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VIBRATION OF ORTHOTROPIC RECTANGULAR PLATE WITH 2D CIRCULAR THICKNESS AND LINEAR DENSITY

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ABSTRACT: The current study computes the modes of frequency of an orthotropic rectangular plate having 2-D (two-dimensional) circular thickness and 1-D (one-dimensional) linear density with variable temperature field at clamped edge condition. Temperature variation on the plate is considered to be parabolic in both the directions. Rayleigh Ritz technique is used to solve the frequency equation of nonhomogeneous orthotropic rectangular plate and frequency values are calculated for first four modes of vibration. The convergence study of modes of frequency of orthotropic rectangle plate is also done at clamped edge condition. The aim of the present study is to present some numerical data in the form of frequency and to demonstrate how different plate parameters variations can be used to control frequency modes. A comparative analysis of the modes of frequency of the current study with the available published results is effectively presented at clamped edge condition, which is also provided in tabular form.

KEY WORDS: Clamped, orthotropic rectangular plate, non-homogeneity, thermal gradient.

1 INTRODUCTION

Scope of vibration of plates have wide range of applications in different branches of science and engineering i.e. nuclear reactors, aeronautical engineering, naval structure, submarines, earth-quake resistors etc. In the mid of nineteen century several authors discussed the behavior of vibration of rectangular plates, rectangular orthotropic plates, elliptical plates by varying plate parameters and large number of research articles has been reported in literature.

Transverse vibrations of orthotropic rectangular plates is studied in [1]. Free vibration analysis in a circular plate made out of isotropic material with and without square cutouts is presented in [2] and solved the system by using a finite element

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method. In [3] and [4], frequency equations for orthotropic rectangular plate is solved and time period of transverse vibration of skew plate with different edge conditions by using Rayleigh Ritz method is implemented in [5]. The vibration problem of viscoelastic orthotropic rectangular plates has been investigated in [6]. Thermal effect on frequencies of an orthotropic rectangular plate of linearly varying thickness was shown in [7]. Analysis of the time period of rectangular plate is calculated in [8] and temperature effect on frequencies of tapered plate is demonstrated in [9]. Thermally induced vibrations of shallow functionally graded material subjected to sudden thermal loading on one surface is examined in [10]. The thermal effect on vibrations of a non-homogeneous orthotropic rectangular plate with thickness variation was proposed in [11] and [12]. Transverse vibrations of orthotropic nonuniform rectangular plates with continuously varying density is discussed in [13]. The new set of beam functions of tapered rectangular plates is presented in [14]. The behavior of an orthotropic viscoelastic rectangular plate under the action of an external periodic load is computed in [15] and resolving equations of the problem were obtained by the Bubnov-Galerkin and numerical method based on the use of quadrature formulas. New analytical solution of plates stiffened by orthogonal beams and using a double finite sine integral transform technique is carried out in [16].

Many researchers discussed about the behavior of orthotropic rectangle plate with different plate parameters viz. thermal gradient, tapering parameters, nonhomogeneity etc., having linear and parabolic variations in one or two dimensions but no one dealt with the impact of $2D$ circular variation in thickness on frequency modes of orthotropic rectangle plate. This aspect prompted us to study the impact of $2D$ circular variation in thickness along with linear density on vibrational frequency orthotropic rectangular plate.

2 Problem geometry and analysis

Consider a nonhomogeneous orthotropic rectangular plate with sides a and b, thickness l and density ρ as shown in Fig. (1). The kinetic energy and strain energy for

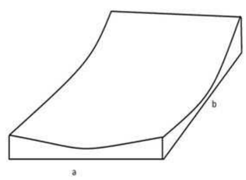

Fig. 1: Orthotropic rectangular plate with $2D$ circular thickness.

vibration of plate are expressed in the following manner as in [18]

(1) $$T_s = \frac{1}{2}\omega^2 \int_0^a \int_0^b \rho l \Phi^2 d\psi d\zeta\,,$$

(2) $$V_s = \frac{1}{2}\int_0^a \int_0^b \Bigg[D_x\Big(\frac{\partial^2\Phi}{\partial\zeta^2}\Big)^2 + D_y\Big(\frac{\partial^2\Phi}{\partial\psi^2}\Big)^2 + 2\nu_x D_y \frac{\partial^2\Phi}{\partial\zeta^2}\frac{\partial^2\Phi}{\partial\psi^2} + 4D_{xy}\Big(\frac{\partial^2\Phi}{\partial\zeta\partial\psi}\Big)^2\Bigg] d\psi d\zeta\,,$$

where Φ deflection function, ω natural frequency, $D_x = E_x l^3/12\,(1-\nu_x\nu_y), D_y = E_y l^3/12\,(1-\nu_x\nu_y)$ are flexural rigidities in x and y directions respectively and $D_{xy} = E_x l^3/12\,(1-\nu_x\nu_y)$ is torsional rigidity.

