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SOLID MECHANICS

A STEADY-STATE HEAT CONDUCTION PROBLEM IN A COMPOSITE SOLID SQUARE TWO-DIMENSIONAL BODY

ISTVÁN ECSEDI, ATTILA BAKSA*

Institute of Applied Mechanics, University of Miskolc,
H-3515 Miskolc, Miskolc-Egyetemváros, Hungary

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ABSTRACT: Studying inhomogeneous structural elements is a very important task. Many studies deal with inhomogeneity of the type whose introduction into an originally homogeneous body does not change its physical fields belonging to the prescribed boundary conditions. This type of inhomogeneity is called neutral inhomogeneity. In the relevant literature, we find several examples of this, mainly in the mathematical theory of elasticity. A prime example is the Saint-Venant's torsion problem of prismatic bars. In the present paper a steady-state heat conduction problem is considered in a two-dimensional square-body. The Cartesian orthotropic solid square body consists of a circular inclusion. The circular inclusion is placed at the center of the square and it has two parts, a core and a coating component, both of which are cylindrically orthotropic. The paper deals with the determination of the thermal conductance of component bodies such that the temperature distribution in originally homogeneous Cartesian orthotropic solid square body does not change. All results of the paper is based on the Fourier's theory of heat conduction in solid body.

KEY WORDS: heat conduction; composite body; anisotropy; cylindrically orthotropic

1 INTRODUCTION

There exists several works on the stead-state heat conductance problems in anisotropic and composite bodies. Some text books such as Carslaw and Jaeger [1], Özisik [2] have devoted the consideration sections of their contains to the heat conduction problem in anisotropic bodies. The exact analytical solutions of anisotropic heat conduction are limited to very simple geometries [2] and for complicated geometries [2] one has to resort to numerical procedures. Kowsary and Arabi [3] studied a two-dimensional anisotropic heat conductance problem in the case of absent of internal

*Corresponding author e-mail: attila.baksa@uni-miskolc.hu

heat generation by Monte Carlo method. For layered composite orthotropic bodies the heat conduction problem is solved by a linear transformation in papers by Poon [4], Poon et al. [5], Ma and Chang [6] or Yan et al. [7]. Mulholland and Gupta [8] studied the heat conduction in a three-dimensional anisotropic body by the use of coordinate transformation of principal axes on conductivity tensor [8]. Clements and Budhi gave solutions for a class steady-state heat conduction problem for anisotropic body by means of boundary element method [9]. Ecsedi and Lengyel presented exact analytical solutions for heat conduction problems in anisotropic two-dimensional solid bodies [10]. A linear coordinate transformation in introduced in [10] to reduce the anisotropic heat conduction problem to an equivalent isotropic one.

Dong, Cui, et. al, developed a novel multi-scale numerical method for the heat conduction problems of composite bodies with diverse periodic configurations in their subdomains [11]. The new contributions of paper Dong, Cui et.al., are the SOTS analyses and the corresponding numerical algorithm. Paper [12] develops a multi-scale thermal/mechanical analysis model which not only can efficiently measure the thermal shock response but also highly reflects the effects of diversiform micro-structures of porous ceramics. The thermal stress intensity factor is derived by finite element/finite difference method and the weight function method in the macro continuum model. Chakib et.al. [13] deal with a multi-scale analysis by using the correctors of a nonlinear heat transmission problem in periodic microstructure which multiple components. The heat flux condition at the interface between the two parts of medium is described by a nonlinear equation [13]. In paper by Gao an Nie, an efficient and accurate Chebyshev expansion method is presented for transient heat conduction problem for nonhomogeneous boundary conditions [14].

For the three-dimensional steady-state heat conduction problem the neutral inhomogeneity with spherical orthotropic inclusions is studied in paper [15], where the volume fraction of the core in all inhomogeneity is the same. The functionally graded material properties of inclusions are also discussed in paper by Ecsedi and Baksa [15]. A steady-state heat conduction problem is considered in a two-dimensional solid body which filled up composite circular inclusions by Ecsedi and Lengyel [16]. The condition of neutral inhomogeneity for constant heat flux vector in the matrix body is derived [16]. The existence of the neutral inhomogeneities in the different boundary-value problems of elasticity is studied by Ru [17], Ru et al. [18], Benveniste and Chen [19] or Ecsedi and Baksa [20].

