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SOLID MECHANICS

FRACTURE ANALYSIS OF REPAIRING CRACKED AIRCRAFT STRUCTURE

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ABSTRACT: The aim of this work is to investigate the repair performance of aircraft structures modeled by 2024-T3 aluminum plate with inclined central cracks under biaxial complex loading. The mixed-mode stress intensity factor (SIF) is greatly affected by biaxial loading. The goal is to reduce the (SIF) level near the crack after repairing, and determine the best orientation of patch fibers regarding the complex load. The results of the study show that the fiber orientation should conform to the patch sequence for better fixation efficiency.

KEY WORDS: Biaxial Complex Loading, Inclined Crack, Stress intensity factors (SIF), Mixed mode, Patch Repair.

NOMENCLATURE

E:	Young modulus
α:	Crack angle
k:	Ratio factor of loading in the X direction relative to the loading in the Y direction
a:	Crack length
SIF:	Stress intensity factors
Y	$=1.12$: geometric factor for a finished plate containing a central crack “$2a$”
σ:	Applied stress intensities
ν:	Volume fraction
G:	Shear modulus

1 INTRODUCTION

Cracks in metallic structures in industrial sectors such as marine, aerospace, aeronautics, automotive and construction civil are a major concern for maintenance engineers. Those industries include aircraft maintenance, which has long been tied to

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the materials used to make them. The process of repairing these structures has historically corresponded to the development of military and civil aviation solutions. The earliest known work on repairing composite materials was done by Jones et al in 1979 [1] proposed the adhesive repair of composite patch for the cracks observed on the metal structure. The technology is based on bonding a sheet of high-performance composite material to the crack area, allowing loads to be transferred from the damaged structure to the interlocking pavement through the adhesive layer. This transfer can reduce the strength of the confinement around the crack front; this slows down the crack growth rate [2, 3]. Several authors have studied the fracture problem of 2024-T3 aluminum sheets by numerical simulations, considering different crack shapes [4–6]. Patching repairs can delay the propagation of cracks, thereby extending the life of structures repaired in this way [7]. Several studies have investigated the effect of different parameters on crack growth behavior. For example, the influence of interlocking pavement size [8], the number of folds [9], asymmetry of the repair structure [10], understanding of material behavior under different loads, contributes to structural engineering and mechanics, strength, deformation, and failure mechanisms of materials and structures [11, 12], the stress of the component before bonding of the composite patch [13], plasticity [14, 15], imperfect bonding of the composite patch [16], or residual stress [17, 18]. Reviewed in the recent literature, Chu et al. [19] investigated the adhesive repair performance of panel cracks under biaxial loading and highlighted the effect of varying the crack inclination and brittle behavior at fracture. The mixed-mode stress intensity factor was used as an objective function for the analysis performance. Salem et al. examined the effect of stiffness ratio on the growth of repaired fatigue cracks, highlighted the importance of considering the stiffness ratio in determining the performance of composite patch repairs [20, 21]. Furthermore, probabilistic elastic-plastic analysis of repaired cracks with bonded composite patches was conducted by Mechab et al [22], in Steel and Composite Structures, they introduced a probabilistic approach to assess the performance of repaired cracks. Serier et al. proposed a new formulation of the J integral for bonded composite repairs in aircraft structures, emphasizing its significance in fracture mechanics analysis [23]. Many works have been carried out in mechanical and civil engineering to analyze the stress intensity factors in this mode that can be used to predict the appearance of a growth crack [24–27]. Crack propagation does not only occur because of pure mode stresses. Indeed, the rupture is generally due to a mixed mode, which associates at least two modes of rupture. This study involves the numerical analysis of a cracked aeronautical structure subjected to complex loading (biaxial tension) repair of composite patches using the 3D finite element method. An energy-nonlinear fracture mechanics method using stress intensity factors was employed to analyze fracture behavior in patched 2024-T3 aluminum panels. This approach minimizes an objective function and predicts fracture behavior, it evaluates the durability and structural strength of repaired materials, commonly utilized in advanced fracture mechanics studies to assess the reliability of repaired structures. Omidi et al. examined the effectiveness of both single and double composite patches in enhancing the structural integrity and load-carrying capacity of the cracked plates. By applying XFEM, were able to accurately capture the complex crack behavior and simulate the interaction between the composite patches and the cracked aluminum plates. They analyzed various factors such as patch size, material properties, and patch orientation to assess their influence on the fracture resistance and stiffness of the repaired plates. The findings of the study provided valuable insights into the performance of composite patch repairs in aluminum structures. The researchers discussed the benefits and limitations of using single and double patches and proposed guidelines for optimizing the repair technique [28]. This study focuses on the selection of the best orientation of fiber in a mechanical material context. Through an extensive examination of various cases, it has been observed that the most effective is when the fibers are aligned with the direction of loads and cracks, particularly in the vertical orientation. It is important to note that in real-world scenarios, the exact application of loads on an aircraft during flight is not always known. Therefore, for academic purposes, the study has selected the most critical and hazardous loading conditions to simulate and analyze. This approach allows for a comprehensive investigation of the fiber orientation's impact on the material's mechanical properties and fracture behavior, aiding in the development of enhanced structural designs and improved performance in aerospace applications.