To solve, Rayleigh Ritz technique is implemented which requires

(3) $$L = \delta(V_s - T_s) = 0.$$

Using Eqs. (1), (2), we get functional equation

(4) $$L = \frac{1}{2}\int_0^a \int_0^b \Bigg[D_x\Big(\frac{\partial^2\Phi}{\partial\zeta^2}\Big)^2 + D_y\Big(\frac{\partial^2\Phi}{\partial\psi^2}\Big)^2 + 2\nu_x D_y \frac{\partial^2\Phi}{\partial\zeta^2}\frac{\partial^2\Phi}{\partial\psi^2} + 4D_{xy}\Big(\frac{\partial^2\Phi}{\partial\zeta\partial\psi}\Big)^2\Bigg] d\psi d\zeta - \frac{1}{2}\omega^2 \int_0^a \int_0^b \rho l \Phi^2 d\psi d\zeta = 0.$$

Now, introducing non-dimensional variable $\zeta_1 = \zeta/a$, $\psi_1 = \psi/a$ together with two dimensional circular thickness and one dimensional linear variation in density as:

(5) $$l = l_0\left(1+\beta_1\left\{1-\sqrt{1-{\zeta_1}^2}\right\}\right)\left(1+\beta_2\left\{1-\sqrt{1-\frac{a^2}{b^2}{\psi_1}^2}\right\}\right), \qquad \rho = \rho_0\,(1+m\zeta_1)\,,$$

where l_0 is thickness at origin, $\beta_1,\ \beta_2 (0 \leqslant \beta_1, \beta_2 \leqslant 1)$ are tapering parameters and m $(0 \leqslant m \leqslant 1)$ is nonhomogeneity.

Two dimensional parabolic temperature distribution is taken as in [3].

(6) $$\tau = \tau_0\left(1-\zeta_1^2\right)\left(1-\frac{a^2{\psi_1}^2}{b^2}\right),$$

where τ and τ_0 denotes the temperature on and at the origin respectively.

For orthotropic materials, modulus of elasticity is given by

$$E_x = E_1\left(1-\gamma\tau\right), E_y = E_2\left(1-\gamma\tau\right), G_{xy} = G_0\left(1-\gamma\tau\right), \tag{7}$$

where E_x and E_y are the Young's modulus in x and y directions, G_{xy} is shear modulus and γ is called slope of variation.

Using Eq. (6), Eq. (7) becomes

$$\begin{aligned} E_x(x) &= E_1\Big[1-\alpha\Big(1-\zeta_1^2\Big)\Big(1-\frac{a^2{\psi_1}^2}{b^2}\Big)\Big],\\ E_y(x) &= E_2\Big[1-\alpha\Big(1-\zeta_1^2\Big)\Big(1-\frac{a^2{\psi_1}^2}{b^2}\Big)\Big],\\ G_{xy(x)} &= G_0\Big[1-\alpha\Big(1-\zeta_1^2\Big)\Big(1-\frac{a^2{\psi_1}^2}{b^2}\Big)\Big], \end{aligned} \tag{8}$$

where $\alpha = \gamma\tau_0\ (0 \le \alpha < 1)$ is called thermal gradient.