This paper deals with the existence condition of neutral inhomogeneity in Cartesian orthotropic two-dimensional square body for a given boundary temperature field.

2 GOVERNING EQUATIONS

Figure 1 shows the two-dimensional homogeneous Cartesian orthotropic square shape body and the Cartesian coordinate system Oxy with its unit vectors $\boldsymbol{e}_x$, $\boldsymbol{e}_y$. The thermal conductance of the homogeneous Cartesian orthotropic two-dimensional body are denoted by K_1 and K_2. The temperature of the solid body is indicated by $T_0 = T_0(x, y)$. The following boundary conditions are prescribed for the homogeneous orthotropic body

$$T_0(a, y) = \frac{t_1}{a}y + t_0, \qquad T_0(-a, y) = -\frac{t_1}{a}y + t_0 \qquad -a \leq y \leq a, \tag{1}$$

$$T_0(x, a) = \frac{t_1}{a}x + t_0, \qquad T_0(x, -a) = -\frac{t_1}{a}x + t_0 \qquad -a \leq x \leq a. \tag{2}$$

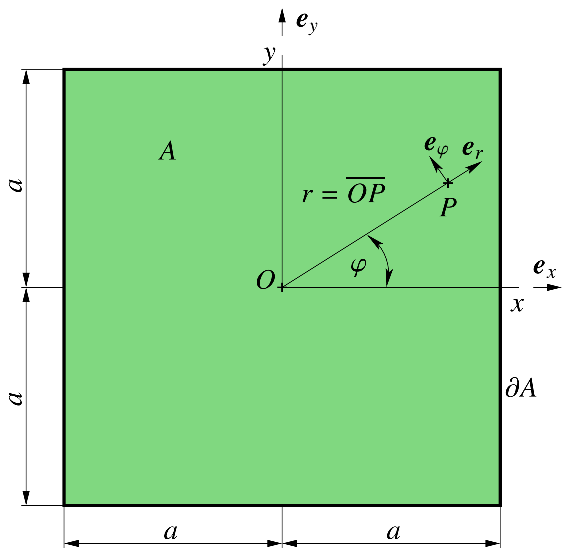

Fig. 1: Two-dimensional square shape domain.

The steady-state temperature field satisfies the Fourier equation

$$K_1 \frac{\partial^2 T_0}{\partial x^2} + K_2 \frac{\partial^2 T_0}{\partial y^2} = 0 \qquad (x, y) \in A. \tag{3}$$

The two-dimensional domain shown in Fig. 1 is denoted by A and its whole boundary curve is ∂A. It is easy to show that in the present problem

$$T_0 = \frac{t_1}{a^2}xy + t_0 \qquad (x, y) \in A \cup \partial A. \tag{4}$$

The heat flux vector $\boldsymbol{q}_0 = \boldsymbol{q}_0(x, y)$ according to the Fourier theory of heat conduction is as follows

$$(5)\qquad \boldsymbol{q}_0(x,y) = -K_1\frac{\partial T_0}{\partial x}\boldsymbol{e}_x - K_2\frac{\partial T_0}{\partial y}\boldsymbol{e}_y = -K_1\frac{y}{a^2}t_1\boldsymbol{e}_x - K_2\frac{x}{a^2}t_1\boldsymbol{e}_y.$$

We introduce a polar coordinate-system $Or\varphi$ as a shown in Fig. 1. The unit vectors of the polar coordinate system are $\boldsymbol{e}_r(\varphi)$ and $\boldsymbol{e}_\varphi(\varphi)$ (Fig. 1).