2 MECHANICAL MODEL

2.1 THE GEOMETRY OF THE MODEL

The geometric model of the composite patch repair plate shown in Fig. 1 is an important tool for understanding the behavior of the plate when subjected to biaxial tension complex loads. The plate is made of 2024-T3 square aluminum and has an inclined crack of length $2a = 62$ mm. The crack is repaired by boron epoxy composite patches on eight layers of the plate, in different stacking sequences. The patch is glued to the board with FM 73 adhesive, as shown in Fig. 2.

2.2 MECHANICAL PROPERTIES

Aluminum 2024-T3 is a high-strength alloy widely used in aerospace and structural applications. Its stress-strain curve (Fig. 2) exhibits distinct characteristics. Initially, the material undergoes elastic deformation, where stress is directly proportional to strain [3]. Beyond the elastic limit, plastic deformation begins, and the material

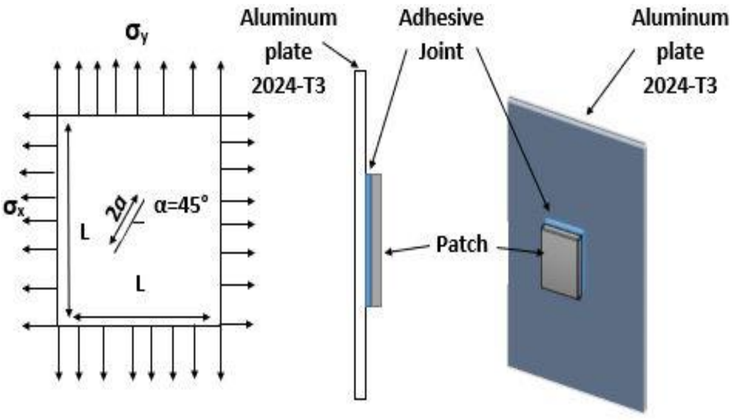

Fig. 1: Geometrical model.

starts to exhibit permanent changes. This region is characterized by strain hardening, where increased stress is required to induce further deformation. Eventually, the curve reaches its ultimate tensile strength, indicating the maximum stress the material can withstand before failure. The final stage is the strain softening phase, where the stress decreases as the material experiences localized necking and eventual fracture. Understanding stress-strain curves is crucial for designing structures and predicting material failure under different loading conditions.

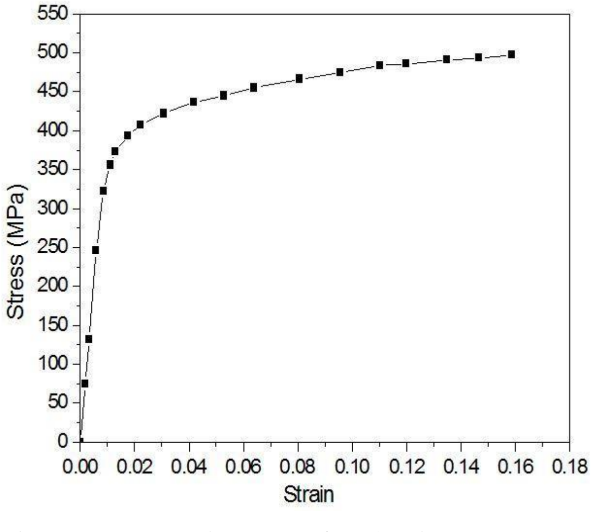

Fig. 2: Stress-strain curves for aluminum 2024-T3.

Table 1: Mechanical properties of the materials used

Properties	Material l (Plate) Aluminum 2024- T3	Material 2 (Composite patch) Boron/epoxy	Material 3 (Adhesive) (FM-73)
Length (mm)	250	50	50
Width (mm)	250	50	50
Thickness (mm)	2	2	0.2
E_1 (MPa)	72000	208000	2550
E_2 (MPa)	—	25000	—
E_3 (MPa)	—	25000	—
ν_{12}	0.33	0.21	0.32
ν_{13}	—	0.21	—
ν_{23}	—	0.21	—
G_{12} (MPa)	—	7200	—
G_{13} (MPa)	—	5500	—
G_{23} (MPa)	—	5500	—

Table 1 provides essential information on the dimensions and mechanical properties of paving slabs and adhesives used for repairs, emphasizing the importance of proper alignment and secure attachment of the patch to the plate, as well as the ability to withstand complex loads. [29]