Using Eqs. (5), (8) and non dimensional variable, the functional in Eq. (4) become

$$\begin{aligned} L = \frac{D_0}{2}\int_0^1\int_0^{\frac{b}{a}} &\Bigg[\Big[1-\alpha(1-\zeta_1^2)\Big(1-\frac{a^2{\psi_1}^2}{b^2}\Big)\Big](1+\beta_1\Psi_1)^3(1+\beta_2\Psi_2)^3\\ &\times\Big\{\Big(\frac{\partial^2\Phi}{\partial\zeta_1^2}\Big)^2+\frac{E_2}{E_1}\Big(\frac{\partial^2\Phi}{\partial\psi_1^2}\Big)^2+2\nu_x\frac{E_2}{E_1}\Big(\frac{\partial^2\Phi}{\partial\zeta_1^2}\Big)\Big(\frac{\partial^2\Phi}{\partial\psi_1^2}\Big)\\ &+4\frac{G_0}{E_1}(1-\nu_x\nu_y)\Big(\frac{\partial^2\Phi}{\partial\zeta_1\partial\psi_1}\Big)^2\Big\}\Bigg]d\psi_1 d\zeta_1\\ &-\lambda^2\int_0^1\int_0^{\frac{b}{a}}\Big[(1+\beta_1\Psi_1)(1+\beta_2\Psi_2)(1+m\zeta_1)\Big]\Phi^2 d\psi_1 d\zeta_1, \end{aligned} \tag{9}$$

where

$$D_0 = \frac{1}{2}\Big(\frac{E_1 l_0^3}{12(1-\nu_x\nu_y)}\Big), \qquad \lambda^2 = \frac{12a^4\rho\omega^2(1-\nu_x\nu_y)}{E_1{h_0}^2},$$

$$\Psi_1 = \{1-\sqrt{1-{\zeta_1}^2}\}, \qquad \Psi_2 = \Big\{1-\sqrt{1-\frac{a^2{\psi_1}^2}{b^2}}\Big\}.$$

We choose deflection function which satisfies all the edge condition as taken in [17]

$$\begin{aligned} \Phi(\zeta,\psi) = &\Big[(\zeta_1)^e(\psi_1)^f(1-\zeta_1)^g\Big(1-\frac{a\psi_1}{b}\Big)^h\Big]\\ &\times\Big[\sum_{i=0}^{n}\Psi_i\Big\{(\zeta_1)(\psi_1)(1-\zeta_1)\Big(1-\frac{a\psi_1}{b}\Big)\Big\}^i\Big], \end{aligned} \tag{10}$$

where Ψ_i, $i = 0, 1, 2, \ldots, n$ are unknowns and the value of e, f, g, h can be 0, 1 and 2 corresponding to given edge condition.

To minimize Eq. (10), we impose the following condition:

$$\frac{\partial L}{\partial \Psi_i} = 0, \quad i = 0, 1, \ldots, n. \tag{11}$$

Solving Eq. (11), we have the following frequency equation:

$$\left| P - \lambda^2 Q \right| = 0\,, \tag{12}$$

where $P = [p_{ij}]_{i,j=0,1,..n}$ and $Q = [q_{ij}]_{i,j=0,1,..n}$ are square matrix of order $(n + 1)$.

3 Numerical Results and Discussion

In this study, authors examined the behavior of orthotropic rectangle plate with 2-D circular variation in thickness and 1-D linear density under 2-D parabolic temperature at clamped boundary conditions. The frequency equation of plate is solved by using Rayleigh-Ritz technique and the frequency modes (first four modes) corresponding to plate parameters i.e., thermal gradient α, tapering parameters β_1, β_2 and nonhomogeneity m for fixed value of aspect ratio $a/b = 1.5$ are obtained. The results have been illustrated with the help of tables.

Table 1 represents the frequency modes(for first four modes) of orthotropic rectangle plate corresponding to tapering parameter β_1 for fixed values of tapering parameter β_2, thermal gradient α and nonhomogeneity m. The values chosen are $\alpha = \beta_2 = m = 0.2$, $\alpha = \beta_2 = m = 0.4$ and $\alpha = \beta_2 = m = 0.6$. From Table 1, authors observed that frequency increases in all four modes with the increasing in tapering parameter β_1 from 0.0 to 1.0. Also, frequency decreases with the increase in value of tapering parameter β_2, thermal gradient α and nonhomogeneity m from 0.2 to 0.6 (i.e, $\alpha = \beta_2 = m = 0.2$ to $\alpha = \beta_2 = m = 0.6$).