The temperature field T_0 and heat flux vector $\boldsymbol{q}_0$ can be expressed in the polar coordinate-system $Or\varphi$ as

$$(6)\qquad \mathcal{T}_0(r,\varphi) = \frac{t_1}{2a^2}r^2\sin 2\varphi + t_0 \qquad (r,\varphi)\in A\cup\partial A,$$

$$(7)\quad \boldsymbol{q}_0(r,\varphi) = -(K_1+K_2)\frac{t_1}{2a^2}r\sin 2\varphi\boldsymbol{e}_r + (K_1\frac{t_1}{a^2}r\sin^2\varphi - K_2\frac{t_1}{a^2}r\cos^2\varphi)\boldsymbol{e}_\varphi \qquad (r,\varphi)\in A\cup\partial A.$$

From Eq. (7) the radial component of the heat flux vector $\boldsymbol{q}_0$ can be obtained as

$$(8)\qquad q_{0r} = -K\frac{t_1}{2a^2}r\sin 2\varphi \qquad K = K_1 + K_2 \qquad (r,\varphi)\in A\cup\partial A.$$

Figure 2 shows the two-dimensional square shape domain with circular inclusion. The circular inclusion is placed at the center of domain A and it contains two different material components. The first component is a coating occupying the hollow circular domain A_1 and the second component is the core occupying the circular domain A_2.

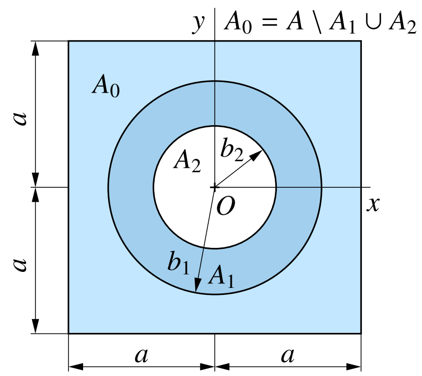

Fig. 2: Two-dimensional square shape domain with circular inclusion.

In the polar coordinate system $Or\varphi$ the two-dimensional domains A_1 and A_2 are defined as

$$A_1 = \left\{ (r,\varphi) \middle| b_2 \leq r \leq b_1,\ \ 0 \leq \varphi \leq 2\pi \right\}, \tag{9}$$

$$A_2 = \left\{ (r,\varphi) \middle| 0 \leq r \leq b_2,\ \ 0 \leq \varphi \leq 2\pi \right\}. \tag{10}$$

In the remaining domain A is denoted by $A_0 = A \setminus A_1 \cup A_2$. The material of the domain A_i $(i = 1, 2)$ is homogeneous cylindrically anisotropic whose thermal conductances are k_{ir} and $k_{i\varphi}$ $(i = 1, 2)$. The material of the domain A_0 is Cartesian orthotropic and its temperature field is given by Eq. (6). It is assumed that there are perfect thermal contact between the different components of the composite two-dimensional body shown in Fig. 2. From this fact it follows that the temperature field and the radial component of heat flux vector are continuous on the whole square shape body. According to Eqs. (6) and (8) we assume that the temperature fields in the core and coating have the following form

$$\begin{aligned} \tau_1(r,\varphi) &= F_1(r)\sin 2\varphi + t_0 \quad (r,\varphi) \in A_1 \\ \tau_2(r,\varphi) &= F_2(r)\sin 2\varphi + t_0 \quad (r,\varphi) \in A_2. \end{aligned} \tag{11}$$

The radial component of the heat flux vectors in the core and coating can be represented as

$$q_{1r}(r,\varphi) = -k_{1r}\frac{\partial \tau_1}{\partial r} = -k_{1r}\frac{\mathrm{d}F_1}{\mathrm{d}r}\sin 2\varphi \quad (r,\varphi) \in A_1, \tag{12}$$

$$q_{2r}(r,\varphi) = -k_{2r}\frac{\partial \tau_2}{\partial r} = -k_{2r}\frac{\mathrm{d}F_2}{\mathrm{d}r}\sin 2\varphi \quad (r,\varphi) \in A_2. \tag{13}$$

The steady-state temperature field in two-dimensional cylindrically orthotropic homogeneous two-dimensional body is described by the following partial differential equation in the cylindrical coordinate system $Or\varphi$ [1,2]

$$k_{ir}\frac{\partial^2 \tau_i}{\partial r^2} + \frac{k_{ir}}{r}\frac{\partial \tau_i}{\partial r} + \frac{k_{i\varphi}}{r^2}\frac{\partial^2 \tau_i}{\partial \varphi^2} = 0 \qquad (r,\varphi) \in A_i \quad (i = 1, 2). \tag{14}$$