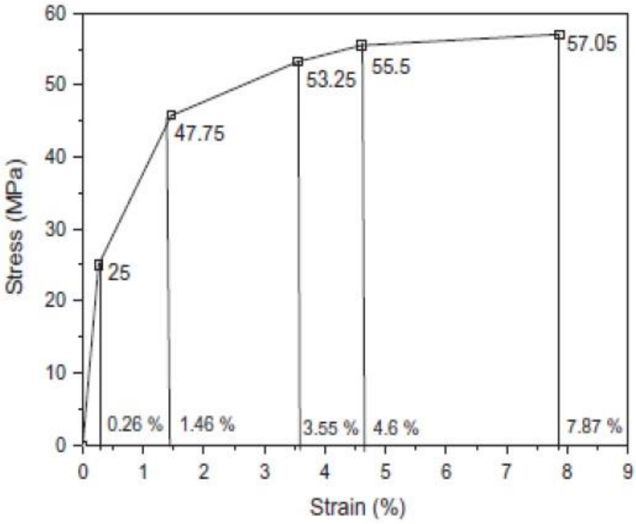

Fig. 3: Multi-linear stress-strain curve of the FM 73 epoxy adhesive.

The patch is glued to the board using FM 73 adhesive, how offers several potential benefits when used to glue a patch to a board. One advantage is its high adhesive strength, ensuring a reliable bond that enhances the structural integrity and longevity of the composite structure. Additionally, FM-73 adhesive is formulated for compatibility with various materials, including the patch material and AL 2024-T3, which helps to prevent issues. The flexibility of FM-73 adhesive allows for slight movement or deformation of the composite structure without compromising the bond. FM-73 adhesive is designed for easy application, with characteristics like suitable viscosity, workable cure time, and convenient dispensing methods, contributing to efficient and effective patch repairs. Figure 3 gives example of multi-linear stress-strain curve of the FM 73 epoxy adhesive [30].

2.3 THE MODEL FORMULATION

In the context of biaxial and mixed-mode loading, cracks exhibit different modes of behavior. Mode I loading occurs when the load is applied perpendicular to the crack planes, according to Rose and al [31], the relationship between the widely applied stress (σ) of the plate and the (SIF) is the following:

$$K_{\mathrm{I}} = Y\sigma\sqrt{\pi a}\,, \tag{1}$$

where both Mode I and Mode II loading coexist, the complex stress field can be determined by combining the contributions of K_{I} and K_{II}. This process of superposition enables the calculation of stress intensity factors in the presence of mixed-mode loading. The stress intensity factors are essential for understanding crack propagation behavior and evaluating the structural integrity of materials. By employing techniques such as the Williams series formula and other analytical [32], or numerical methods, engineers and researchers can analyze the behavior of cracks and predict their growth under different loading scenarios. Understanding stress intensity factors and their relationships is vital for assessing structural integrity and implementing appropriate measures to prevent catastrophic failures.

$$K_{\mathrm{I}} = \frac{\sigma_x\sqrt{\pi a}}{2}\left[(1+k)-(1-k)\cos 2\alpha\right], \tag{2}$$

$$K_{\mathrm{II}} = \frac{\sigma_y\sqrt{\pi a}}{2}\left[(1-k)\sin 2\alpha\right], \tag{3}$$

$$k = \frac{\sigma_x}{\sigma_y}\,. \tag{4}$$

2.4 THE FRACTURE MECHANICS CRITERIA

In the case of aluminum 2024-T3 under a biaxial complex load, the applied stress values are $\sigma_x = 100$ MPa and $\sigma_y = 150$ MPa. The critical value, which represents the maximum allowable stress for the material, once this stress exceeds the critical limit critical of the (SIF), it becomes impractical, because it may have surpassed the threshold for safe reparation, the significance of these parameters and their implications on the structural integrity of the aluminum component. By considering the stress values, their relation to the critical value, and the specific material properties of aluminum 2024-T3, it is possible to analyze and assess the behavior and potential failure modes of the structure under these biaxial complex loading conditions.

3 NUMERICAL SOLUTION BY FEM

The use of C3D8 elements (8-node linear bricks) based on fracture mechanics and finite element modeling of cracked plates is an effective way to model the structure of a repair patch. The mesh used is based on the finite element method and is shown in Fig. 4, which clearly shows the shape of the 2024-T3 patch. The stress intensity factor value (SIF) was extracted using the domain integration method in ABAQUS [33], which provides a high level of accuracy for rough 3D models. A regular grid was used for all structures, with a total of 30683 elements for the repair structure, 9820 for the patch, and 2800 for the adhesive. In order to account for the geometric singularities caused by the inclined crack in the center of the plate, the mesh was refined around the crack and remained unchanged throughout the calculation process to avoid the influence of mesh on the results. The perfect connection between slabs and interlocking paving was created by connecting the nodes of the elements, which ensured that the structure and the composite sheet had the same mesh.