Table 2 displays the frequency for first four modes corresponding to tapering parameter β_2 for fixed values of thermal gradient α and tapering parameter β_2 i.e.,

Table 1: Modes of frequencies of clamped orthotropic rectangle plate corresponding to tapering parameter β_1 for aspect ratio $a/b = 1.5$

β_1	$\alpha = \beta_2 = m = 0.2$				$\alpha = \beta_2 = m = 0.4$				$\alpha = \beta_2 = m = 0.6$			
	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4
0.0	16.2110	62.2269	139.759	318.940	15.5970	59.0903	132.851	313.616	15.0624	56.1845	126.804	310.024
0.2	16.9172	64.5269	144.810	334.398	16.2862	61.2983	137.686	329.278	15.7361	58.3041	131.505	326.122
0.4	17.6774	66.9729	150.195	351.90	17.0255	63.6365	142.964	345.848	16.4560	60.5451	136.725	341.095
0.6	18.4853	69.5478	156.056	368.675	17.8091	66.0932	148.736	360.723	17.2169	62.8920	142.255	357.326
0.8	19.3348	72.2352	162.151	387.047	18.6313	68.6528	154.670	378.380	18.0137	65.3333	147.927	376.599
1.0	20.2206	75.0153	168.513	407.106	19.4872	71.3032	160.479	402.314	18.8417	67.8569	153.650	400.087

Table 2: Modes of frequencies of clamped orthotropic rectangle plate corresponding to tapering parameter β_2 for aspect ratio $a/b = 1.5$

β_2	$\alpha=\beta_1=m=0.2$				$\alpha=\beta_1=m=0.4$				$\alpha=\beta_1=m=0.6$			
	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4
0.0	16.0949	61.8537	138.820	315.426	15.3436	58.2678	130.879	303.492	14.6631	54.8877	123.393	295.154
0.2	16.9172	64.5269	144.810	334.398	16.1523	60.8700	136.610	323.916	15.4580	57.407	129.291	313.429
0.4	17.8091	67.3872	151.264	356.081	17.0255	63.6365	142.964	345.848	16.3125	60.0840	135.568	334.686
0.6	18.7597	70.4105	158.105	379.560	17.9530	66.5505	149.872	366.756	17.2169	62.8920	142.255	357.326
0.8	19.7596	73.5582	165.481	403.693	18.9258	69.5808	156.986	391.951	18.1631	65.8077	149.098	384.840
1.0	20.8005	76.8121	173.113	430.625	19.9363	72.7098	164.456	418.113	19.1440	68.8084	156.730	406.506

$\alpha = \beta_1 = m = 0.2$, $\alpha = \beta_1 = m = 0.4$ and $\alpha = \beta_1 = m = 0.6$. Frequency increases in Table 2 for all four modes corresponding to tapering parameter β_2, which is varied from 0.0 to 1.0. Frequency decreases with the increase in value of tapering parameter β_1, thermal gradient α and nonhomogeneity m from 0.2 to 0.6 (i.e, $\alpha = \beta_2 = m = 0.2$ to $\alpha = \beta_2 = m = 0.6$) but there is an instant increment of frequency of tapering parameter β_2 at 0.4 then the increment of frequencies is smooth, which is as shown in Table 2.

The following facts have been observed in Tables 1 and 2:

- Frequency increases for first four modes corresponding to both the tapering parameters.
- Rate of decrement in Table 1 is slower as compared to Table 2.
- It is also observed in Table 1 that the tapering parameter β_1 shows smooth rate of increment of frequency modes but along β_2 there is an instant increment of frequency of tapering parameter at 0.4 which is shown by Table 2 then the increment of frequencies is smooth.

In Table 3, the parameters chosen for frequency of first four modes corresponding to thermal gradient α for fixed values of tapering parameters β_1, β_2 and nonhomogeneity m are $\beta_1 = \beta_2 = m = 0.2$, $\beta_1 = \beta_2 = m = 0.4$ and $\beta_1 = \beta_2 = m = 0.6$.