Substitution of Eq. (11) into Eq. (14) gives an ordinary differential equation for the function $F_i = F_i(r)$ $(i = 1, 2)$

$$\frac{\mathrm{d}^2 F_i}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}F_i}{\mathrm{d}r} - \frac{k_i^2}{r^2}F_i = 0, \qquad (i = 1, 2) \tag{15}$$

where

$$k_i = 2\sqrt{\frac{k_{ir}}{k_{i\varphi}}} \quad (i = 1, 2). \tag{16}$$

The solution of the second order ordinary differential Eq. (15) can be represented as

(17) $$F_1(r) = C_1 r^{k_1} + C_2 r^{-k_1} \qquad b_2 < r < b_1,$$

(18) $$F_2(r) = C_3 r^{k_2} + C_4 r^{-k_2} \qquad 0 < r < b_2.$$

The function $F_2 = F_2(r)$ is bounded at $r = 0$, from this fact it follows that

(19) $$C_4 = 0.$$

The combination of Eq. (11) with Eqs. (17) and (18) gives

(20) $$\tau_1(r, \varphi) = \left(C_1 r^{k_1} + C_2 r^{-k_1}\right) \sin 2\varphi + t_0 \qquad (r, \varphi) \in A_1,$$

(21) $$\tau_2(r, \varphi) = C_3 r^{k_2} \sin 2\varphi + t_0 \qquad (r, \varphi) \in A_2.$$

Substitution of Eqs. (20) and (21) into Eqs. (12) and (13) gives the expressions of radial component of heat flux vector in the inclusion

(22) $$q_{1r} = -\kappa_1 \left(C_1 r^{k_1 - 1} C_2 r^{-k_1 - 1}\right) \sin 2\varphi \qquad (r, \varphi) \in A_1,$$

(23) $$q_{2r} = -\kappa_2 C_3 r^{k_2 - 1} \sin 2\varphi \qquad (r, \varphi) \in A_2,$$

where

(24) $$\kappa_i = k_{ir} k_i = 2\sqrt{k_{ir} k_{i\varphi}} \qquad (i = 1, 2).$$

3 Formulation of the Condition of Existence of Neutral Inhomogeneity

The thermal contact between the different components of the composite square body is perfect that is the following four equations must be satisfied

(25) $$\tau_1(b_1, \varphi) = \tau_0(b_1, \varphi) \qquad 0 \le \varphi \le 2\pi,$$

(26) $$q_{1r}(b_1, \varphi) = q_{0r}(b_1, \varphi) \qquad 0 \le \varphi \le 2\pi,$$

(27) $$\tau_1(b_2, \varphi) = \tau_2(b_2, \varphi) \qquad 0 \le \varphi \le 2\pi,$$

(28) $$q_{1r}(b_2, \varphi) = q_{2r}(b_2, \varphi) \qquad 0 \le \varphi \le 2\pi.$$

The detailed form of the system of Eqs. (25)-(28) are as follows:

(29) $$C_1 b_1^{k_1} + C_2 b_1^{-k_1} - t_1 \frac{b_1^2}{2a^2} = 0,$$

$$-\kappa_1 C_1 b_1^{k_1} + \kappa_1 C_2 b_1^{-k_1} + K t_1 \frac{b_1^2}{2a_2} = 0, \tag{30}$$

$$C_1 b_2^{k_1} + C_2 b_2^{-k_1} - C_3 b_2^{k_2} = 0, \tag{31}$$

$$-\kappa_1 C_1 b_2^{k_1} + \kappa_1 C_2 b_2^{-k_1} + \kappa_2 C_3 b_2^{k_2} = 0. \tag{32}$$

The solution of the system of Eqs. (29) and (30) for the constant C_1 and C_2 is

$$C_1 = \frac{t_1}{4a^2} \frac{b_1^2(\kappa_1 + K)}{\kappa_1 b_1^{k_1}}, \qquad C_2 = -\frac{t_1}{4a^2} \frac{b_1^2(-\kappa_1 + K)}{\kappa_1 b_1^{-k_1}}. \tag{33}$$