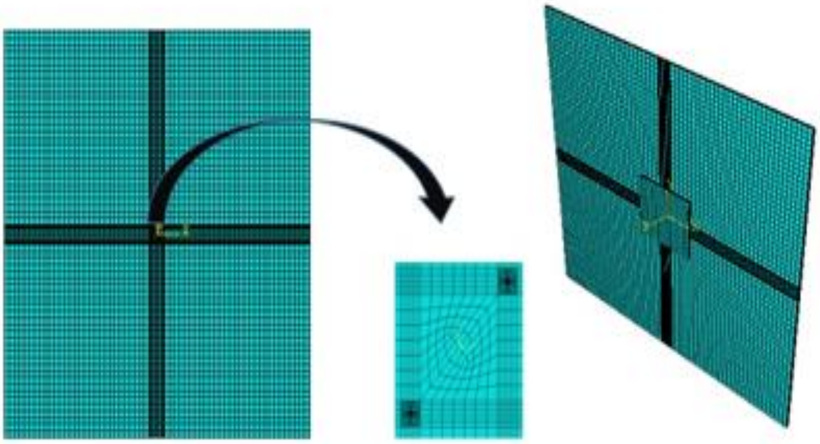

Fig. 4: Meshing of the model.

4 RESULTS AND DISCUSSION

4.1 VALIDATION OF THE NUMERICAL MODEL

Before embarking on the numerical study, it is imperative to validate the model that has been developed in this research. This validation can be achieved by comparing the results obtained from the model with those predicted by the analytical equation (1). Figure 5 depicts a comparative analysis of the variation of the Stress Intensity Factor (SIF) under a constant applied stress of $\sigma_x = 150$ MPa, as reported in reference [29]. To ensure the accuracy of our approach, we conducted an in-depth investigation focusing on the behavior of a central crack under the influence of pure opening mode K_{I}. This choice of crack configuration enables us to develop a precise methodology for analyzing and understanding crack propagation. By validating the model through the analytical equation and conducting a thorough study on the central crack, we can enhance our understanding of the stress and strain distribution at the crack tip. This comprehensive analysis will enable us to make informed decisions and draw meaningful conclusions from the subsequent numerical study.

These results show that the numerical and analytical models give almost identical results. This behavior clearly demonstrates that the model developed and the imposed boundary conditions in this study are reliable and allow for a better analysis of the mechanical behavior of the crack-repaired structure, allowing a reliable analysis of the effect of the repair performance.

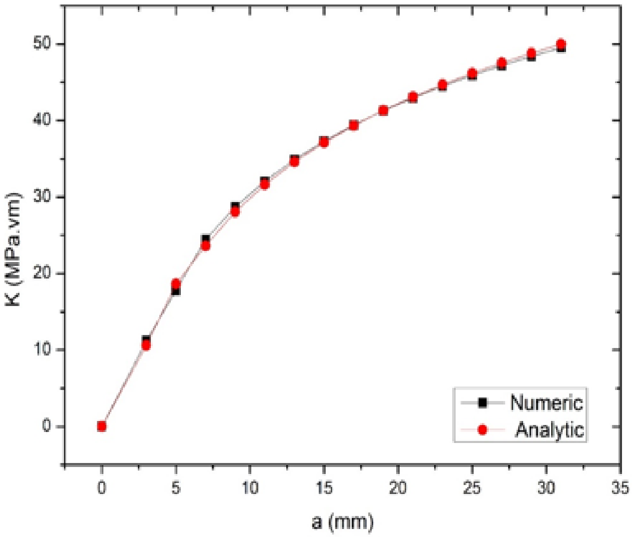

Fig. 5: The analytical-numerical validation of the model.

4.2 NUMERICAL STUDY

After successfully validation of analytical model, we proceeded with a comprehensive numerical study using Abaqus software [33], to investigate the behavior under biaxial loading conditions. The numerical simulations allowed us to simulate and analyze the complex interactions between multiple applied loads and their effects on the structural integrity, crack propagation, and overall performance of the reparation. The numerical study provided a deeper understanding of the system's response to biaxial loading, enabling us to make informed decisions and recommendations for the performance of the structure under real-world operating conditions.

In Fig. 6, the influence of identical biaxial complex loading under $\sigma_x = \sigma_y = 150$ MPa on the mixed mode K_{I} and K_{II} is depicted. The results reveal an intriguing observation: the K_{I} mode experiences a sudden variation, while the K_{II} mode tends to approach zero. This finding implies that the traction following the x-axis does not affect the K_{II} mode. To understand this phenomenon, we need to consider the behavior of the material under the complex loading conditions. When one side of the material is stretched along the x-axis, the other side is simultaneously compressed. As a result, the material experiences a one-way pull, leading to an abrupt opening in the K_{I} mode. On the other hand, the K_{II} mode, which is sensitive to the traction following the y-axis, tends towards zero because the complex loading condition counteracts the applied traction. To gain a deeper understanding of the behavior of K_{I} and