Table 3: Modes of frequencies of clamped orthotropic rectangle plate corresponding to thermal gradient α for aspect ratio $a/b = 1.5$

α	$\beta_1=\beta_2=m=0.2$				$\beta_1=\beta_2=m=0.4$				$\beta_1=\beta_2=m=0.6$			
	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4
0.0	17.6726	67.6599	151.849	346.756	18.5222	69.9404	156.720	368.909	19.4779	72.5262	162.819	388.794
0.2	16.9172	64.5236	144.867	334.284	17.7932	66.8765	149.997	357.562	18.7651	69.5005	156.312	378.519
0.4	16.1217	61.2117	137.456	321.463	17.0255	63.6404	142.914	345.948	18.0141	66.3018	149.463	368.059
0.6	15.2778	57.6743	129.738	307.861	16.2112	60.1838	135.518	333.779	17.2169	62.8908	142.266	357.304
0.8	14.3742	53.8731	121.405	293.848	15.3390	56.4640	127.633	321.238	16.3619	59.2159	134.652	346.260
1.0	13.3937	49.7141	112.494	279.033	14.3917	52.3983	119.181	308.252	15.4316	55.1951	126.565	334.873

respectively. Table 3 illustrates that frequency decreases for all four modes with the increase in numerical value of α from 0.0 to 1.0. With the increase in value of tapering parameters β_1, β_2 and nonhomogeneity m from 0.2 to 0.6 (i.e., $\beta_1 = \beta_2 = m = 0.2$, $\beta_1 = \beta_2 = m = 0.4$ and $\beta_1 = \beta_2 = m = 0.6$.), frequency decreases for all four modes, which is demonstrated by Table 3.

In Table 4, authors describe the frequency for first four modes corresponding to nonhomogeneity m for fixed values of tapering parameters β_1, β_2 and thermal gradient α i.e., $\beta_1 = \beta_2 = \alpha = 0.2$, $\beta_1 = \beta_2 = \alpha = 0.4$ and $\beta_1 = \beta_2 = \alpha = 0.6$ respectively. From Table 4, it is found that frequency decreases for all four modes with the increase in nonhomogeneity m from 0.0 to 1.0. Frequency values almost coincide in Table 4 for 2nd, 3rd and 4th frequency modes with the increase in value of tapering parameters β_1, β_2 and thermal gradient α from 0.2 to 0.6 (i.e., $\beta_1 = \beta_2 = \alpha = 0.2$ to $\beta_1 = \beta_2 = \alpha = 0.6$).

Table 4: Modes of frequencies of clamped orthotropic rectangle plate corresponding to nonhomogeneity m for aspect ratio $\frac{a}{b} = 1.5$

m	$\alpha = \beta_1 = \beta_2 = 0.2$				$\alpha = \beta_1 = \beta_2 = 0.4$				$\alpha = \beta_1 = \beta_2 = 0.6$			
	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4
0.0	17.7471	67.7064	151.864	352.102	18.6654	69.8130	157.082	377.783	19.6609	71.9172	162.662	412.507
0.2	16.9172	64.5260	144.825	334.369	17.7890	66.5116	149.515	360.758	18.7344	68.4890	154.936	391.489
0.4	16.1939	61.7549	138.700	318.839	17.0255	63.6365	142.955	345.869	17.9277	65.5126	148.183	373.182
0.6	15.5560	59.3166	133.094	307.217	16.3525	61.1041	137.391	329.946	17.2169	62.8907	142.271	357.292
0.8	14.9881	57.1415	128.256	295.673	15.7536	58.8531	132.319	317.448	16.5845	60.5644	136.917	344.022
1.0	14.4781	55.1907	123.835	286.123	15.2159	56.8306	127.812	306.203	16.0170	58.4729	132.166	332.724

From Tables 3 and 4, following conclusions can be made:

- Frequency modes decreases from 0.0 to 1.0 corresponding to thermal gradient and non homogeneity respectively for fixed parameters.
- Behavior of increment and decrement in both tables is almost the same.

4 Convergence Study

In this section, convergence study on frequency modes of orthotropic rectangle plate at clamped edge condition, when the order of approximation increased for all values

Table 5: Convergence study of modes of frequency of orthotropic rectangle plate at clamped edge condition

	$N = 2$	$N = 3$	$N = 4$	$N = 5$
λ_1	16.9543	16.9543	16.9543	16.9543
λ_2	68.1543	65.7956	65.7620	65.7620

of plate parameters in the ranges specified i.e., $E = 0.04, E1 = 1, E2 = 0.32, G = 0.09, \beta_1 = \beta_2 = m = \alpha = 0$ is reported in tabular form (refer Table 5). It is also seen from the table that the modes of frequency of orthotropic rectangle plate converges up to four decimal places in the fifth approximation.