Substitution the expression C_1 and C_2 into Eq. (31) gives the result for C_3

$$C_3 = \frac{t_1}{4a^2} = \frac{(\kappa_1 + K) b_2^{k_1} b_1^{-k_1+2} - (-\kappa_1 + K) b_2^{-k_1} b_1^{k_1+2}}{\kappa_1 b_2^{k_2}}. \tag{34}$$

The combination of Eq. (33) with Eq. (32) leads to an another expression of C_3 which is denoted by $\widetilde{C}_3$

$$\widetilde{C}_3 = \frac{t_1}{4a^2} \frac{(\kappa_1 + K) b_1^{2-k_1} b_2^{k_1} + (-\kappa_1 + K) b_1^{2+k_1} b_2^{-k_1}}{\kappa_2 b_2^{k_2}}. \tag{35}$$

It is very easy to show that for

$$\kappa_1 = \kappa_2 = K \tag{36}$$

the expression of C_3 and $\widetilde{C}_3$ are the same, that is

$$C_3 = \widetilde{C}_3 = \frac{t_1}{2a^2} b_1^{2-k_1} b_2^{k_1-k_2} \tag{37}$$

and in this case for C_1 and C_2 we have

$$C_1 = \frac{t_1}{2a^2} b_1^{2-k_1} \qquad C_2 = 0. \tag{38}$$

The condition of the existence of neutral inclusion (36) can be formulated in terms of thermal conductance as

$$K = 2\sqrt{k_{1r} k_{1\varphi}} = 2\sqrt{k_{2r} k_{2\varphi}}. \tag{39}$$

The temperature field in the inclusion according to Eqs. (37) and (38) can be represented as

$$\tau_1(r,\varphi) = \frac{t_1}{2a^2} b_1^{2-k_1} \sin 2\varphi + t_0 \qquad (r,\varphi) \in A_1, \tag{40}$$

(41) $$\tau_2(r,\varphi) = \frac{t_1}{2a^2} b_1^{2-k_1} b_2^{k_1-k_2} \sin 2\varphi + t_0 \qquad (r,\varphi) \in A_2.$$

The radial components of the heat flux vector are obtained as follows:

(42) $$q_1(r,\varphi) = -\kappa_1 \frac{t_1}{2a_2} b_1^{2-k_1} r^{k_1-1} \sin 2\varphi \qquad b_2 \leq r \leq b_1 \quad 0 \leq \varphi \leq 2\pi,$$

(43) $$q_2(r,\varphi) = -\kappa_2 \frac{t_1}{2a_2} b_1^{2-k_1} b_2^{k_1-k_2} r^{k_2-1} \sin 2\varphi \qquad 0 \leq r \leq b_2 \quad 0 \leq \varphi \leq 2\pi.$$

The temperature field over the whole two-dimensional square shape domain is given by the following formula

(44) $$\tau(r,\varphi) = (H(r) - H(r-b_2))\,\tau_2(r,\varphi) + (H(r-b_2) - H(r-b_1))\,\tau_1(r,\varphi) + H(r-b_1)\tau_0(r,\varphi),$$

where $H = H(r)$ is the Heaviside unit step function

(45) $$H(r) = 0 \quad \text{if} \quad r < 0, \qquad H(r) = 1 \quad \text{if} \quad r > 0.$$

Similar formula gives the expression of the radial component of the heat flux vector in the whole square shape domain

(46) $$q_r = (H(r) - H(r-b_2))\,q_{2r}(r,\varphi) + (H(r-b_2) - H(r-b_1))\,q_{1r}(r,\varphi) + H(r-b_1)q_{0r}(r,\varphi).$$

The boundary curve ∂A of the square shape domain in the polar coordinate system is described by the equation $r = R(\varphi)$ where

(47) $$\begin{aligned} R(\varphi) = {} & (H(\varphi) - H(\varphi-\varphi_1))\,R_1(\varphi) + (H(\varphi-\varphi_1) - H(\varphi-\varphi_2))\,R_2(\varphi) \\ & + (H(\varphi-\varphi_2) - H(\varphi-\varphi_3))\,R_3(\varphi) + (H(\varphi-\varphi_3) - H(\varphi-\varphi_4))\,R_4(\varphi) \\ & + (H(\varphi-\varphi_4) - H(\varphi-\varphi_5))\,R_5(\varphi) + (H(\varphi-\varphi_5) - H(\varphi-\varphi_6))\,R_6(\varphi) \\ & + (H(\varphi-\varphi_6) - H(\varphi-\varphi_7))\,R_7(\varphi) + H(\varphi-\varphi_7 R_8(\varphi) \qquad 0 \leq \varphi \leq 2\pi. \end{aligned}$$