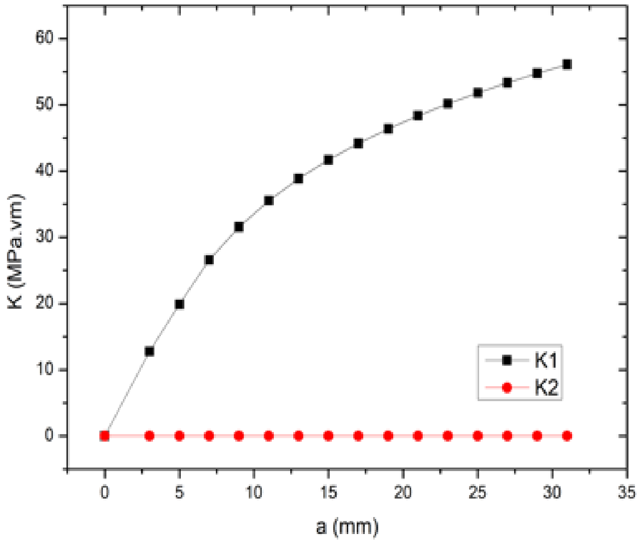

Fig. 6: Variation of SIF under same biaxial complex charges $\sigma_x = \sigma_y = 150$ MPa.

K_{II} modes, it is necessary to apply different tractions. By varying the tractions along different axes, we can explore the effects of complex loading on the mixed mode K_{I} and K_{II}. This comprehensive study will provide insights into how different loading conditions impact the opening behavior in the K_{I} mode and the closing behavior in the K_{II} mode. Additionally, it is crucial to consider the angle made by the notch in the material. The angle of the notch can significantly influence the results. Varying the angle of the notch while applying different tractions will allow us to investigate the combined effects of complex loading and notch angle on the mixed mode K_{I} and K_{II}. This comprehensive analysis will enhance our understanding of how biaxial complex loading affects fracture behavior under different conditions.

The variation of K_{I} mode under multi biaxial complex load refers to the changes in the SIF values associated with the opening behavior of a crack or notch in response to the complex loading conditions. Depending on the specific combination and magnitudes of the applied stresses, the K_{I} mode SIF may increase, decrease, or exhibit other complex behavior.

When the variation of K_{II} mode under multi biaxial complex load refers to the changes in the SIF values associated with the sliding or closing behavior of a crack or notch in response to the complex loading conditions. Depending on the nature and combination of the applied stresses, the K_{II} mode SIF may increase, decrease, or even approach zero.

Figures 7 and 8 present the variation of the mixed mode in a biaxial complex

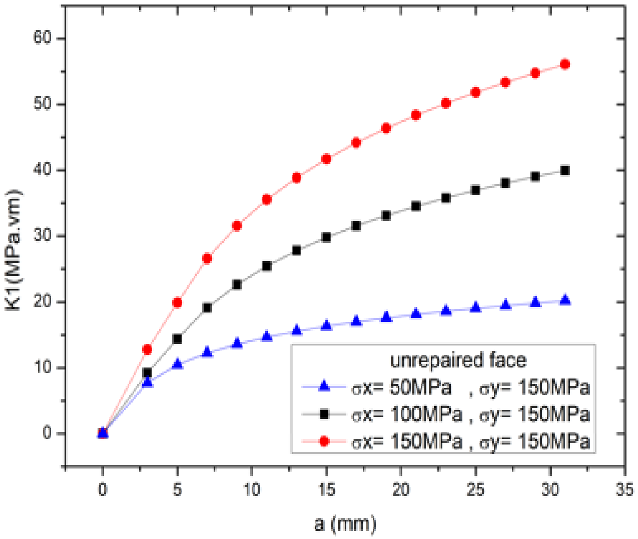

Fig. 7: Variation of SIF (K_{I}) under multi biaxial complex load.

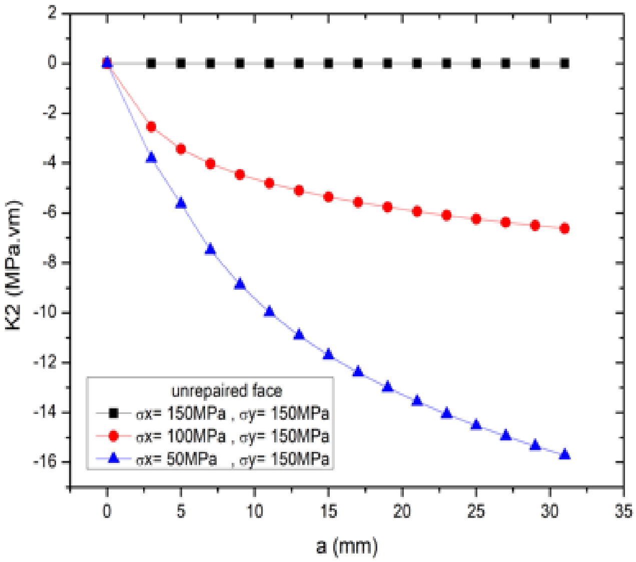

Fig. 8: Variation of SIF (K_{II}) under multi biaxial complex load.

