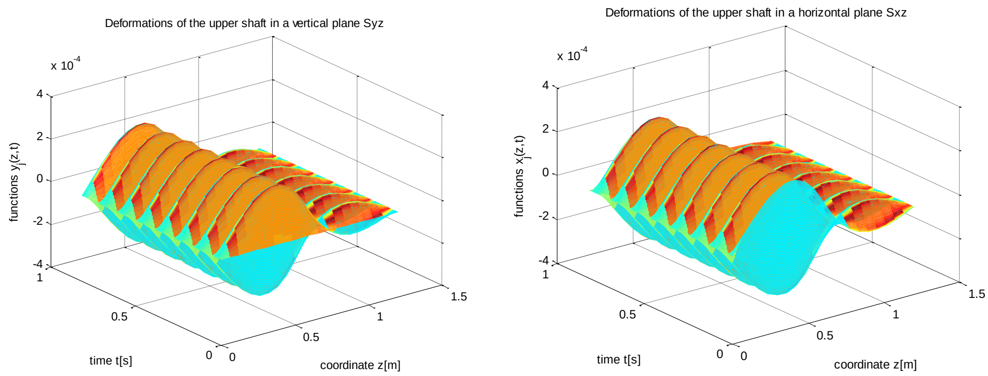

Fig. 9: Upper shaft – spatial deformations in a vertical and horizontal planes.

6 Conclusion

In this paper, the influence of the static and dynamic loads on the upper shaft of big band saw machines when the leading wheel is mounted between the two bearing supports is analyzed. For this purpose, dynamic model is built. The deformations of the shaft as a result of the static loads are investigated. Theoretical expressions for calculating the static deformations are obtained. The transverse vibrations of the upper shaft as a result of the dynamic loads have been investigated. The deformations of the shaft due to the dynamic loads have also been studied and theoretical expressions for calculating these deformations are obtained. Plane and spatial diagrams are drawn, showing the change of the deformations of the upper shaft when changing different parameters.

With the help of the obtained theoretical dependencies and diagrams different tasks can be solved by using known expressions from the technical literature [12–14]. Some of the most important tasks are strength and deformation check of the shaft, reduction of the shaft deformations, achievement of minimum overall dimensions of the machines, vibration control, etc.

References

[1] L. Hlásková, K. Orlowski, Z. Kopeck, M. Jedinák (2015) Sawing processes as a way of determining fracture toughness and shear yield stresses of wood. *BioResources* **10**(3) 5381-5394.

[2] P. Moradpour, F. Scholz, K. Doosthoseini, A. Tarmian (2016) Measurement of wood cutting forces during using piezoelectric dynamometer. *Drvna Industrija* **67**(1) 79-84.

[3] T. Krenke, S. Frybort, U. Müller (2017) Determining cutting force parameters by applying a system function. *Journal Machining Science and Technology* **21**(3) 436-451.

[4] P. OBRESHKOV (1995) "Woodworking Machines". Publishing House BM (in Bulgarian).

[5] B. MARINOV (2018) "Dynamic and Shock Processes in Some Classes of Woodworking Machines. Analysis and Optimization". Omniscriptum Publishing Group-Germany/LAP LAMBERT Academic Publishing.

[6] B. MARINOV (2021) Full dynamic reactions in the main links of big band saw machines. *Global Journal of Engineering and Technology Advances* **9**(3) 92-104.

[7] ZH. GOCHEV (2005) "Handbook for Exercise of Wood Cutting and Woodworking Tools". Publishing House in LTU (in Bulgarian).

[8] B. CHESHANKOV (1992) "Theory of the Vibrations". Publishing House of Technical University – Sofia (in Bulgarian).

[9] R. PETERS (2006) "Band Saw Fundamentals: The Complete Guide". Hearst Communications Inc.

[10] WALKER TURNER (2013) "A Comprehensive Handbook on Uses and Applications of the Band Saw and Jig Saw". Literary Licensing LLC.

[11] Š. BARCÍK (2003) Experimental cutting on the table band-sawing machine. *European Journal of Wood and Wood Products* **61**(4) 313-320

[12] R. DREIZLER, C. LÜDDE (2010) "Theoretical Mechanics: Theoretical Physics 1". Springer, Berlin, Heidelberg.

[13] B. DUPEN (2014) "Applied Strength of Materials for Engineering Technology". Indiana University – Purdue University Fort Wayne.

[14] Y. CHEN, X.G. WANG, C. SUN, F. DEVINE (2003) Active Vibration Control with State Feedback in Woodcutting. *Journal of Vibration and Control* **9**(6) 645-664.