Here

(48) $$R_1 = R_1(\varphi) = \frac{a}{\cos\varphi} \qquad R_2 = R_2(\varphi) = \frac{a}{\sin\varphi},$$

(49) $$R_3 = R_3(\varphi) = \frac{a}{\sin\varphi} \qquad R_4 = R_4(\varphi) = -\frac{a}{\sin\varphi},$$

(50) $$R_5 = R_5(\varphi) = -\frac{a}{\cos\varphi} \qquad R_6 = R_6(\varphi) = \frac{a}{\cos\left(\frac{3\pi}{2} - \varphi\right)},$$

(51) $$R_7 = R_7(\varphi) = \frac{a}{\cos\left(\varphi - \frac{3\pi}{2}\right)} \qquad R_8 = R_8(\varphi) = -\frac{a}{\cos\left(2\pi - \varphi\right)}.$$

4 Numerical Example

The following data are used in the numerical example (see Fig. 2):

$$a = 0.35\,\mathrm{m}, \quad b_1 = 0.25\,\mathrm{m}, \quad b_2 = 0.2\,\mathrm{m}, \quad t_1 = 300\,\mathrm{K}, \quad t_0 = 0, \quad K_1 = 30\,\frac{\mathrm{W}}{\mathrm{mK}},$$
$$K_2 = 50\,\frac{\mathrm{W}}{\mathrm{mK}}, \quad k_{1r} = 25\,\frac{\mathrm{W}}{\mathrm{mK}}, \quad k_{1\varphi} = 64\,\frac{\mathrm{W}}{\mathrm{mK}}, \quad k_{2r} = 50\,\frac{\mathrm{W}}{\mathrm{mK}}, \quad k_{2\varphi} = 32\,\frac{\mathrm{W}}{\mathrm{mK}}.$$

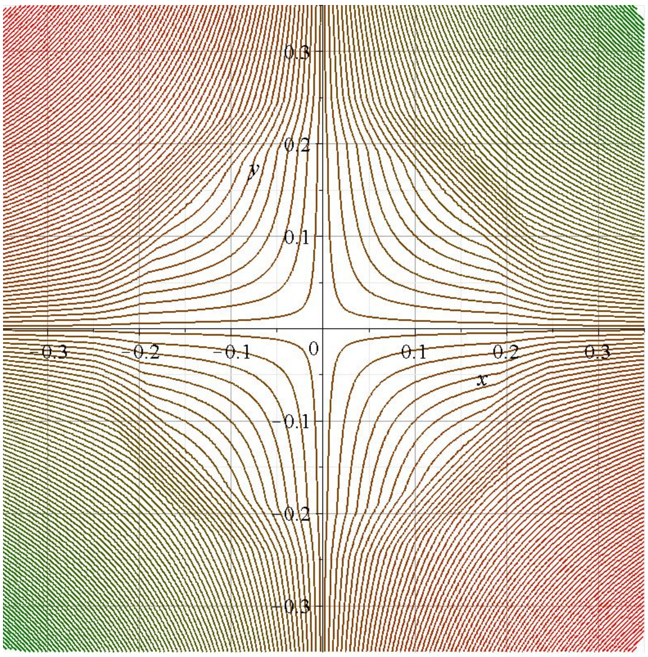

Fig. 3: Contour lines of the temperature function.