loading most commonly encountered and the most dangerous, the result of the modes of openings (mode I) and sliding (mode II), it is well known that the mixed mode fracture toughness depends on the actual mode of crack propagation and the fracture parameter used. When the crack subjected to complex loading in mixed mode, the crack tip displacement can be broken up into two parts: a horizontal displacement corresponding to the loading in mode I, and a vertical displacement due to slip in the crack tip corresponding to the loading in mode II. Thus, crack propagation occurs in the direction that corresponds to the maximum of the opening component or the maximum of the shear component. For that the case of $\sigma_x = 100$ MPa and $\sigma_y = 150$ MPa is selected.

Table 2 presents the different composite patch stacking sequences for optimal use in crack repair. This optimal use is obtained when one manages to select in a

Table 2: The different cases of patch stacking sequences

Case	Stacking sequences
[A]	[00°/45°/-45°/90°/90°/45°/-45°/00°]
[B]	[90°/45°/-45°/00°/00°/45°/-45°/90°]
[C]	[45°/90°/-45°/00°/00°/-45°/90°/45°]

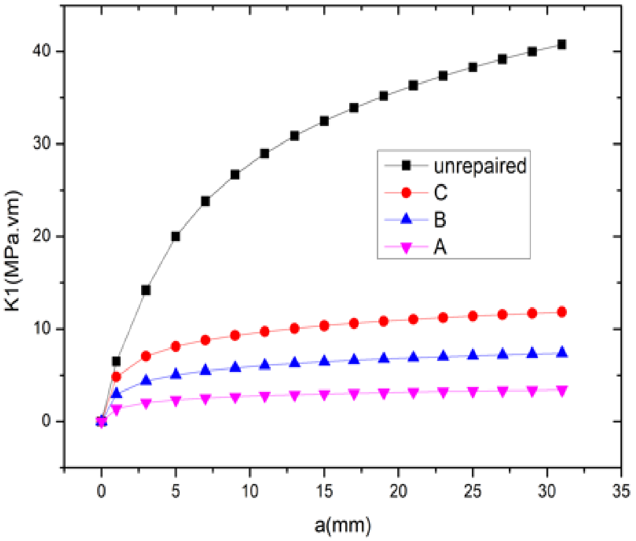

Fig. 9: SIF at the tip of the crack (mode I) under different stacking sequences.

judicious way the type of stacking sequence allowing reducing as much as possible the stress intensity factor in mixed mode. The dimensions as well as the shape of the patch are assumed constant. The results show that the repair by composite patch of sequence [A] leads to a very low fracture energy at the crack tip compared to the sequence [B] and [C]. This reduction is very significant and can exceed 70% when the folds are oriented in sequence [A]. To confirm this observation, we compare the fracture energies at the crack tip in mode I and in mixed mode.

Figures 9 and 10 respectively show the (SIF) of the cracks stressed in mode I and in mode II. This study is carried out for the two failure modes by patches by dimensions bonded by the FM 73 with thickness.

The increase in the size of the crack leads to very significant deformations at its tip that are directly related to the (SIF), the importance of which depends on the stacking sequence. We find that there is a critical crack size equal to a=31 mm beyond which the fracture energy at the crack tip slowly decreases. From this critical value the asymptotic behavior is observed on the variations of the fracture energy. This phenomenon is more marked when the crack is repaired by a composite patch of sequence [A]. The other two sequences behave almost the same as the one at [A], but with higher fracture energy levels. This is probably due to the effect of the longitudinal modulus, the values of which are close. A crack stressed in opening leads to higher energy levels than those obtained for an inclined crack. In this position, the

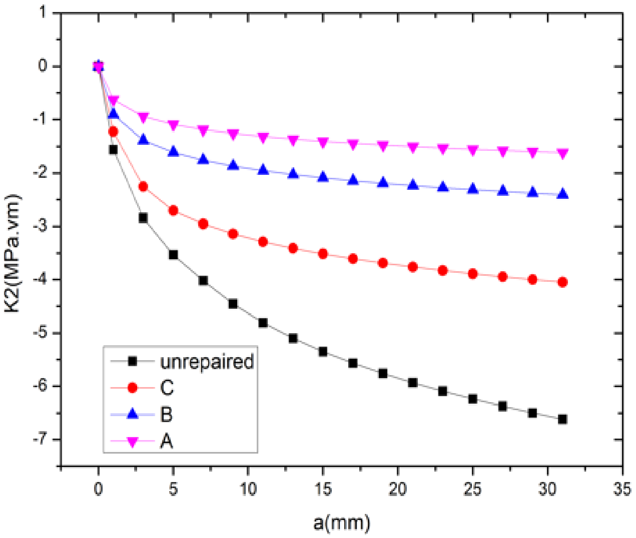

Fig. 10: SIF at the tip of the crack (mode II) under different stacking sequences.

lips of the crack are requested in opening and shearing. Thus, the two failure modes coexist at the crack tip. The stress and strain fields are in turn divided into normal and shear stresses and strains and the crack propagates in mixed mode I+II where mode I dominates mode II, this causes a decrease in stress intensity factor.