5 Comparison of Results

The results (vibrational frequency) of present study is compared with [19] corresponding to nonhomogeneity m for fixed value of tapering parameter β_1, β_2 and thermal gradient α. In Table 6, authors describe the comparison of nonhomogeneity in which present study dealt with linear nonhomogeneity and [19] dealt with circular nonhomogeneity.

Table 6: Comparison of frequency modes of the present study (orthotropic rectangular plate) and obtained in [19] corresponding to nonhomogeneity m for aspect ratio $a/b = 1.5$. Bold values are obtained from [19]

m	$\alpha = \beta_1 = \beta_2 = 0.2$				$\alpha = \beta_1 = \beta_2 = 0.4$				$\alpha = \beta_1 = \beta_2 = 0.6$			
	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4
0.0	17.7471	67.7064	151.864	352.102	18.6654	69.8130	157.082	377.783	19.6609	71.9172	162.662	412.507
	17.2013	**65.9667**	**148.314**	**335.442**	**17.4856**	**66.1290**	**148.314**	**348.144**	**17.7821**	**66.1528**	**149.118**	**362.262**
0.2	16.9172	64.5260	144.825	334.369	17.7890	66.5116	149.515	360.758	18.7344	68.4890	154.936	391.489
	16.9440	**64.8663**	**145.634**	**331.105**	**17.2222**	**65.0210**	**145.943**	**341.903**	**17.5128**	**65.0309**	**146.362**	**355.401**
0.4	16.1939	61.7549	138.700	318.839	17.0255	63.6365	142.955	345.869	17.9277	65.5126	148.183	373.182
	16.6979	**63.8335**	**143.210**	**323.724**	**16.9703**	**63.9628**	**143.544**	**334.381**	**17.2554**	**63.9650**	**143.309**	**348.519**
0.6	15.5560	59.3166	133.094	307.217	16.3525	61.1041	137.391	329.946	17.2169	62.8907	142.271	357.292
	16.4622	**62.8448**	**140.797**	**318.704**	**16.7292**	**62.9605**	**141.119**	**328.422**	**17.0089**	**62.9572**	**141.309**	**341.897**
0.8	14.9881	57.1415	128.256	295.673	15.7536	58.8531	132.319	317.448	16.5845	60.5644	136.917	344.022
	16.2363	**61.8978**	**138.611**	**313.065**	**16.4980**	**62.0028**	**138.814**	**323.043**	**16.7728**	**61.9888**	**139.107**	**334.293**
1.0	14.4781	55.1907	123.835	286.123	15.2159	56.8306	127.812	306.203	16.0170	58.4729	132.166	332.724
	16.0193	**60.9914**	**136.613**	**306.693**	**16.2762**	**61.0890**	**136.707**	**316.661**	**16.5461**	**61.0639**	**136.860**	**329.374**

From Table 6, authors concluded that:

- Frequency modes in both the study decreases but frequencies of present study are less as compared to [19].
- Rate of decrement of frequencies of present study are slower as compared to [19].
- Frequency value increases in both the study, when combined values of tapering parameter β_1, β_2 and thermal gradient α is varied from 0.2 to 0.6 (i.e., $\beta_1 = \beta_2 = \alpha = 0.2$ to $\beta_1 = \beta_2 = \alpha = 0.6$) but rate of increment of frequencies in [19] is fast as compared to present study.

Table 7 provides the comparison of frequency modes of present study with [3] corresponding to tapering parameter β_1 without thermal gradient α and nonhomo-

Table 7: Comparison of frequency modes of the present study (orthotropic rectangular plate) and obtained in [3] corresponding to tapering parameters β_1, β_2, for aspect ratio $a/b = 1.5$; $\alpha = m = 0.0$. Bold values are obtained from [3]