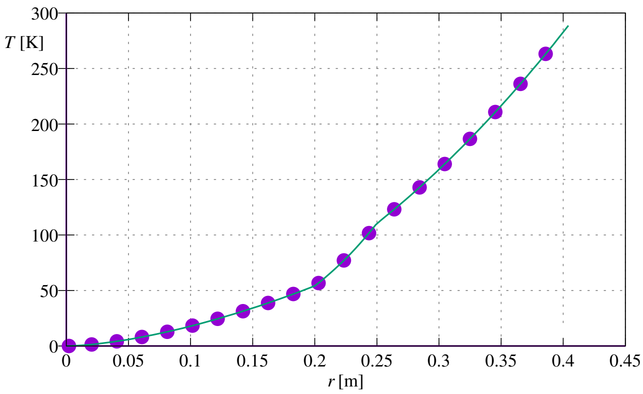

Fig. 4: The plot of function $T(r, \pi/6)$ for $0 \leq r \leq R_0$.

The contour lines of the temperature function $\tau = \tau(r, \varphi)$ for $0 \leq r \leq R(\varphi)$ and $0 \leq \varphi \leq 2\varphi$ are shown in Fig. 3.

The plot of $\tau(r, \pi/6)$ for $0 \leq r \leq R(\pi/6)$ is presented in Fig. 4.

Figure 5 shows the contour lines of the radial component of the heat flux vector $q_r = q_r(r, \varphi)$ for $0 \leq r \leq R(\varphi)$ and $0 \leq \varphi \leq 2\pi$.

The graph of the function of the radial component of the heat flux vector $q_r(r, \pi/6)$ for $0 \leq r \leq R(\pi/6)$ is given in Fig. 6.

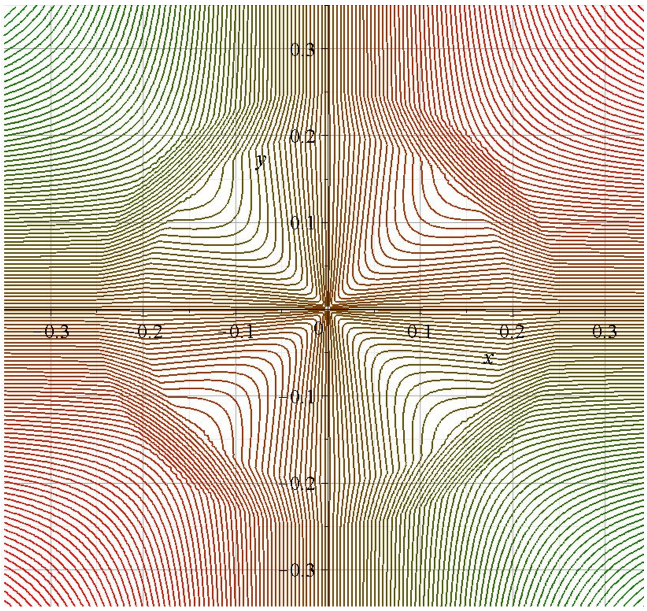

Fig. 5: Contour lines of the radial component of the heat flux vector.

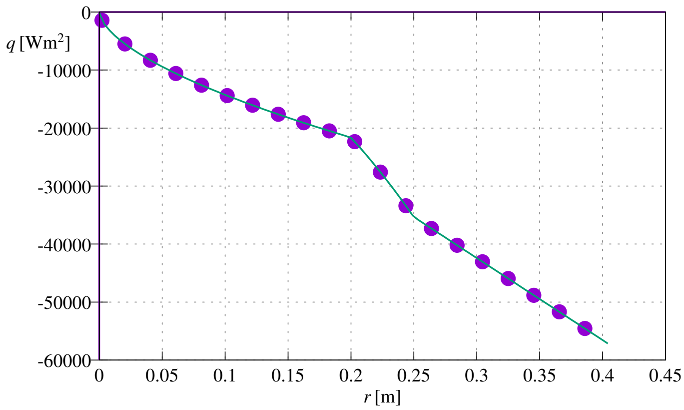

Fig. 6: The plot of function $q_r(r, \pi/6)$ for $0 \leq r \leq R$.

5 Conclusions

A steady-state heat conduction problem is analysed in a two-dimensional composite square shape domain. The Cartesian orthotropic square domain consists of a circular inclusion which has two parts, a core and a coating component. Both components of inclusion are cylindrically orthotropic. Paper gives the condition of existence of neutral inhomogeneity. A numerical example illustrates the application of the presented solution of the steady-state heat conduction problem.

Conflict of interest

The authors declare that they have no conflict of interest.

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