The analysis of the previous results shows that patch repair considerably reduces the fracture at the crack tip.

5 CONCLUSIONS

The external environment of an aeronautical structure is often the source of much damage. These structures are subjected in service to mechanical stresses giving rise to stress states that are generally multiaxial and of variable amplitude. This study falls within the context of the repair of aeronautical structures by composite patch in order to slow down the propagation of cracks. A way to repair of metal structures with elastic behavior by composite patch has been identified. This involves minimizing the stress intensity factor (SIF) at the tip of the crack repaired by a bonded patch. Modeling by the finite element method makes it possible to precisely analyze the stress intensity factor. These fields are directly related to the fracture energy at the crack tip. The main results show the (beneficial) effect of different sequences on the failure behavior of a cracked structure, stressed in mixed mode. Composite material patch considerably reduces the mechanical energy highly concentrated at the crack tip by slowing its propagation speed. The performance of the composite patch repair depends not only on the mechanical properties of the adhesive and its thickness but also at the stacking sequence. The rigidity of the patch plays is a very important role in the performance of the repairs. There is an optimal stacking sequence leading to the minimization of fracture energy at the crack tip. The most important composite parameter having a significant influence on the fracture energy behavior at the crack tip is none other than the longitudinal Young's modulus of the composite. The performance of the structure repaired by bonding is improved when the patch has a longitudinal Young's modulus of high value. The difference between Young's modules of the different plies generates a significant variation in the stress intensity factor.

REFERENCES

[1] R. JONES, R.J. CALLINAN (1981) A design study in crack patching. *Fibre Science and Technology* **14**(2) 99-111.

[2] M. SALEM, M. BERRAHOU, B. MECHAB, B.B. BOUIADJRA (2021) Analysis of the adhesive damage for different patch shapes in bonded composite repair of corroded aluminum plate under thermo-mechanical loading. *Journal of Failure Analysis and Prevention* **21**(4) 1274-1282.

[3] M. SALEM, B.B. BOUIADJRA, B. MECHAB, K. KADDOURI (2015) Elastic-plastic analysis of the J integral for repaired cracks in plates. *Advances in Materials Research* **4**(2) 087.

[4] M. ABDELMADJID, Z.M. EL SALLAH, S. ABDERAHMANE, A. KACI DJAFAR, Z. RACHID (2020). Comparative study of the repair of cracked plates with two different composite patches. *Frattura ed Integrità Strutturale* **14**(53) 187-201.

[5] B.B. BOUIADJRA, M.F. BOUANANI, A. ALBEDAH, F. BENYAHIA, M. ES-SAHEB (2011) Comparison between rectangular and trapezoidal bonded composite repairs in aircraft structures: a numerical analysis. *Materials & Design* **32**(6) 3161-3166.

[6] M. RAMJI, R. SRILAKSHMI (2012) Design of composite patch reinforcement applied to mixed-mode cracked panel using finite element analysis. *Journal of Reinforced Plastics and Composites* **31**(9) 585-595.

[7] M. SALEM, B. MECHAB, M. BERRAHOU, B. BACHIR BOUIADJRA, B. SERIER (2019) Failure analyses of propagation of cracks in repaired pipe under internal pressure. *Journal of Failure Analysis and Prevention* **19**(1) 212-218.

[8] K.H. CHUNG, W.H. YANG (2003) A study on the fatigue crack growth behavior of thick aluminum panels repaired with a composite patch. *Composite Structures* **60**(1) 1-7.

[9] H. HOSSEINI-TOUDESHKY, G. SADEGHI, H.R. DAGHYANI (2005) Experimental fatigue crack growth and crack-front shape analysis of asymmetric repaired aluminium panels with glass/epoxy composite patches. *Composite Structures* **71**(3-4) 401-406.

[10] D.-C. SEO, J.-J. LEE (2002):. Fatigue crack growth behavior of cracked aluminum plate repaired with composite patch. *Composite Structures* **57**(1-4) 323-330.

[11] Z.M. El Sallah, B. Smail, S. Abderahmane, B.B. Bouiadjra, S. Boualem (2016) Numerical simulation of the femur fracture under static loading. *Structural Engineering and Mechanics* **60**(3) 405-412.