β_1	$\beta_2 = 0.0$				$\beta_2 = 0.2$				$\beta_2 = 0.4$			
	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4
0.0	16.95	65.76	147.67	330.94	17.79	68.52	153.75	349.10	18.70	71.47	160.37	370.62
	16.95	**65.76**	**147.62**	**331.06**	**18.72**	**72.57**	**162.87**	**336.16**	**20.62**	**79.75**	**179.06**	**404.79**
0.2	17.68	68.17	152.99	345.29	18.53	70.99	159.18	365.23	19.47	74.02	166.19	384.89
	18.70	**72.51**	**162.74**	**365.47**	**20.66**	**80.01**	**179.72**	**402.92**	**22.75**	**87.92**	**197.44**	**447.67**
0.4	18.46	70.74	158.58	362.56	19.34	73.63	165.22	380.50	20.30	76.73	172.18	402.97
	20.55	**79.52**	**178.58**	**402.24**	**22.69**	**87.75**	**196.99**	**445.86**	**24.99**	**96.42**	**216.62**	**491.60**
0.6	19.29	73.44	164.80	379.01	20.20	76.40	171.30	400.66	21.19	79.58	178.57	422.41
	22.46	**86.74**	**194.72**	**441.44**	**24.81**	**95.71**	**214.74**	**490.83**	**27.32**	**105.15**	**236.24**	**540.18**
0.8	20.17	76.27	171.16	398.11	21.10	79.30	178.01	418.13	22.12	82.55	185.42	441.64
	24.43	**94.12**	**211.45**	**480.09**	**26.98**	**103.84**	**233.07**	**534.76**	**29.70**	**114.08**	**256.76**	**583.03**
1.0	21.09	79.20	177.66	420.14	22.05	82.31	184.69	441.66	23.09	85.64	192.12	467.78
	26.44	**101.60**	**228.13**	**523.88**	**29.20**	**112.09**	**252.24**	**574.42**	**32.14**	**123.13**	**277.07**	**636.38**

β_1	$\beta_2 = 0.0$				$\beta_2 = 0.2$				$\beta_2 = 0.4$			
	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4
0.0	19.67	74.60	167.63	392.06	20.70	77.88	175.05	416.77	21.77	81.27	182.81	444.64
	22.61	**87.21**	**195.91**	**446.28**	**24.67**	**281.07**	**213.26**	**488.14**	**26.77**	**102.68**	**230.72**	**535.54**
0.2	20.47	77.22	173.33	410.12	21.53	80.58	181.20	433.21	22.63	84.05	189.07	462.66
	24.94	**96.15**	**215.60**	**495.77**	**27.21**	**104.59**	**234.81**	**542.83**	**29.54**	**113.18**	**254.79**	**586.30**
0.4	21.33	80.01	179.67	427.41	22.41	83.44	187.70	451.76	23.54	87.00	195.61	484.46
	27.40	**105.42**	**236.88**	**542.97**	**29.89**	**114.66**	**258.27**	**589.11**	**32.43**	**124.08**	**279.32**	**646.76**
0.6	22.24	82.93	186.19	448.74	23.35	86.45	194.49	473.02	24.51	90.09	202.82	504.36
	29.94	**114.96**	**258.89**	**588.64**	**32.65**	**125.03**	**281.26**	**649.90**	**35.43**	**135.27**	**304.27**	**713.30**
0.8	23.20	85.98	193.34	467.40	24.34	89.58	201.51	497.03	25.53	93.31	210.26	527.27
	32.55	**124.70**	**280.49**	**646.30**	**35.49**	**135.59**	**304.94**	**712.36**	**38.51**	**146.69**	**330.13**	**777.14**
1.0	24.20	89.15	200.41	492.67	25.37	92.83	208.96	521.30	26.59	96.64	218.32	547.30
	35.21	**134.57**	**302.60**	**705.55**	**38.39**	**146.31**	**329.18**	**773.05**	**41.64**	**158.27**	**356.68**	**838.26**

geneity m. In Table 7, authors demonstrated the comparison of present study dealt with circular thickness in $2 - D$ and [3] dealt with linear thickness.

From Table 2, authors concluded that:

- Frequency modes in both the study increases but frequencies of present study are less as compared to [3].
- Rate of increment of frequencies of present study are slower as compared to [3].
- Frequency value decreases in both the study, when tapering parameter β_2 is varied from 0.0 to 1.0 but rate of decrement of frequencies in [3] is fast as compared to present study.

6 Conclusions

On the basis of the above results and discussion, the following facts can be concluded:

- Similar behavior of increment of frequencies is observed in Tables 1 and 2 corresponding to both the tapering parameters.
- When authors compared frequency values in Table 4, they found that frequency values in third mode are almost constant but coincide in second and forth modes.
- Numerical value is maximum in Table 3 without thermal effect.
- Rate of increment is much slower as compared to rate of decrement in Tables 1 and 2.

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