[12] M. Abdelmadjid, Z.M. El Sallah, S. Abderahmane, A. kaci Djafar, Z. Rachid (2020) Comparative study of the repair of cracked plates with two different composite patches. *Frattura ed Integrità Strutturale* **14**(53) 187-201.

[13] A. Bassetti (2001) Lamelles précontraintes en fibres carbone pour le renforcement de ponts rivetés endommagés par fatique. No. THESIS. EPFL.

[14] P. Colombi (2005) Plasticity induced fatigue crack growth retardation model for steel elements reinforced by composite patch. *Theoretical and Applied Fracture Mechanics* **43** 63-76.

[15] W. Oudad, B.B. Bouiadjra, M. Belhouari, S. Touzain, X. Feaugas (2009) Analysis of the plastic zone size ahead of repaired cracks with bonded composite patch of metallic aircraft structures. *Computational Materials Science* **46**(4) 950-954.

[16] S. Naboulsi, S. Mall (1997) Fatigue crack growth analysis of adhesively repaired panel using perfectly and imperfectly composite patches. *Theoretical and Applied Fracture Mechanics* **28**(1) 13-28.

[17] A. Albedah, B.B. Bouiadjra, L. Aminallah, M. Es-Saheb, F. Benyahia (2011) Numerical analysis of the effect of thermal residual stresses on the performances of bonded composite repairs in aircraft structures. *Composites Part B: Engineering* **42**(3) 511-516.

[18] L. Aminallah, T. Achour, B.B. Bouiadjra, B. Serier, A. Amrouche, X. Feaugas, N. Benssediq (2009). Analysis of the distribution of thermal residual stresses in bonded composite repair of metallic aircraft structures. *Computational Materials Science* **46**(4) 1023-1027.

[19] C.-H. Chue, L.-C. Chang, J.-S. Tsai (1994) Bonded repair of a plate with inclined central crack under biaxial loading. *Composite Structures* **28** 39-45.

[20] M. Salem, R. Mhamdia, B. Mechab, B. Bachir Bouiadjra (2023) Effect of the stiffness ratio on the growth of repaired fatigue cracks with composite patch. *Mechanics Based Design of Structures and Machines* Published online: 28 Mar.

[21] N.C.M. Ibrahim, B. Serier, B. Mechab (2018) Analysis of the crack-crack interaction effect initiated in aeronautical structures and repaired by composite patch. *Frattura ed Integrità Strutturale* **46** 140-149.

[22] B. Mechab, M. Chama, K. Kaddouri, D. Slimani (2016) Probabilistic elastic-plastic analysis of repaired cracks with bonded composite patch. *Steel and Composite Structures* **20** 1173-1182.

[23] N. Serier, B. Mechab, R. Mhamdia, B. Serier (2016) A new formulation of the J integral of bonded composite repair in aircraft structures. *Structural Engineering and Mechanics* **58** 745-755.

[24] V.I. Rizov (2012) Fracture in composites – An Overview (Part I). *Journal of Theoretical and Applied Mechanics, Sofia* **42**(2) 3-42.

[25] V.I. Rizov, A.S. Mladensky (2013) Strain energy release rate determination in the case of mode ii crack in overhanging bilayered composite beam. *Journal of Theoretical and Applied Mechanics, Sofia* **43**(3) 87-96.

[26] A.S. Mladensky, V.I. Rizov (2012) Analysis of mixed mode ii/iii crack in bilayered composite beam. *Journal of Theoretical and Applied Mechanics, Sofia* **42**(4) 41-52.

[27] V.I. Rizov, A.S. Mladensky (2012) Analysis of mode ii crack in bilayered composite beam. *Journal of Theoretical and Applied Mechanics, Sofia* **42**(2) 67-78.

[28] M.J. Omidi, M. Falah, D. Taherifar (2014) 3-D fracture analysis of cracked aluminum plates repaired with single and double composite patches using XFEM. *Structural Engineering & Mechanics* **50**(4) 525-539.

[29] M. Baghdadi, B. Serier, M. Salem, B. Zaoui, K. Kaddouri (2019) Modeling of a cracked and repaired Al 2024T3 aircraft plate: effect of the composite patch shape on the repair performance: Effect of the composite patch shape on the repair performance. *Frattura ed Integrità Strutturale* **13**(50) 68-85.

[30] C.S. Ban, Y.H. Lee, J.H. Choi, J.H. Kweon (2008) Strength prediction of adhesive joints using the modified damage zone theory. *Composite Structures* **86** 96-100.

[31] L.R.F. Rose (1981) An application of the inclusion analogy for bonded reinforcements. *International Journal of Solids and Structures* **17** 827-38.

[32] S. Mirlohi, M. Aliha (2013) Crack growth path prediction for the angled cracked plate using higher order terms of Williams series expansion. *Engineering Solid Mechanics* **1**(3) 77-84.

[33] ABAQUS (2014) Analysis User's Manual, Version 6.14. Dassault Systemes Simulia.